"Geometry and Topology of Complex Singularities"

15-19 April 2013, CIRM, Marseille

Titles and abstracts

Courses

The courses consist of four lectures of 50 minutes each

Norbert A'Campo

Geometry of singularities of functions

The geometry of a polynomial function singularity is very rich. In this course, we will study properties of different natures : topological, combinatoric, holomorphic, symplectic, etc.

Walter Neumann

Local bilipschitz geometry of complex normal varieties

The germ of a point in a complex variety has an "inner metric" given by arclength for some embedding of the germ in an affine space. The metric is *bilipschitz canonical*: up to bilipschitz equivalence it does not depend on the embedding.

For curves the metric is uninteresting (the germ of a point is "metrically conical": bilipschitz equivalent to a metric cone). It is only in the last few years, starting with examples of Birbrair and Fernandes, that it has become clear that in higher dimensions the metric is rarely metrically conical. For surface germs a complete description of this failure of conicalness was given by Birbrair, Neumann and Pichon in terms of a canonical "thick-thin" decomposition of the germ. It was subsequently developed into a complete classification of the local bilipschitz geometry in terms of a rich set of local invariants related to 3-manifold topology.

A stronger metric (already non-trivial for curves) is the "outer metric" (distance in the ambient affine variety). It is also canonical, and it determines the inner metric, and it is classified for surface germs in terms of a richer set of local invariants.

The minicourse will describe these classifications and illustrate them with examples. Applications to equisingularity, L_p-cohomology, etc., and the extent to which the results extend to higher dimensions will also be described.

Jonathan Wahl

Deformations and smoothings of complex normal surface singularities

Suppose $f(x, y, z)$ is a complex polynomial which has a zero and an isolated critical point at the origin. The germ $(V, 0)$ of its zero set has an isolated normal complex singularity at 0. Topologically, it is the cone over its neighborhood boundary, the compact 3-manifold which is the intersection of V with a small sphere. The Milnor fibre M of f is the intersection of $f^{-1}(\delta)$ with a small ball; it is a compact 4-manifold with boundary. Milnor's classical work proves that M is simply-connected, and has the homotopy type of a bouquet of a certain number μ of 2-spheres. μ is called the Milnor number.

A hypersurface singularity is easily "smoothed", or more generally "deformed", because it is defined by a single equation, whose coefficients may be arbitrarily perturbed so that the new polynomial can be set equal to 0. But it is both natural and important to consider a general normal surface singularity $(V, 0)$. These arise for instance in taking quotients of C^2 by a finite group, or taking cones over curves embedded in projective space. In this situation, it is very difficult to know what kinds of deformations can occur. Many of these singularities are not even "smoothable". One way to understand the existence of a smoothing is through the 4-manifold M with boundary that arises as before. However, such M are topologically more complicated than in the hypersurface case; and, the same singularity could have many distinct Milnor fibres.

These themes will be explored in the lectures, especially the issue of smoothings whose Milnor fibre has the same rational homology as a disk.

One-hour talks

Enrique Artal Bartolo

Quasi-projectivity of fundamental groups of algebraic links

A finitely presented group is said to be quasi-projective if it is the fundamental group of a quasi-projective smooth variety. Taking a work of Arapura as starting point several authors have found special properties of these groups in terms of characteristic varieties. We are going to use these properties as obstructions in order to prove that the fundamental group of an algebraic link is quasi-projective if and only if its topological type can be realized by a quasi-homogeneous equation. We will discuss the proof and the consequences for fundamental groups of normal surface singularities. This is a joint work (in progress) with J.I. Cogolludo, D. Matei and S. Papadima.

Eva Bayer-Flückiger

Isometries of quadratic spaces

Michel Boileau

L-spaces, Graph manifolds and taut foliations

We will discuss the notion of L-spaces introduced by Ozsvath and Szabo, and in particular the conjecture that an aspherical integral homology 3-sphere has a non-trivial Heegaard-Floer homology, and the existence of a taut foliation on such a manifold. We will consider the case of a graph manifold. This is a joint work with Steve Boyer.

Clément Caubel

Interactions between contact topology and singularities : a survey

Javier Fernandez de Bobadilla

Boundary of Milnor fibres of real and complex singularities

We prove that if a real analytic function defined from $\mathbb{C}^3$ to $\mathbb{C}$ is the product of a holomorphic and a a antiholomorphic function, and has an isolated critical value at the origin, then it has a Milnor fibration and the boudary of the Milnor fibre is a Waldhausen 3-manifold. This is a joint work with A. Menegon and generalises results of Nemethi-Szilard and Michel-Pichon-Weber in the holomorphic case.

Lê Dung Trang

Generalized relative polar curve

Hélène Maugendre
Polar quotients

Andràs Némethi
Reduction Theorem for Lattice Cohomology

We review some basic properties of the lattice cohomology associated with links of normal surface singularities and its connection with the combinatorial Poincaré series and Heegaard-Floer and Seiberg-Witten theories. We state the Reduction Theorem which allows to reduce the rank of the lattice to the number of "bad vertices" of the plumbing graph. We list several applications: e.g. Seifert links, or links of super-isolated singularities.

Patrick Popescu-Pampu
Françoise Michel and the Seifert forms

Maria Pe Pereira
Nash Problem for surfaces

Dmitry Stepanov
Local tropicalization (joint work with Patrick Popescu-Pampu)

Given a subvariety X of algebraic torus $(K^*)^n$ over a field K, the usual (non-local) tropicalization procedure puts in correspondence to X a piecewise-linear object called the tropicalization of X. We develop a local theory of tropicalization that puts a piecewise linear object (a conical complex) in correspondence to an ideal in the formal power series ring $K[[x_1, ..., x_n]]$, or, in other words, to a formal germ of singularity. In our talk, we shall give three equivalent definitions of the local tropicalization, and indicate the main steps of the proof of its piecewise-linear structure. We expect that the local tropicalization reflects some properties of the singularity, and thus can be used in the local study of complex singularities.

Bernard Teissier
Topological invariance by equisingularity