Projective subdynamics of multidimensional subshifts
N. Aubrun

These talks will be devoted to the study of multidimensional susbhifts through their subdynamics, and I will present recent results in this area. We will first study subdynamics of sofic subshifts and prove that they are exactly effectively closed subshifts (Hochman, then Aubrun and Sablik and Durand, Romaschenko and Shen). Subdynamics of subshifts of finite type are much more complicated, and there is no complete characterization known in the general case. Nevertheless it is possible to exhibit some classes that can be obtained that way (Pavlov and Schraudner and Guillon). Opting for a different approach, we will finally see that if we impose some irreducibility condition on the SFT, then its subdynamics are completely known (Schraudner).

On the sets of real vectors recognized by finite automata in multiple bases

B. Boigelot

This course studies the properties of finite automata recognizing real numbers, or more generally real vectors, encoded positionally in an integer base. Such automata are used, in particular, as symbolic data structures for representing the sets that are definable in the first-order additive theory of mixed integer and real variables. They also lead to a simple decision procedure for this arithmetic.
After introducing automata over real numbers and vectors, we study their expressive power and establish that a restricted form of infinite-word automaton is sufficient for recognizing the sets that are definable in mixed integer and real additive arithmetic. We then address the problem of characterizing the sets that are simultaneously recognizable by finite automata in multiple bases, which leads to generalizations of Cobham's and Semenov's theorems to automata operating over the real domain.
In addition, the techniques used for obtaining our main result lead to valuable insight into the internal structure of automata recognizing sets of vectors definable in mixed integer and real additive arithmetic. This structure can be exploited in order to improve the efficiency of data structures and decision procedures for this arithmetic.

Matchbox Manifolds
A. Clark

We shall give an introduction to and overview of matchbox manifolds, which are compact connected spaces that locally have the structure of the product of a disk and a totally disconnected space. These spaces arise naturally in the study of locally finite tilings. We shall explain our results with Hurder characterising the matchbox manifolds with equicontinuous dynamics and shall relate equicontinuity to the topological structure of the underlying matchbox manifold. We shall explain recent work with Hurder and Lukina that relates the dynamical and topological structures of matchbox manifolds in the general case. We shall also show how shape theory can be effectively used in the study of matchbox manifolds, highlighting recent work with Hunton on Euclidean tiling spaces and codimension one attractors. We will finish by indicating how one can combine all these results to establish a programme for the spectral analysis of general matchbox manifolds.

A short introduction to the atomic structures of real quasicrystals

D. Gratias

One of the very early and basic questions posed to crystallographers after the discovery of quasicrystals by D. Shechtman was the determination of their atomic structures. We propose here a short chronological survey of the various progresses that have been made in that field from the first model of Ilan Blech to the present sophisticated models of the binary icosahedral phases recently discovered by the group of A-P Tsai.
We will see that the major milestones in that field have successively been the indexing of the diffraction patterns followed by the discovering of stable quasicrystalline phases that opened the studies of single grain diffraction using X-rays and neutrons. This lead to the unexpected result that the atomic surfaces used to describe the real structures were located only at a few simple special positions in the configuration space of the cut method. This lead to surprisingly simple descriptions in that space.
To exemplify these aspects, we will discuss the main common features of the real structures of the P and F-type icosahedral phases and show how a natural hierarchy of high symmetry atom clusters emerges that form the elementary blocks used to build the atomic skeleton of these solids.

Expansion Factors and Rigidity for (Abstract) Self-Affine Tiling Spaces

J. Kwapisz

I will outline a new proof of Thurston-Kenyon-Solomyak theorem about expansion factors of self-affine tilings being Perron integral linear transformations. (This resolves the stubborn non-diagonalizable case.)
I will also discuss topological rigidity of tiling spaces and discreteness of the group of topological symmetries of a tiling.
These results make use of finite-local complexity and its well known consequence: the lattice property (asserting that translation vectors between tiles of the same type generate a group of finite rank). I will show that this property is purely topological and, in a nod to Bourbaki, self-affine tiling spaces can be equated with *phase stable* expansive self-affine $\mathbb{R}^d$-actions, erasing the need for postulating a geometric structure (like that of a tile, Delone set, or G-solenoid). A step in this program depends on Hausdorff dimension estimates for expansive actions established in collaboration with Hannah Sobek.

