Microlocal analysis and spectral theory

Titles and Abstracts

J.-M. Bony: Deux ou trois choses que je sais de Johannes

L. Boutet de Monvel: Asymptotic equivariant Toeplitz index
In this lecture I wish to describe the asymptotic equivariant index of Toeplitz operators, and how it behaves in case of a torus or $SU(2)$ action.

N. Burq: Microlocal analysis of the Dirichlet-Neumann operator in rough domains, application to the study of the water-waves system
We present some recent results where we give a microlocal description of the Dirichlet-Neumann operator in domains. These results are non trivial as soon as the domain is smoother than Lipshitz. We also prove a pseudo-local property of this operator. This work allows us to obtain some new results on the analysis of the water-waves system. This is a joint work with T. Alazard and C. Zuily.

M. Dimassi: Upper bound for the counting function of interior transmission eigenvalues
In this lecture, we give an estimate of the number of eigenvalues, counting with multiplicities, with modulus less than t^2 when t is large for the interior transmission eigenvalue problem. Joint work with V. Petkov.

F. Faure: Spectral bands and semiclassical zeta function of contact Anosov flows
Contact Anosov flows are models of "fully chaotic dynamics". A well known example is the geodesic flow on a Riemannian manifold N with (possibly non constant) negative curvature. If X is a contact Anosov vector field on a compact manifold M and $V \in C^\infty(M)$, it is known that the differential operator $A = -X + V$ has some discrete spectrum in specific Sobolev spaces called Ruelle-Pollicott resonances. These eigenvalues control the asymptotic expansion of dynamical correlation functions. We show that the eigenvalues of A are restricted to vertical bands and in the gaps between the bands, the resolvent of A is bounded uniformly with respect to $|\operatorname{Im}(z)|$. With the special choice $V(x) = 1/2\,\operatorname{div}(X/\operatorname{unstable}(x))$, the external band concentrates on the imaginary axis as and these eigenvalues coincide with the zeroes of the semiclassical zeta function (also called the Gutzwiller Voros zeta function). In the case of constant curvature, this zeta function is the Selberg zeta function shifted by 1/2. We interpret these results saying that quantum dynamics emerges from classical correlation functions. Joint work with J. Sjöstrand and M. Tsujii.

C. Gérard: Construction d'états de Hadamard par le calcul pseudo-différentiel
Nous donnons une nouvelle construction, basée sur le calcul pseudo-différentiel d'états de Hadamard quasi-libres pour des champs de Klein-Gordon sur une classe d'espaces-temps dont la métrique se comporte bien à l'infini spatial. Nous construisons en particuliers tous les états de Hadamard purs à covariance pseudodifférentielle, et étudions leurs changements par des transformations symplectiques. Nous donnons aussi une nouvelle construction d'états de Hadamard sur des espaces-temps globalement hyperboliques arbitraires. Travail en collaboration avec Michal Wrochna.

B. Helffer: From hypoellipticity for operators with double characteristics to semi-classical analysis of magnetic Schrödinger operators

In 1972, J. Sjöstrand was sitting in the same office as me and completing his paper: Parametrices for pseudodifferential operators with multiple characteristics. Important tools were Microlocal Analysis and also the introduction of a Grushin's problem (already present in his PHD thesis). During 40 years this technique has been used successfully in many situations. This applies in particular in the analysis of magnetic wells where some of the questions could appear as a rephrasing of questions in hypoellipticity. We would like to present some of these problems and their solutions and then discuss a few open or solved problems in the subject, including non self-adjoint problems.

F. Hérau: Tunnel effect for semiclassical random walk

We study the semigroup associated to a system of particules walking randomly at a scale of order h with respect to a probability measure with several attractive wells. Using a semi-classical approach, we give a precise description of the low lying spectrum. In particular, we exhibit some supersymmetric structure for the operator, and give a precise description of the tunneling effect between the wells, corresponding to metastable states of the system and exponentially close to 1 eigenvalues. This is a joint work with L. Michel (Nice) and J.-F. Bony (Bordeaux).

M. Hitrik: Spectral clusters and Weyl laws for non-selfadjoint operators in two dimensions

We study the distribution of eigenvalues for non-selfadjoint perturbations of selfadjoint semi-classical operators in dimension two, assuming that the bicharacteristic flow of the unperturbed part is either periodic or completely integrable. In the periodic case, when the strength of the perturbation is not too large, the spectrum displays a cluster structure and we obtain a complete asymptotic description of individual eigenvalues inside suitable subclusters. In both the completely integrable and periodic cases, under analyticity assumptions, we obtain a Weyl law for the distribution of the imaginary parts of eigenvalues. This talk is based on joint works with Michael Hall and Johannes Sjöstrand.

