Conference in honor of Etienne Pardoux.
Marseille, February 18-22, 2013.
Abstracts

BALLY Vlad (Univ. Paris Est) : A Hörmander type condition of order one for Wiener functionals.

We consider a Wiener functional $F = (F_1, ..., F_n)$ such that each F_i is four times differentiable in Malliavin sense and $\sigma(W_t, t \leq T)$ measurable. We assume that

$$\inf_{|\xi|=1} \sum_i \left\langle D_T^j F, \xi \right\rangle^2 + \sum_{i,j} \left\langle D_T^j D_T^i F - D_T^i D_T^j F, \xi \right\rangle^2 > 0$$

$D_T^j F$ is not precisely defined but we replace it with an appropriate limit. We prove that under the above non degeneracy condition the law of F is absolutely continuous with respect to the Lebesgue measure. If $F_i \in D^\infty$ then the density is of class C^∞.

This is a joint work with Lucia Caramellino.

BISMUT Jean-Michel (Univ. Paris 11) : Hypoelliptic Laplacian and Langevin process.

If X is a Riemannian manifold, the hypoelliptic Laplacian L_b is an operator acting on the total space $\mathcal{X}$ of the tangent bundle of X, that is supposed to interpolate between the elliptic Laplacian (when $b \to 0$) and the geodesic flow (when $b \to +\infty$). Up to lower order terms, L_b is a weighted sum of the harmonic oscillator along the fibre TX and of the generator of the geodesic flow. The operator L_b is a geometric version of a Fokker-Planck operator. Its probabilistic counterpart is a Langevin process. In this deformation, there are conserved quantities. In some cases, the full spectrum of the elliptic Laplacian is preserved by the deformation. In the talk, I will explain the underlying analytic and probabilistic aspects of the construction.

CARLEN Eric (Rutgers Univ.) : Probabilistic methods in kinetic theory : The Kac walk for large N.

The Kac walk is a N particle walk in a continuous state space that was introduced by Mark Kac to study the non-linear Boltzmann equation in the large N limit by considering marginals of his walk. Much progress has been made in recent years on obtaining information on the Kac walk that is uniform in N, and therefore yields information about the Boltzman equation.

This talk will present recent joint work with Carvalho and Loss, as well as work by a number of other authors.

COMETS Francis (Univ. Paris 7) : An information transmission model : combining Galton-Watson tree and the Coupon Collector.

We model the transmission of a message on the complete graph with n vertices and limited resources. The vertices of the graph represent servers that may broadcast the message at random. Each server has a random emission capital that decreases at each emission. Quantities of interest are the number of servers that receive the information before the capital of all the informed servers is exhausted and the exhaustion time. We establish limit theorems (law of large numbers, central limit theorem and large deviation principle), as $n \to \infty$, for the proportion of visited vertices before exhaustion and for the total duration. The analysis relies on a construction of the transmission procedure as a dynamical selection of successful nodes in a Galton-Watson tree with respect to the success epochs of the coupon collector problem.

Joint work with Francois DELARUE et Rene SCHOTT.

EL KAROUI Nicole (Ecole Polytechnique) : TBA

ETHERIDGE Alison (Univ. of Oxford) : Modelling natural selection.

HAIRER Martin (Univ. of Warwick) : Regularity Structures.

The classical way of measuring the regularity of a function is by comparing it in the neighbourhood of any point with a polynomial of sufficiently high degree. Would it be possible to replace monomials by functions with less regular behaviour or even by distributions ? It turns out that the answer to this question has surprisingly far-reaching consequences for building solution theories for semilinear PDEs with very rough input signals, revisiting the age-old problem of multiplying distributions of negative order, and understanding renormalisation theory. As an application, we build the natural "Langevin equation" associated with Φ^4 Euclidean quantum field theory in dimension 3.

KURTZ Thomas (Univ. of Wisconsin) : Strong and weak solutions for general stochastic models and time-change equations for diffusion processes.