On Pisot conjecture, Meyer sets, Delone sets without finite local complexity

J.-Y. Lee

Pisot substitution conjecture, so called Pisot conjecture, has been an interesting problem in tiling dynamics. There are various notions of coincidences which are equivalent to the Pisot conjecture. The equivalent relations between some of them are known, but not for all of them. We will discuss what is known and what is not known about their relations.

Meyer property is one of important properties in the tiling setting which helps us to understand the various tiling structures. Especially on substitution tilings with the Meyer property, we know a lot more about the structures. We will talk about some of known results on Meyer sets.

A lot of study on tiling dynamics has been done on tilings under the assumption of finite local complexity (FLC). Now we would like to look at the tilings without FLC. There are various questions which arise in this case. Under what conditions the tiling dynamics has unique ergodicity? Under what conditions the tiling dynamics has discrete spectrum? and so on. We discuss these questions and give some answers to them.

Physical properties of locally specified nonperiodic structures bases
J. Socolar

Given a tiling model that is known to have a nonperiodic ground state, one can ask whether the ordered state can be realized via some growth process or annealing protocol. I will discuss this question in two contexts: the problem of growing a Penrose tiling and the problem of relaxation to the ordered state in a recently discovered limit-periodic tiling. For the Penrose tiling, an understanding of defects associated with noncrystalline symmetry is required. For the limit-periodic case, ordering can occur through an infinite sequence of phase transitions. A scaling theory describing an infinite sequence of equilibrium phase transitions works quite well, but a straightforward extension to dynamical properties appears to fail for interesting reasons.

Aujogue Jean-Baptiste
Computation of the Ellis semigroup for canonical cut and project point sets.

In this talk we are interested in the study of the enveloping semigroup for particular dynamical systems which we call 'rotations on a cut up torus'. This setting arises in the context of Delone dynamical systems, and here more precisely, for canonical cut and project point sets. In the theory of Delone sets the question after the nature of the dynamical spectrum is one of the main questions. Quite recently it has been shown that if a dynamical system is 'tame', a criterion expressed in terms of the Ellis semigroup, then its dynamical spectrum is purely discrete. Although pure discreteness for the spectrum for the cut and project point sets is a well-known fact, we show here that cut up torus are tame, through an explicit computation of the associated Ellis enveloping semigroup.

Bartlett Alan
Spectral Analysis of Substitution Tilings

We illustrate an algorithm that is effective in excluding Lebesgue components from the spectrum of a primitive $\mathbb{Z}^d$ substitution of constant length. More specifically, we compute a finite collection of ergodic measures generating the maximal spectral type of a given substitution. In this way, it is trivial to verify singularity to Lebesgue spectrum. This work is based on methods developed in Queffelec's Substitution Dynamical Systems - Spectral Analysis.

Cyr Van
Nonexpansive $\mathbb{Z}^2$-subdynamics and Nivat's conjecture

For a finite alphabet A and $\eta\colon \mathbb{Z} \to A$, the Morse-Hedlund Theorem states that η is periodic if and only if there exists $n \in \mathbb{N}$ such that the block complexity function $P_\eta(n)$ satisfies $P_\eta(n) \leq n$, and this statement is naturally studied by analyzing the dynamics of a $\mathbb{Z}$-action associated with η. In dimension two, a well-known conjecture of M. Nivat states that if there exist $n, k \in \mathbb{N}$ such that the $n \times k$ rectangular complexity $P_\eta(n,k)$ satisfies $P_\eta(n,k) \leq nk$, then η is periodic. In this talk I will discuss recent joint work with B. Kra in which we show that if there exist $n, k \in \mathbb{N}$ such that $P_\eta(n,k) \leq nk$, then the periodicity of η is equivalent to a statement about the expansive subspaces of this action. As a corollary, we show that if there exist $n, k \in \mathbb{N}$ such that $P_\eta(n,k) \leq \frac{nk}{2}$, then η is periodic. This proves a weak form of Nivat's conjecture.

