TOPOLOGY and GEOMETRY of SINGULAR SPACES

$CIRM-29th \ October-2nd \ November \ 2012$

SCHEDULE and ABSTRACTS

MONDAY

9h-9h10 : Opening session by the organizers.

9h10-10h : Andrew du PLESSIS (University of Aarhus)

Title : Stratification, resolution, and stable unfoldings of map-germs on singular varieties **Abstract :** Let $V \subset \mathbb{C}^n$ be open, let $X \subset V$ be an analytic variety, let $S \subset X$ be a finite set, and let $f : (X; S) \to (\mathbb{C}^p; 0)$ be a germ of analytic map. I will sketch how to construct topologically versal unfoldings of such germs under very mild conditions, conditions so mild that the germs for which they do not hold form a subset of infinite codimension. There are several ingredients :

(1) Critical sets for analytic maps on singular varieties via stratifications

- (2) Stratifications and fine resolutions
- (3) Jacobian ideal-sheaves in the contexts of (1) and (2)
- (4) Variations on Thom-Mather theory
- (5) Critical value stratifications.

I will briefy describe these, and how they fit together to prove the result claimed.

10h-10h30 : COFFEE BREAK

10h30-11h20 : LE DUNG Trang (Université d'Aix-Marseille)

Title : Le Polyèdre d'effondrement

Abstract : Une fonction analytique complexe sur un espace analytique réduit X avec point critique isolé en un point x de X possède une fibration localement triviale dans un voisinage du point x sur X. La fibre générale de cette fibration s'effondre sur la fibre spéciale qui passe par le point x. En dehors d'un polyèdre, appelé Polyèdre d'effondrement, cet effondrement est un homéomorphisme avec la fibre spéciale en dehors du point singulier isolé x.

11h30-12h20 : Helmut HAMM (Muenster University)

Title : Functions on real analytic spaces and stratified Morse theory

Abstract : Ordinary Morse theory has been generalized about thirty years ago by M. Goresky and R. MacPherson to functions on stratified spaces : stratified Morse theory. The latter can also be applied to cases where the original hypothesis fails, by change of the original function f. Apart from the comparison of $\{f \le a\}$ for different regular values a, we may allow critical values or look at $\{f = a\}$ instead of $\{f \le a\}$.

Goresky and MacPherson concentrate on the topological type of spaces, i.e. work up to homeomorphism (more precisely, strata preserving homeomorphism). If one wants to pass to cohomology usually some extra information is needed. This is already the case for cohomology with constant coefficients, not to speak of cohomology of constructible sheaves. As for the latter it is easier to start directly in the framework of cohomology of sheaves, as J. Schürmann has done.

12h30-13h30 : LUNCH

13h30-15h30 : POSTER SESSION

15h30-16h10 : Victor GORYUNOV (University of Liverpool)

Title : Local order 1 invariants of fronts in \mathbb{R}^3

Abstract : Invariants under consideration are those whose increments in generic homotopies are completely defined by the local bifurcations of the fronts. I will give a geometrical description of the generators of the invariant spaces, both over integers and *mod* 2, in two settings in \mathbb{R}^3 – for co-oriented fronts and for oriented fronts.

16h10-16h40 : COFFEE BREAK

16h40-17h20 : Dirk SIERSMA (University of Utrecht) **Title :** *Extremal Configurations of Polygonal Linkages* Joint work with Gaiane Panina, George Khimshiashvili and Alena Zhukova.

Abstract : We study several functions on the moduli spaces of Closed Polygons and Open Arms in two and three dimensional space. Examples of these functions are : length of connecting edge of an open arm, the signed area of a planar polygon or arm, volumes and signed area of projections in the 3-dimensional case. We determine the critical points of those functions and relate them to geometrical criteria (e.g. the vertices have to lie on a circle or a line, etc ...). We also treat the Morse theory and compute Morse indices from geometric data. The influence of bending and reflecting along diagonals will be discussed too and its relation to periodic behaviour.

17h30-18h10 : Ursula LUDWIG (Freiburg University)

Title : The Witten deformation for singular spaces and radial Morse functions

Abstract : The Witten deformation is an analytical method proposed by Witten in the 80's which, given a Morse function $f: M \to \mathbb{R}$ on a smooth compact Riemannian manifold M, leads to a proof of the famous Morse inequalities.

