Workshop The geometry of the Frobenius automorphism.
25 - 29 March 2013, CIRM (Luminy, France).

Elisabeth Bouscaren/Jean-Benoît Bost (Orsay)
Box 1: *Introduction: Statement of the theorem, link with model theory and difference geometry*

This talk will present the statement of the theorem and its variations, explain how to get from one version to the others, and the motivations behind this study. Some model-theoretic consequences will be explained as well.

Alice Medvedev (UC Berkeley)
Box 0: *Introduction to Difference algebra*

This talk will present the very basic definitions of difference algebra, and those needed to present the detailed plan of the proof.

Jean-Benoît Bost/Elisabeth Bouscaren (Orsay)
Box 2: *Detailed structure of the proof*

This will introduce many of the tools and concepts needed in the proof, and state precisely the main steps of the proof.

Zoé Chatzidakis (CNRS - Université Paris Diderot)
Box 3: *More difference algebra*

This talk will present more of the material needed in difference algebra. It might have several installments during the meeting.

Piotr Kowalski (Wroclaw University)/Françoise Point (FNRS - U. of Mons)
Box 4: *First upper bound lemma*

The aim of this talk is to count points *without multiplicities* on certain difference varieties. We will first introduce the functors M_q, which specialise difference varieties to the particular case where the automorphism is the q-Frobenius $\phi_q : x \mapsto x^q$. The main result states that if the difference scheme X has reduced total dimension $\leq e$ over Y, then for some B, for any $q \gg 0$, for any point $y \in M_q(Y)(L)$, the difference scheme $M_q(X)_y(L)$ has $\leq Bq^e$ points.

Françoise Point (FNRS - U. of Mons)/Piotr Kowalski (Wroclaw University)
Box 5: *Second upper bound lemma*

The aim of this talk is to count points *with multiplicities* on certain difference varieties as above. As we count the multiplicities, we cannot use Noetherian induction, and some commutative algebra is needed. The aim is to prove Corollaries 9.21 to 9.24.

François Loeser (UPMC)
Box 14: *Applications*

I will present some applications of Hrushovski's theorem:

– Jacobi's bound for algebraic difference equations (Hrushovski)
– algebraic dynamics (Fakhruddin, Amerik)
– simply connected varieties in characteristic p (Esnault-Mehta)

Immanuel Halupczok (Münster)
Box 6: *Valued difference fields*

We will introduce transformal valued fields, i.e., the right notion of valued fields K with automorphism. We will mainly be interested in such K which are "ω-increasing" (a condition about how σ acts on the value group) and which are of transformal transcendence degree 1 over a trivially valued subfield.

For the completion of such fields, we will prove a variant of Hensels lemma (6.25), and we will give a precise description of the structure of such a completion (6.31). The final goal is to prove 6.35. To a field extension L/K, we associate $rk_{val}(L/K) := \dim_{\mathbb{Q}}(val(L)/val(K)) + trdeg(res(L)/res(K))$. 6.35 is an inequality of the form $rk_{val}(LK'/KK') \leq rk_{val}(L/K)$ (under suitable assumptions).

James Freitag and William Johnson (UC Berkeley)
Box 7: *Co-analysability and inertial dimension*

This talk gives a version (7.18-7.19) of the first upper bound lemma (box 4) in the special setting of transformal valued fields. The exponent in the asymptotic bound is improved by replacing total dimension with an "inertial dimension" arising from "co"-analyzability over the residue sort (7.10). This inertial dimension can in turn be controlled by the valuative rank

rk_{val} of Box 6. Proposition 7.16 roughly asserts that the inertial dimension of a definable set is bounded above by the maximum of $rk_{val}(K(c)/K)$ as c ranges over the definable set. Therefore, asymptotic sizes of sets can be bounded above by proving bounds on valuative rank. In this talk, we will clarify the definition of co-analyzability and give a few examples. We will suggest slight modifications of the definition of co-analyzability due to problems and ambiguities present in Hrushovski's original definition.

Martin Hils (Université Paris Diderot Paris 7)
Box 8: *Key bound for transformal specialisations*

In the talk, we present the proof of the key proposition which is used to bound the number of bad points when performing transformal specialisations. We will use the results on co-analysability and inertial dimension in omega-increasing valued difference fields which are presented in the talks on Wednesday.

Damian Rössler (CNRS - U. Toulouse)
Box 9: *The smooth projective separable case*

This is a description of a variant of the proof of Th. 11.2 in Hrushovski's paper. Th. 11.2 is an analog of the main result (Th. 1.1), where the number of intersection points is replaced by a virtual intersection number and the underlying variety is assumed to be smooth and projective.

Damian Rössler (CNRS - U. Toulouse)
Box 10: *Reduction to the smooth projective separable case*

We shall describe the proof of Lemma 10.30 in Hrushovski's paper. In this lemma, he shows that we may assume that the ambient variety in his main theorem (Th. 1.1) can be embedded in a smooth compactification. A weak form of resolution of singularities, due to A.-J. de Jong, is needed here.

François Charles (U. of Rennes)
Box 11: *Basics on intersection theory and the moving lemma*

We will give a general introduction to intersection theory in algebraic geometry, and will describe the role played by moving lemmas in this classical case, giving a sketch of proof.

Jean-Benoît Bost (Orsay)
Box 12: *Moving Lemma, the difference case.*

We will present the appropriate version of the moving lemma in the difference case.

Thomas Scanlon (UC Berkeley)
Box 13: *Conservation of numbers*

With this talk we complete the proof of the main theorem following closely the presentatation in Chapter 12 of Hrushovski's manuscript. Using the geometric interpretation of the difference equations, the main theorem is reduced to the study of intersections of the form $\Gamma \cap \Phi_q$ where $\Gamma \subseteq X \times X^{\phi_q}$ is a correspondence between a variety X and its Frobenius transform and Φ_q is the graph of the q-power Frobenius morphism on X. Earlier sections give an estimate of the form $(\Gamma \cdot \Phi_q) = \delta q^{\dim(X)} + O(q^{\dim(X)-\frac{1}{2}})$ for the virtual intersection number. The crux of this proof is a comparison of this virutal intersection number with the actual number of points in the intersection $\Gamma \cap \Phi_q$. This is achieved using a moving lemma for difference schemes and bounds on the size of some exceptional sets coming from the theory of co-analysis for sets definable in transformal valued fields.