Domenjoud Eric
Facet connectedness of discrete hyperplanes

The arithmetical hyperplane with normal vector $v \in \mathbb{R}^d \backslash \{0\}$, shift $\mu \in \mathbb{R}$ and thickness $\omega \in \mathbb{R}$ is the subset of $\mathbb{Z}^d$ defined by $P(v, \mu, \omega) = \{x \mid x \in \mathbb{Z}^d,\ 0 \leq \langle v, x\rangle + \mu < 0\}$. The connecting thickness of v with shift μ, $\Omega(v, \mu)$, is the infimum of the values of ω for which $P(v, \mu, \omega)$ is connected. We study the behaviour of the fully subtractive algorithm used to compute $\Omega(v, \mu)$ and we establish conditions under which $P(v, \mu, \Omega(v, \mu))$ is connected.

Dunham Douglas
An Algorithm to Create Hyperbolic Escher Tilings

Many of the Dutch artist M.C. Escher's patterns are based on the regular tilings $\{p, q\}$ by regular p-sided polygons meeting q at each vertex. Most of Escher's patterns were Euclidean, using the tilings 4,4, 6,3, and 3,6. In this presentation we will show a combinatorial algorithm for drawing patterns in the hyperbolic plane based on the tilings p,q, where (p-2)(q-2) $>$ 4 in order for the tiling to be hyperbolic. The algorithm has been implemented in a program that draws patterns in the Poincaré disk model of the hyperbolic plane. The algoritm draws the contents of the tiles from the center outward in an even way, leaving no ragged edges; it is also efficient, in that it draws each tile only once.

Ei Hiromi
Stepped surfaces and Rauzy fractals for some automorphisms on the free group of rank 3

For substitutions satisfying the irreducible unimodular Pisot condition, the ways to construct stepped surfaces which give quasi-periodic tilings and Rauzy fractals are well known, but we have little knowledge about them induced from automorphisms on the free group. In this talk, we discuss the automorphisms on the free group with three letters which are conjugate to substitutions, and give the stepped surfaces and Rauzy fractals. This is a joint work with P. Arnoux, M. Furukado and S. Ito.

Fernique Thomas
When periodicity enforces aperiodicity

We are interested in the digitizations of two-dimensional planes in a higher dimensional Euclidean space (which can be seen as rhombus tilings) of finite type, that is, which are characterized by a finite set of finite forbidden patterns (also called local rules). We introduce the notion of subperiod, that are rational periods of projections of planes, and show that if a plane is characterized by its subperiods, then its digitization is of finite type (we also conjecture that the converse does hold!). The point is that this does not happen only for rational planes, but for many algebraic ones, thus yielding a way to obtain a wide family of aperiodic tilings. We easily retrieve, for example, the Pavlovitch-KlÃ©man local rules for the generalized Penrose tilings, or the Socolar local rules for the 7-fold tilings. An interesting degenerated case is when its subperiods do not characterize a plane, but nevertheless still enforce planarity, yielding forbidden patterns which enforce a whole family of planes. This is for example the case of the Ammann-Beenker tilings. This method does not rely on an (eventual) hierarchical structure, nor on computational properties (as in the talk proposed by Mathieu Sablik, which would perfectly complete this one...), but only on geometric considerations (naturally expressed in terms of Grassmann coordinates) and yield simple forbidden patterns.

Frettlöh Dirk
Substitution tilings with and without infinitely many orientations

The Pinwheel tiling is the most prominent example of a substitution tiling with tiles in infinitely many orientations. In the first part of this talk we briefly show further examples together with their - individual or common - properties. In the second part we show that in any tiling with finitely many tile types, all being centrally symmetric convex polygons, the tiles always appear in finitely many orientations only. The second part is joint work with Edmund Harriss.