The aim of this talk is to present a generalization of the Witten deformation to a singular space X with cone-like singularities and radial Morse functions. As a result one gets Morse inequalities for the L^2 -cohomology, or dually for the intersection homology of the singular space X. Moreover, as in the smooth theory, one can relate the Witten complex, i.e. the complex generated by the eigenforms to small eigenvalues of the Witten Laplacian, to an appropriate geometric complex (a singular analog of the smooth Morse-Thom-Smale complex). Radial Morse functions are inspired from the notion of a radial vector field on a singular space. Radial vector fields have first been used by Marie-Hélène Schwartz to define characteristic classes on singular varieties. They also appear in the work of David Trotman, who has studied the Poincaré-Hopf theorem for radial vector fields.

18h20-19h : Aleksandra NOWEL (Gdansk University)

Title : Counting branches of the set of self-intersections of a real analytic germ from \mathbb{R}^2 to \mathbb{R}^3 Joint work with Zbigniew Szafraniec

Abstract : Let $u : (\mathbb{R}^2, 0) \to (\mathbb{R}^3, 0)$ be an analytic germ with an isolated critical point at 0 and only transverse self-intersections. We want to count the number of branches of its self-intersections set or, equivalently, the number of branches of its double point curve $D^2(u)$.

19h30-20h30 : DINNER

TUESDAY

9h10-10h : Jean-Paul BRASSELET (Université d'Aix-Marseille) **Title :** Sur les stratifications de l'ensemble de Jelonek (d'après Thuy) **Abstract :**

10h-10h30 : COFFEE BREAK

10h30-11h20 : Adam PARUSINSKI (Université de Nice)

Title : A Weight Filtration and Additive Invariants for Real Algebraic Varieties Joint work with Clint McCrory

Abstract : For each real algebraic variety we define a filtration on the complex of semialgebraic chains with closed support that induces on Borel-Moore homology with $\mathbb{Z}/2$ coefficients an analog of Deligne's weight filtration for complex algebraic varieties.

The weight complex is acyclic for smooth blowups and additive for closed inclusions.

As a corollary we obtain a new construction of the virtual Betti numbers, which are additive invariants of real algebraic varieties, and we show their invariance by a large class of mappings that includes regular homeomorphisms and Nash diffeomorphisms. These invariants are used to classify real analytic function germs.

11h30-12h20 : Clint McCRORY (University of Georgia)

Title : A weight filtration for real algebraic varieties II : classical homology Joint work with Adam Parusinski

Abstract : A weight filtration of the the complex of semialgebraic chains with compact support and $\mathbb{Z}/2$ coefficients on a nonsingular real algebraic variety X is defined in terms of a compactification W of X such that $W \setminus X$ is a divisor with normal crossings. This is analogous to Deligne's 1971 definition of the weight filtration for nonsingular complex varieties. The extension theorem of Guillen and Navarro Aznar is then used to define the weight filtration for singular real algebraic varieties.

In this talk I will sketch the construction of the filtration for nonsingular varieties, list its properties, and describe some elementary examples.

12h30-13h30 : LUNCH

13h30-15h30 : POSTER SESSION

15h30-16h10 : Georges COMTE (Université de Savoie)

Title : Additive invariants of definable sets

Abstract : Additive invariants such as the volume or the Euler-Poincaré characteristic (with compact support) play a central role in convex geometry, differential geometry and of course in singularity theory (specially in the complex case where they provide discrete measures of the nature of the singularity). I will present possible generalizations of these classical additive invariants to tame real geometry as well as non archimedean tame geometry.

16h10-16h40 : COFFEE BREAK

16h40-17h20 : Vincent GRANDJEAN (UFC Fortaleza)

Title : Collapsing locus of definable Hausdorff limits

Joint work in progress with Lev Birbrair and Alexandre Fernandes (UFC Fortaleza)

Abstract : Following the recent remarkable work of Birbrair, Neumann and Pichon on the thick and thin decomposition of complex normal surface singularities, we investigate the eventuality of such a decomposition for the Hausdorff limits of definable families.

17h30-18h10 : Patrice ORRO (Université de Savoie) Title : Abstract :

18h20-19h : Guillaume VALETTE (Polish Academy of Science, IMPAN)

Title : On the local geometry of definably stratified sets

Joint work with David Trotman

Abstract : We will focus on stratifications of subsets which are definable in an o-minimal structure. If many results about the geometry of semialgebraic or subanalytic sets go over polynomially bounded o-minimal structures, the situation is more delicate when one deals with non polynomially bounded o-minimal structures. I will give several equisingularity results on definable sets (polynomially bounded or not) which are generalizations of some known results about subanalytic geometry.