C.-Y. Hsiao: Szego kernel asymptotics for high power of CR line bundles

Let X be a CR manifold of dimension $2n-1$, $n \geq 2$, and let L^k be the k-th tensor power of a positive CR line bundle over X. Let $\Box_{b,k}$ be the Kohn Laplacian for functions with values in L^k and let Π_k be the orthogonal projection onto $\mathrm{Ker}\,\Box_{b,k}$. Assume that the Levi form of X has at least one negative and one positive eigenvalues. Then the semi-classical characteristic manifold Σ of $\Box_{b,k}$ is always degenerate at some point of the cotangent bundle of X. In this work, we establish microlocal asymptotic expansions for Π_k in the non-degenerate part of Σ under certain assumptions. As an application, we obtain Kodaira embedding Theorems for generalized torus CR manifolds.

T. Kawai: Linear differential operators of infinite order - another interaction of microlocal analysis and exact WKB analysis

Microlocal analysis and exact WKB analysis are cognate and mutually complementary subjects. When they interact, the outcome is substantially important. I report here another new example of such interactions. In a paper (Funkcial. Ekvac 53 (2010)) dedicated by T. Koike,

Y. Takei and me to Professor J. Sjöstrand on his sixtieth birthday, finding the connection formula for WKB solutions of

$$\Big(\frac{d^2}{dx^2} - \eta^2\Big(\frac{1}{x} + \frac{\beta}{x^2}\eta^{-1}\Big)\Big)\psi = 0,$$

β constant, was mentioned as an open problem.

We have recently succeeded in giving a concrete description of the formula with the help of a differential operator of infinite order, an important tool in microlocal analysis. Furthermore, the technique has turned out to be effective in dealing with the alien derivative of WKB solutions at their fixed singular point for the "boosted" Whittaker equation and the "boosted" Legendre equation.

An important step of our reasoning is to compute the alien derivative of the exponentiated Voros coefficients for such equations with the help of infinite order differential operators. Concrete manipulation of resurgent functions with essential singularities seems to be interesting. Joint work with S. Kamimoto and T. Koike.

F. Klopp: Interacting electrons in a random environment: a simple one-dimensional model

In this talk, we will present a simple model of one dimensional interacting electrons in a disordered environment and describe its thermodynamic limit. I.e. we consider N interacting electrons located in a random background and restricted to a box Λ of volume $|\Lambda|$. We study the limit of the ground state and of the ground state energy (per particle) of this quantum system when N and $|\Lambda|$ go to infinity in such a way that $N/|\Lambda|$ converges to a fixed positive density, say, ρ. The density of particles ρ is our main parameter to control the thermodynamic limit; it will be assumed to be small. The results were obtained in collaboration with N. Veniaminov.

G. Lebeau: Strichartz estimates inside a convex domain

We shall explain how the construction of a precise parametrix for waves inside a strictly convex domain with Dirichlet boundary condition leads to Strichartz estimates with a loss of 1/6 derivatives with respect to the flat case. This is a joint work with 0. Ivanovici, R. Lascar and F. Planchon.

A. Melin: Asymptotic expansions for Fourier transforms over spherical caps

The function $|x|^{-m}J_m(|x|)$, where $m = (n-2)/2$ and J_m is the Bessel function of order m, is proportional to the integral

$$\psi_n(x) = \int_{S^{n-1}} e^{i\langle x,\omega\rangle}\, d\omega$$

when $n \geq 2$ and $x \in \mathbf{R}^n$. This function solves Helmholtz equation $(\Delta + 1)u = 0$, and its asymptotic behaviour as x tends to infinity results from classical expansions for the Bessel functions or, more directly, from application of the stationary phase method. Other solutions to Helmholtz equation are obtained when S^{n-1} is replaced by any measurable subset M. When M is a spherical cap this leads to the study of the function

$$\psi_n(x,t) = \int_{S_t^{n-1}} e^{i\langle x,\omega\rangle}\, d\omega,$$

where $S_t^{n-1} = \{\omega \in S^{n-1};\, \omega_n > t\}$ and $t \in [-1,1]$. When x tends to infinity the major contributions to the integral is due to the stationary point of the function $\omega \mapsto \langle x,\omega\rangle$ in S_t^{n-1}

and the stationary point of its restriction to the boundary of that set. The analysis of the asymptotic behaviour as x tends to infinity of $\psi_n(x,t)$ is quite easy when these stationary points are well separated. In my talk I shall try to derive an asymptotic expansion that is as uniform as possible when the stationary points come together. The functions used in this approach may be viewed as Bessel type functions in two variables and turn up already in the case $n = 2$.

The motivation for studying these functions come from the Faddeev approach to scattering theory. In that approach solutions to the Lippmann-Schwinger equation, that are perturbations of nonsymmetric solutions to Helmholtz equation, are used in order to factorize the scattering matrix by upper and lower triangular matrices.