Furukado Maki
Non-Pisot substitutions from Rauzy induction of 4 interval exchange transformations and quasi-periodic tilings

Gabriel Olivier
Cyclic cohomology and generalized crossed products

Given a compact space X together with an homeomorphism σ and a line bundle over X, we can form a discrete dynamical system which we study via a C*-algebra. The latter fits within the framework of Generalized Crossed Products (GCP), a particular kind of Cuntz-Pimsner algebras. We start by describing the K-theory of GCP, which is determined by a Pimsner-Voiculescu-like exact sequence. Under certain analytical assumptions, we show that a similar six-term exact sequence exists for periodic cyclic cohomology. These two exact sequences are compatible, when coupled using Chern-Connes pairing. Finally, we discuss the analytical assumptions of the theorem and illustrate this theory by the concrete case of quantum Heisenberg manifolds.

Gähler Franz

A modified dual-tiling approach to the Anderson-Putnam method

The cohomology of substitution tiling spaces is usually computed with the Anderson-Putnam method, using the substitution action on an approximant cell complex. As many substitutions don't force the border, the approximant complex usually has to be constructed from collared tiles. Unfortunately, especially for triangle tilings, there may be very many collared tile types, and for 3d tetrahedra tilings even hopelessly many. We present here an alternative approach to the problem. Instead of working with the original tiling, we pass to a dual tiling, which is mutually locally derivable from the original one, and thus has the same cohomology. The induced substitution on the dual tiling always forces the border, so that collared tiles can be avoided. Together with other simplifications, this results in a modified Anderson-Putnam method requiring far less tile types, with which we have been able to compute the cohomology of even the 3d icosahedral Danzer tiling (composed of tetrahedra). We will illustrate the new method and its requirements with a number of 2d examples, including several triangle and rhombus tilings. This is joint work with John Hunton and Gregory Maloney.

Gautero François
Dynamics of tilings, invariant measures and asymptotic Thurston semi-norm

The problem of deciding whether a given finite set of tiles can tile the Euclidean plane is known to be an undecidable problem. An aim of this work is to translate this undecidability in a purely topological and geometrical way. When non-empty, the set of tilings of the Euclidean plane constructed from the given finite set of tiles inherits a natural structure of compact metric space: this is a compact laminated space with transverse structure a Cantor set, equipped with an action of $\mathbb{R}^2$ on the leaves of the lamination. There is then a non-empty set of invariant measures: each of these measures defines a certain homology-class in the second homology group of a branched surface constructed from the given set of tiles. The aim of the current work is to characterize, among all the homology classes, those coming from invariant measures on the laminated space. This is done by the introduction of a kind of Thurston semi- norm: the homology-classes coming from invariant measures are exactly those on which this semi-norm vanishes.

Grimm Uwe
The squiral tiling and its diffraction

The Thue-Morse system is a paradigm of singular continuous diffraction in one dimension. Here, we consider a planar system constructed by a bijective block substitution rule, which is equivalent to the squiral inflation rule. For balanced weights, its diffraction is purely singular continuous. The diffraction measure is a two-dimensional Riesz product that can be calculated explicitly.

Hejda Tomas
Lazy representations, substitutions and tilings associated with complex Pisot numbers

For a complex Pisot number β and a finite set of integer digits A, we consider the subset $\{\sum_{k=0}^{n} a_k \beta^k | n \in \mathbb{N}, a_k \in A\}$ of the set $\mathbb{Z}[\beta]$. Here, a complex Pisot number β is a non-real algebraic integer such that $|\beta| > 1$ and all its Galois conjugates except $\bar{\beta}$ are inside the unit circle. We show how to generate this subset by a substitution on finitely many letters such that each point is generated exactly once, using so-called lazy representations. For a sufficiently large alphabet A this subset is Delone, and we then assign a tiling of the complex plane to the substitution. With the help of this tiling, we define expansions of arbitary complex numbers in our numeration system. Parts of the results are true for a larger family of beta, namely all algebraic numbers having no Galois conjugates on the unit circle.