19h30-20h30 : DINNER

WEDNESDAY

9h10-10h : Hans BRODERSEN (University of Oslo)

Title : On sufficiency of jets

Abstract: A jet $z \in J^r(n, p)$ is topologically right-left sufficient in $C^r(n, p)$ if, for every C^r -germ $f: (\mathbb{R}^n, 0) \to (\mathbb{R}^p, 0)$ with $j^r f(0) = z$ we can find germs homeomorphisms $h: (\mathbb{R}^n, 0) \to (\mathbb{R}^n, 0)$ and $k: (\mathbb{R}^p, 0) \to (\mathbb{R}^p, 0)$ such that $k \circ f \circ h = z$.

We will review some recent results on sufficiency of jets $(\mathbb{R}^2, 0) \to (\mathbb{R}^2, 0)$ and then discuss conditions for sufficiency of jets $(\mathbb{R}^2, 0) \to (\mathbb{R}^3, 0)$.

10h-10h30 : COFFEE BREAK

10h30-11h20 : Terry WALL (University of Liverpool)

Title: Unfoldings in the flat singularity theory of plane curves

Abstract : Flat singularity theory is the study of germs of plane curves where vanishing of curvature is taken into account. For example, we say that germs C, C' with respective tangent cones (i.e. sets of lines through the origin) D and D' are flat isosingular if there is a diffeomorphism germ taking C to C' and D to D'. While the theory of equivalence relations for germs of curves and their classification can be carried through much as for ordinary singularity theory, the theory of versality for unfoldings does not go over.

I will describe attempts to establish some of the usual properties of transversal unfoldings in flat singularity theory by direct methods.

11h30-12h20 : Les WILSON (University of Hawaii) **Title :** A Survey on the Trotman works

12h30-13h30 : LUNCH

13h30-19h30 : FREE AFTERNOON

19h30-20h30 : DINNER

THURSDAY

9h10-10h : Satoshi KOIKE (Hyogo University)

Title : Sequence Selection Property and Bi-Lipschitz Homeomorphism

Abstract : In the previous joint paper with Laurentiu Paunescu, we proved that the dimension of the common direction set of two subanalytic subsets is preserved by a bi-Lipschitz, homeomorphism, provided that their images are also subanalytic. One of the main ingredients of the proof is the sequence selection property, denoted by (SSP) for short. This property is not invariant under a bi-Lipschitz homeomorphism but it is well suited. Our aim is to establish the geometry of sets satisfying (SSP) with bi-Lipschitz transformations. In this talk, we mention some fundamental results on (SSP), e.g. weak transversality theorem in the singular case, (SSP) structure preserving theorem, and so on.

10h-10h30 : COFFEE BREAK

10h30-11h20 : Jim DAMON (University of North Carolina)

Title : Topology of Nonisolated and Non-Complete Intersection Matrix Singularities

Abstract : Matrix singularities arise as the nonlinear sections (i.e. pullbacks) via a holomorphic mapping of the variety of singular matrices in a space of matrices (which may be $m \times m$ matrices which are symmetric, or skew-symmetric) or $m \times p$ general matrices. Except in low dimensions, matrix singularities are highly singular, and may have vanishing topology in many dimensions. Even in low dimensions certain singularities are not complete intersections. An approach which applies to both cases uses the theory of prehomogeneous spaces. In the case of representations of solvable linear algebraic groups, we identify conditions which yield linear free divisors.

We determine the topology of these divisors (their complements, Milnor fibers and links). Also, we use a generalization of a method introduced by Lê-Greuel to compute a form of the vanishing topology for the matrix singularities by placing the variety of singular matrices in a configuration of such linear free divisors.

This approach includes isolated non-complete intersection singularities and allows us to obtain algebraic formulas for Milnor numbers of isolated Cohen- Macaulay surface singularities and difference of Betti numbers $b_3 - b_2$ for isolated Cohen-Macaulay 3-fold singularities.

