École d'hiver de probabilités
Semaine 1 : Mécanique statistique de l'équilibre

	Lundi	**Mardi**	**Mercredi**	**Jeudi**	**Vendredi**
09h45-10h45	Velenik	Velenik	Velenik	Velenik	Velenik
11h00-12h00	Presutti	Nachtergaele	Kupiainen	Marchand	Donati Martin
13h30-15h00	TD	TD		TD	14h00 - 15h00
15h15-16h00	Pfister	Ioffe		Procacci	Péché
16h00-16h45	Giuliani	Bissacot		Lacoin	15h00 - 16h00
17h00-17h45	Campanino	Ndreca		Théret	Scoppola

Y. Velenik (5 lectures): An introduction to statistical mechanics (+ 3 complement and exercise sessions).

E. Presutti: Phase transitions in particle systems and microstructures, in particular about particles models with short range interactions plus long range Kac potentials.

C. Pfister: One dimensional models.

A. Giuliani: Universality for non-integrable 2D Ising models: energy critical exponents and central charge.

We investigate a non solvable two-dimensional Ising model with nearest neighbor plus weak finite range interactions. We rigorously establish two properties of the critical theory:
(1) We prove and compute the existence of a scaling limit for the multipoint energy correlations, as the lattice spacing "a" goes to zero and the temperature goes to the critical one, with explicit bounds on the finite-"a" corrections.
(2) For ferromagnetic perturbations, we prove that at the critical temperature the finite size corrections to the free energy are universal, in the sense that they are exactly independent of the interaction. The corresponding central charge, defined in terms of the coefficient of the first subleading term to the free energy, as proposed by Affleck and Blote-Cardy-Nightingale, is constant and equal to 1/2.

These are two of the very few cases where the predictions of conformal field theory can be rigorously verified starting from a microscopic non solvable statistical model.

M. Campanino: Ornstein behavior for truncated correlation functions.

B. Nachtergaele: Gapped quantum phase and quantum phase transitions.

D. Ioffe: Stochastic representations of quantum spin states.

R. Bissacot: The Lovasz Local Lemma, the Lattice gas and the Shearer's measure.

We combine the connection discovered by Scott and Sokal em 2005 between the famous Lovasz Local Lemma used by the combinatorialists and the partition function of the lattice gas, with the criterion to the convergence of Cluster Expansion proved by Fernandez and Procacci in 2007 to obtain an improvement of this classical tool used in the Probabilistic Method. An introduction to the Probabilistic Method will be given as well to theory of lattice gases. The idea is to expose some of the main results of each topic and explain in details the connection between them, some recent applications and open problems.

S. Ndreca: Queues with Exponentially Delayed Arrivals

We study a discrete time queueing system where deterministic arrivals have i.i.d. exponential delays ξ_i. The standard deviation σ of the delay is finite, but its value is much larger than the deterministic unit service time; it turns out that the arrivals are negatively autocorrelated. We find the bivariate generating function for the system, and we solve the resulting boundary value problem in terms of a power series expansion in a parameter related to σ^{-1}. We also prove the analyticity of the generating function with respect to this parameter. The model, motivated by air and railway traffic, has been proposed by Kendall and others many decades ago, but no solution of it has been found so far.

A. Kupiainen: Logarithmically Correlated Random Energy Models

R. Marchand: Boolean percolation in high dimension.

In boolean percolation, random balls with iid random radii with common law ν are thrown with intensity λ in the space R^d, and we focus on the critical intensity $\lambda_c(\nu)$ required to percolate with these balls. It has been conjectured that the best way to percolate, i.e. to minimize the critical intensity, should be to use balls with constant radius. We prove that it is not the case, at least in high dimension: any non-degenerate distribution of radii, when correctly renormalized with respect to the dimension, is more efficient than constant radii as soon as the dimension is large enough.

A. Procacci: Percolation on infinite graphs and isoperimetric inequalities

We consider the Bernoulli bond percolation process (with parameter p) on infinite graphs and we give a general criterion for bounded degree graphs to exhibit a non-trivial percolation threshold based either on a single isoperimetric inequality if the graph has a bi-infinite geodesic, or two isoperimetric inequalities if the graph has not a bi-infinite geodesic. This new criterion extends previous criteria and brings together a large class of amenable graphs (such as regular lattices) and non-amenable graphs (such trees). We also study the finite connectivity in graphs satisfying the new general criterion and show that graphs in this class with a bi-infinite geodesic always have finite connectivity functions with exponential decay as p is sufficiently close to one. On the other hand, we show that there are graphs in the same class with no bi-infinite geodesic for which the finite connectivity decays sub-exponentially (down to polynomially) in the highly supercritical phase even for p arbitrarily close to one.

H. Lacoin: Counting self-avoiding paths on an infinite supercritical percolation cluster.

The self-avoiding walk on $\mathbb{Z}^d$ has been introduced by Flory and Ott as a natural model for polymers. In spite of the apparent simplicity of the model, mathematicians understanding is very far of being complete, in particular in low dimension (d=2,3,4). For this reason the disordered version of the model: Self-avoiding walk in a random potential has not received much attention from the mathematical community. On the other hand the model has received some attention in the Physics literature and some conjectures have been formulated. Our aim is to approach the problem by studying the asymptotic of the partition function. A particular case of interest is the one where the environment is given by supercritical Bernouilli percolation. We obtained so far two results: that when $d = 2$ the model is is never self-averaging even for small dilution in the sense that the number of open path is typically exponentially smaller than its average, and that the same phenomenon occur just above the percolation threshold in high dimension.

M. Théret: Maximal flow and minimal cut set in the first passage percolation.

C. Donati Martin: TBA.

S. Péché: TBA.

E. Scoppola: Sampling from a Gibbs Measure with Pair Interaction by Means of PCA.

We consider the problem of approximate sampling from the finite volume Gibbs measure with a general pair interaction. We exhibit a parallel dynamics (Probabilistic Cellular Automaton) which efficiently implements the sampling. In this dynamics the product measure that gives the new configuration in each site contains a term that tends to favour the original value of each spin. This is the main ingredient that allows one to prove that the stationary distribution of the PCA is close in total variation to the Gibbs measure. The presence of the parameter that drives the "inertial" term mentioned above gives the possibility to control the degree of parallelism of the numerical implementation of the dynamics.