Jolivet Timo
Countable fundamental groups of Rauzy fractals

All the currently known examples of Rauzy fractals have a fundamental group which is either trivial, or uncountable and difficult to describe. This suggests a dichotomy between these two cases. We show that this dichotomy doesn't hold by providing examples of Rauzy fractals with nontrivial and countable fundamental groups. Moreover, we prove that every countable fundamental group of a Rauzy fractal is isomorphic to a free group of finite rank. We also work towards giving a full description of the fundamental group for some particular examples where it is uncountable but not too complicated. Lastly, we provide examples of substitutions which are conjugate by a free group automorphism but which which have different Rauzy fractal fundamental groups. Joint work with Benoît Loridant and Jun Luo.

Kalouguine Pavel
An ansatz for eigenstates in quasicrystalline potentials

An ansatz for eigenstates in quasicrystalline potentials is proposed. The states are parameterized by Floquet multipliers associated with the generators of the first cohomology group of the hull of the potential. All known examples of hierarchical eigenstates in quasicrystalline potentials are covered the proposed solution. The completeness of the eigenstates given by the ansatz would imply the existence of "blind spots" — small regions weakly electrically connected with the bulk of the quasicrystal.

Kanel-belov Alexey, Ivanov-pogodaev Ilya
Finitely presented nil-semigroups and aperiodic tilings.

The talk is devoted to the new construction method for algebraic objects using various aperiodic tilings. We use this method to costruct an finitely presented infinite nil-semigroup, answering on the Shevrin problem. The method is based on considering the paths on some tiling as non trivial elements of a semigroup. Also, the structure of tiling induces the relations in the semigroup. The tiling can be presented by the finite number of rules, so the semigroup would have the finite number of defining relations. These facts correspond to Goodmann-Strauss theorem about aperiodic hierarchical tilings. There are no periodic paths on the tiling so there are no periodic words in the semigroup. The subject is related to other Burnside type problems in groups and rings.

Kim Dong Han
Sturmian colorings of regular trees

In this talk, we study subword complexity $b(n)$ of colorings of regular trees. We classify colorings of subword complexity $b(n) \leq n + 2$ into periodic colorings, eventually periodic colorings and Sturmian colorings, and study them. We further classify Sturmian colorings of regular trees by their type set. We show that Sturmian colorings are all induced from an infinite of bi-infinite sequence on a quotient ray of the tree, and that Sturmian colorings of bounded type is induced from an eventually periodic sequence on a quotient ray. This is a joint work with Seonhee Lim

Minervino Milton
Escaping unimodularity for Pisot numeration and tilings

Tiles associated to irreducible Pisot substitutions, which are not necessarily unimodular, live in a representation space made by algebraic completions of a number field, hence it consists of a mixed product of Euclidean and p-adic spaces. We review some of the equivalent definitions of these tiles arising from numeration, duality and model sets settings, in order to describe then their topological properties. Using these tiles we construct a natural extension for the greedy beta-transformations and we show that we can parametrize it using Euclidean intersection tiles. We prove that a certain weak finiteness property, called (W), is equivalent in having either that the natural extension provides a periodic tiling, or that the tiles provide an aperiodic tiling, or that the intersection tiles induce a weak tiling of the Euclidean part of the representation space. Finally we discuss some examples and number-theoretical applications.

Mitrofanov Ivan
Multidimensional morphic words, embedding types and some decidability problems.

We define a construction of embedding types and use it to determine algorithmically some properties of multidimensional morphic words.

Monteil Thierry
Kari-Culik tile sets are too aperiodic to be substitutive.

We will propose invariants to quantify the level of aperiodicity of a tile set. We will deduce that no substitution rule the tile sets of Kari and Culik.

Nakaishi Kentaro
Pisot conjecture and Rauzy fractals

We show that there exists a discrete flow $(\tilde{X}, \tilde{T}, \mu)$ which is measure-theoretically isomorphic to the domain exchange system (X, T, v) by Arnoux-Ito. This domain exchange flow $(\tilde{X}, \tilde{T}, \mu)$ turns out to be minimal and an isometry for some metric on $\tilde{X}$.By a classical theorem of Halmos and von Neumann, such a system is topologically conjugate to a minimal rotation on a compact abelian group. Since the normalised Lebesgue measure v on X naturally induces the normalised Haar measure on $\tilde{X}$, it follows that (X, T, v) has discrete spectrum. Which proves Pisot conjecture for irreducible, unimodular Pisot substitutions with strong coincidence.