Typically, a stochastic model relates stochastic "inputs" and, perhaps, controls to stochastic "outputs." A general version of the Yamada-Watanabe and Engelbert theorems relating existence and uniqueness of weak and strong solutions of stochastic equations will be given in this context. A notion of "compatibility" between inputs and outputs is critical in relating the general result to its classical forebears. Time-change equations for diffusion processes provide an interesting example. Such equations arise naturally as limits of analogous equations for Markov chains. For one-dimensional diffusions they also are essentially given in the now-famous notebook of Doeblin. Although requiring nothing more than standard Brownian motions and the Riemann integral to define, the question of pathwise uniqueness remains unresolved. To prove weak uniqueness, the notion of compatible solution is employed and the martingale properties of compatible solutions used to reduce the uniqueness question to the corresponding question for a martingale problem or an Ito equation.

LE GALL Jean-François (Université Paris-Sud et Institut universitaire de France) : The harmonic measure of critical Galton-Watson trees.

We consider simple random walk on a critical Galton-Watson tree conditioned to have height greater than n. It is well known that the cardinality of the set of vertices of the tree at generation n is then of order n. We prove the existence of a constant β belonging to $(0, 1)$ such that the hitting distribution of the generation n in the tree by random walk is concentrated with high probability on a set of cardinality approximately equal to n^β. In terms of the analogous continuous model, the dimension of harmonic measure of a level set of the tree is equal to β, whereas the dimension of any level set is equal to 1.

This is a joint work with Nicolas Curien.

LIONS Pierre-Louis (Collège de France) : TBA

LYONS Terry (Univ. of Oxford) : Cubature rough paths and the patched particle filter.

Many important algorithms involve transporting measures forward and it is an empirical fact that methods that approximate the measure by an empirical measure work effectively. In this talk we explain why Monte Carlo works badly in high dimensions (like 2 or 3) and explain other algorithms that out perform it.

This is a joint work with Wonjung Lee.

MATTINGLY Jonathan (Duke Univ. Durham) : TBA

MELEARD Sylvie (Ecole Polytechnique) : Stochastic modeling of Darwinian evolution : a stochastic multi-resources chemostat model.

We consider a stochastic model describing the Darwinian evolution of a polymorphic population with mutation and selection. The interactions between individuals occur by way of competition for resources whose concentrations depend on the current state of the population. Our aim is to model the successive fixations of successful mutants in the population and further its diversification on an evolutionary time scale. We prove, starting from a birth and death model, that, when advantageous mutations are rare and the population size large enough, the population process behaves on the mutation time scale as a jump process moving between successive equilibria. The main idea is a time scale separation : the time scale for the selection process to eliminate disadvantaged types has to be much smaller than the mutation time scale. Essential technical ingredients are the study of a generalized system of ODE's modeling a finite number of biological populations in a competitive interaction due to multi-resources and a fine description of the invasion and fixation of mutants using branching processes

This is a joint work with Pierre-Emmanuel Jabin and Nicolas Champagnat.

NUALART David (Univ. of Kansas) : Density formulas and applications to stochastic partial differential equations.

In this talk we will review explicit formulas for the density of random variables which are measurable with respect to an underlying Gaussian process, using the techniques of Malliavin calculus. As an application we will discuss upper and lower Gaussian bounds for the one-dimensional stochastic heat equation and we establish the Holder continuity of a solution to a nonlinear stochastic partial differential equation, arising from the asymptotic behavior of a particle system in a random environment.

PENG Shige (Shandong Univ.) : BSDE, PDE and Nonlinear Expectations v.s. Kigntian Uncertainty.