S. Nakamura: Applications of phase space analysis to scattering theory for discrete Schrödinger operators
We discuss applications of microlocal ideas to the scattering theory of discrete Schrödinger operators. In particular, we consider the existence of modified wave operators for discrete Schrödinger operators with long-range perturbations, and microlocal properties of scattering matrix for Schrödinger operators with short-range perturbations.

F. Nier: Boundary value problems for the geometric Kramers-Fokker-Planck equation
I will present in this talk a general theory of boundary value problems for the geometric Kramers-Fokker-Planck equation. After a brief presentation I will review some difficulties and provide a general result concerned with functional analysis and subelliptic estimates. The strategy will be explained and details will be given about the one dimensional problem. Applications include the semigroup approach of Langevin stochastic processes, kinetic theory and the hypoelliptic Laplacian introduced by J.M. Bismut.

G. Perelman: Blow up dynamics near the ground state for the energy critical NLS
In this talk I will review some recent results related to the construction of type II blow up solutions for the energy critical NLS.

G. Popov: Isospectral deformations, Mather's β-function and spectral rigidity
Consider a family of Laplace-Beltrami operators corresponding to a smooth deformation of Riemannian metrics on a compact manifold with or without boundary. Suppose that the initial metric is either completely integrable or close to a non-degenerate completely integrable metric (K.A.M. system). If the deformation is isospectral we prove that the values of Mather's β-function at Diophantine vectors of rotation corresponding to K.A.M. tori is constant along the deformation. As an application we obtain infinitesimal rigidity of Liouville billiard tables. The proof is based on a construction of quasi-modes associated with K.A.M. tori.

D. Robert: Spectral estimates and random weighted Sobolev inequalities for Schrödinger operators
In this talk we shall consider Schrödinger operators: $H = -\Delta + V$ in $L^2(\mathbb{R}^d)$, $d \geq 1$, for confining smooth potentials satisfying $V(x) \approx |x|^{2k}$, $k > 0$, at infinity. From estimates on the spectral function of H, L^r-Sobolev-type estimates are obtained for the L^2-scale Sobolev spaces associated to H. Using randomization methods, inspired from recent works by Burq-Tzvetkov and Burq-Lebeau, these estimates can be almost surely improved. We discuss these

different approaches of randomization. The probabilistic estimates obtained here are applied to discuss local and global well-posedeness for super-critical non-linear Schrödinger equations, with and without potentials, for random initial data. This is a joint work, in progress, with A. Poiret and L. Thomann.

G. Uhlmann: Seeing through space time
We consider inverse problems for the Einstein equation with a time-depending metric on a 4-dimensional globally hyperbolic Lorentzian manifold. We formulate the concept of active measurements for relativistic models. We do this by coupling Einstein equations with equations for scalar fields.

The inverse problem we study is the question, do the observations of the solutions of the coupled system in an open subset U of the space-time with the sources supported in U determine the properties of the metric in a larger domain. To study this problem we define the concept of light observation sets and show that these sets determine the conformal class of the metric. This corresponds to passive observations from a distant area of space which is filled by light sources. This is joint work with Y. Kurylev and M. Lassas

S. Vũ Ngọc: Eigenvalue clusters for magnetic Laplacians in 2D
In the late 1970's, there has been many works on the cluster structure of the spectrum of Schrödinger operators when the principal symbol has a periodic flow. These operators with "periodic bicharacteristics" were studied using a microlocal refinement of the averaging method. In the case of a magnetic Laplacian, even if the principal symbol does not have a periodic flow, the existence of the well-known "cyclotron" motion makes it possible to employ a similar method, and yields very precise asymptotics for the quantum eigenvalues. This is joint work with Nicolas Raymond.

W. M. Wang: Nonlinear Fourier series and applications to PDE
We describe a new method to analyze space-time Fourier series. This method is motivated by solving nonlinear PDE's when conservation laws are not useful. We shall discuss its applications to the nonlinear Schrödinger equations and possibly also some other equations.

X. P. Wang: The Kramers-Fokker-Planck equation with a short-range potential
This talk is concerned with the Kramers-Fokker-Planck equation with a potential whose gradient tends to zero at the infinity. We prove a pseudospectral estimate at high energies. For short-range potentials in dimension three, we show that the limiting absorption principles hold at low-energies. As consequence, we obtain optimal time-decay estimates of the solutions.

M. Zworski: Numerical and experimental advances in chaotic scattering
Fractal Weyl laws for quantum resonance of classically chaotic systems were first suggested in a 1990 paper by Sjöstrand, where the first fractal upper bounds were proved. They have been studied numerically in many different settings, and recently first experimental results were produced by Kuhl et al. I will survey some of these new results, as well as numerical and experimental results on the resonance gap in chaotic scattering.