Renault Jean
Combinatorial tilings and their C*-algebras

Combinatorial tilings lead to generalized symbolic dynamical systems. Various constructions, due to A. Connes, J. Kellendonk and others, attach C*-algebras to combinatorial tilings. I will present some of these constructions and their relations. The pentagonal substitution tiling will be used to illustrate these constructions. This last part is joint work with M. Ramirez-Solano.

Sablik Mathieu

Local Rules for Computable Planar Tilings

A promising approach to obtain local rules for aperiodic tilings is the one opened by Leonid Levitov. He considered non-periodic planar tilings, that are digitizations of irrational vector spaces, and searched algebraic conditions on vector space parameters for the existence of local rules. This approach led to numerous results, but no complete characterization of aperiodic planar tilings has yet been obtained. The aim of this talk is to move a step forward in the above approach by enriching geometric methods with calculability, in the spirit of the first works on aperiodic tile sets (Hao Wang, Raphael Robinson...). The main result states that a planar tiling admits local rules if and only if it is a digitization of a vector space whose parameters are computable.

Sirvent Victor
Symmetries in Rauzy fractals

We explore geometrical symmetries of the Rauzy fractals and its relation to symbolic symmetries, i.e. symmetries of the languages that define the fractals. The geometrical symmetries studied here are reflections through a point, i.e. its center of symmetry. We show that for unimodular Pisot substitutions so that the abelianization of the set of proper prefixes is symmetric in R, there is a symmetrical subset of the Rauzy fractal. This subset corresponds to the maximal symbolic system in the paths of the prefix automaton such that is invariant under the involution defined by the symmetries of the prefixes.

Skripchenko Alexandra
Interval identification systems of order 3 and plane sections of triply periodic surfaces

The notion of interval identification systems is a generalization of interval exchange transformations and interval translation mappings. The same objects have also appeared in the theory of R-trees as an instrument fordescribing the leaf space of a band complex. We study dynamical properties of such systems (including behavior of orbits) and applications of interval identification systems of order 3 to remaining open questions in Novikov's problem of asymptotic behavior of plane sections of triply periodic surfaces. Our main tools include the Rauzy induction and the Rips machine for band complexes.

Tamura Jun-ichi
A new algorithm of continued fraction and the generation of stepped surface

We give a new algorithm of slow continued fraction expansion of dimension 2 related to any real cubic number field. Using our algorithm, we can find the generators of tiling substitutions for any stepped surface for any cubic direction.

Thieullen Philippe
The Frenkel-Kontorova model for almost-periodic environments of Fibonacci type

The Frenkel-Kontorova model describes how a chain of atoms minimizes its total energy on interaction with a substrat. We consider in a joint work with E. Garibaldi and S. Petite the case where the environment is almost periodic. A minimizing configuration in the Aubry sense may not detect the configurations at the lowest energy. We introduce a stronger notion of calibrated configurations and prove in some cases, like the Fibonacci case, their existence. The main tool we use is the Mather set, the set of minimizing measures.

Verger-gaugry Jean-louis
Parry numbers and non-Parry numbers

The canonical decomposition of the RÃ©nyi-Parry germ of curve associated with a real algebraic number beta > 1, by the theory of Puiseux, leads to characterize Parry numbers and to a possible classification of algebraic numbers which are not Parry numbers. The geometry of the zeros and the poles of the dynamical zeta function of the beta-transformation and the Parry Upper function are investigated by the Puiseux series of the minimal polynomial of beta, involved in the germ.

Yasutomi Shin-ichi
On the generation of a stepped surface by the modified Jacobi-Perron algorithm

We give a condition for the expansion obtained by the modified Jacobi-Perron algorithm to get whole part of stepped surface. The condition can be described in terms of finite automata.