In 1921 Frank Knight has been clearly classified two types of uncertainties : the first one is for which the probability is known ; the second one, now called Knightian uncertainty, is for cases where the probability itself is also uncertain. The situation with Knightian uncertainty has become one of main concerns in the domain of data processing, economics, statistics, and specially in measuring and controlling financial risks. A long time challenging problem is how to establish a theoretical framework comparable to the Kolmogorov's one for probability, to treat these more complicated situations with Knightian uncertainties. The objective of the theory of nonlinear expectation rapidly developed in recent years is to solve this problem. This is an important program. Some fundamental results have been established such as law of large numbers, central limit theorem, martingales, G-Brownian motions, G-martingales and the corresponding stochastic calculus of Ito's type, nonlinear Markov processes, as well as the calculation of measures of risk in finance. But still so many deep problems are still to be explored. This new framework of nonlinear expectation is naturally and deeply linked to nonlinear partial differential equations (PDE) of parabolic and elliptic types. These PDEs appear in the law of large numbers, central limit theorem, and nonlinear diffusion processes in the new theory, and inversely, almost all solutions of linear, quasilinear and/or fully nonlinear PDEs can be expressed in term of the nonlinear expectation of a function of the corresponding (nonlinear) diffusion processes. Moreover, a new type of 'path-dependent partial differential equations' have been introduced which provide a PDE tool to study a stochastic process under a nonlinear expectation. Numerical calculations of these path dependent PDE will provide the corresponding backward stochastic calculations.

PIATNITSKI Andrey (Narvik Univ. & Lebedev Inst. : Homogenization of surface and length energies for spin systems.

SOUGANIDIS Panagiotis (Univ. of Chicago) : TBA.

TALAY Denis (INRIA Sophia) : Is Etienne forward or backward ?

I will present two recent results obtained with various co-authors which rely, one the one hand, on the Pardoux and Veretennikov papers on ergodic diffusions and elliptic partial differential equations and, on the other hand, on the theory of backward stochastic differential equations.

VERETENNIKOV Alexander (Univ. of Leeds) : On McKean-Vlasov equations.

New results on existence and uniqueness and on asymptotic behaviour of solutions of stochastic McKean-Vlasov equations will be presented.

WAKOLBINGER Anton (Goethe Univ., Frankfurt) : Continuous branching with an asymmetric competition.

We consider a branching population with a linear ("left to right") order between contemporaneous individuals which is passed on to their offspring, and with pairwise fights which are always won by the individual to the left. In the diffusion limit with weak competition, the total population size follows a Feller branching diffusion with logistic drift, and the genealogy is described by a reflected Brownian motion with a "local time drift". We discuss the process indexed by the ancestral mass, and the corresponding Girsanov transforms. As an interesting by-product, we get insight into the distribution of the stochastic integral $\int_0^1 L_s^e(e_s)de_s$, where e is a normalized Brownian excursion.

This is joint work with Etienne Pardoux.

References :

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– E. Pardoux and A Wakolbinger, From exploration paths to mass excursions - variations on a theme of Ray and Knight, in : Surveys in Stochastic Processes, J. Blath, P. Imkeller, S. Roelly (eds.), EMS 2011, pp 87-106.

ZEITOUNI Ofer (Weizman Inst & Univ. of Minnesota) : Einstein relation for the metropolis algorithm on a disordered tree and a conjecture of Aldous.

T he limiting speed of the right-most particle in a one-dimensional (discrete time, binary) branching random walk with negative-mean increments can be easily expressed in terms of the large deviations of the increments. Sampling efficiently (in a computational sense) particles that escape with positive velocity is however harder. In 1998, Aldous analyzed the Metropolis algorithm for sampling such particles, proved the existence of a temperature above which the asymptotic speed (= efficiency) of the algorithm vanishes, and conjectured, among other things, that for temperatures slightly below the threshold, the asymptotic velocity of the algorithm is positive, as soon as the asymptotic velocity of the BRW is positive. This part of Aldous' conjecture is equivalent to the validity of Einstein's relation for a certain random walk on a disordered tree. In joint work with Pascal Maillard, we establish the conjecture, in two steps. First we show, using homogenization techniques and regeneration estimates, that the asymptotic variance at the critical value

is strictly positive. Then, adapting an approach of Lebowitz-Rost (as in recent work of Gantert, Mathieu and Piatnitski), we establish the Einstein relation.