11h30-12h20 : Masahiro SHIOTA (Nagoya University)

Title : To avoid Vector Fields in Singularity Theory

Abstract : In singularity theory we frequently use the method of the integration of C^{∞} vector fields because the flows obtained by the integration are of class C^{∞} . If we have a locally finite family of local vector fields such that the flows have some good properties, then we can paste them by a partition of unity of class C^{∞} keeping the properties after integration. However, we cannot, in general, apply this method in the semialgebraic category. Indeed, the flow induced by the integration of a semialgebraic C^k vector field is rarely semialgebraic, $k = 1, ..., \infty$. Many years ago M. Coste and I introduced a method, which avoids integrations of vector fields, is constructive and hence also works in any o-minimal structure. Many results are obtained by this method, e.g., semialgebraic versions of Thom's first and second isotopy lemmas. Now I introduce another method, which also avoid integrations but is close to the usual method of pasting by a partition of unity.

12h30-13h30 : LUNCH

13h30-15h30 : POSTER SESSION

15h30-16h10 : Edward BIERSTONE (Fields Institute, Toronto)

Joint work with Franklin Vera Pacheco

Title : Transversality and resolution of singularities

Abstract : We will discuss the problem of resolution of singularities of an algebraic variety X, except for simple normal crossings singularities (or, more generally, the problem for a pair (X, D), where D is a divisor on X). It turns out that transversality, as opposed to general normal crossings, is needed. The results resonate with ideas of David Trotman, in a different context.

16h10-16h40 : COFFEE BREAK

16h40-17h20 : Markus PFLAUM (University of Colorado)

Title : On the stratification theory of orbit and inertia spaces of proper Lie groupoids

Joint work with C. Farsi, H. Posthuma, Ch. Seaton and X. Tang

Abstract : In the talk, the stratification theory of the orbit space and the inertia space of a proper Lie groupoid will be examined. We construct an explicit Whitney stratification of the orbit space and, under the assumption that the groupoid is the action groupoid of a compact Lie group action, a Whitney stratification of the inertia space. As a consequence, it is proved that both the orbit and the inertia space are triangulable differentiable stratified spaces. In addition, de Rham type theorems are shown, and some applications are given.

17h30-18h10 : André GALLIGO (Université de Nice)

Title: Equidistribution of the roots of a sparse polynomial system

Joint work with Carlos D'Andrea and Martin Sombra from Barcelona

Abstract : We consider a system of Laurent polynomials $f_1, \ldots, f_n \in \mathbb{C}[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$ whose coefficients are not too big with respect to its facet resultants, with a finite number of solutions. For large degrees, we show that the solutions in the algebraic torus $(\mathbb{C}^{\times})^n$ of the system are approximately equidistributed near the unit polycircle. This generalizes to the multivariate case a classical result due to Erdös and Turán (1950). We apply this result to study, when the degrees tend to infinity, the asymptotic distribution of the roots of systems of Laurent polynomials over \mathbb{Z} and for random systems of sparse polynomials over \mathbb{C} .

18h20-19h00 : Jean-Jacques RISLER (Université Pierre et Marie Curie, Paris)

Title: Curvature of Real Algebraic Varieties

Abstract: Let $X \subset \mathbb{C}^{n+1}$ be an algebraic hypersurface (resp. a Milnor fiber of a germ of hypersurface singularity at 0) which is real (invariant by complex conjugation), $X_{\mathbb{R}}$ its real part. Smith Theory implies the following Thom-Smith inequality between the sum of Betti numbers (mod 2) of $X_{\mathbb{R}}$ and X:

$$\sum_{i\geq 0} h_i(X_{\mathbb{R}}) \leq \sum_{i\geq 0} h_i(X).$$

We prove in several cases listed below a similar inequality where the sum of Betti numbers is replaced by the total curvature.

Local case : Let K be the Lipshitz-Killing Curvature function on a smooth deformation X_t of a real singularity $X \subset (\mathbb{C}^{n+1}, 0)$, k the one of the real part $X_t^{\mathbb{R}}$, $X_{t,\varepsilon} = X_t \cap B_{\varepsilon}$, σ_k the volume of the k-dimensional sphere. Then we have

$$\frac{\sigma_{2n}}{\sigma_n} \lim_{\varepsilon \to 0} \int_{X_{t,\varepsilon}} |k| \le \lim_{\varepsilon \to 0} \int_{X_{t,\varepsilon}} |K|$$

Case of Amoebas : If $A \subset (\mathbb{C}^*)^2$ is a real smooth algebraic curve, the Amoeba $\mathcal{A} \subset (\mathbb{R})^2$ is the image of A under the map $Log : (x, y) \mapsto Log(|x|, |y|)$. A bound is given for the total curvature of the Real Amoeba, and this one is sharp if and only if the curve A is "Harnack" in the sense of Mikhalkin (joint work with M. Passare).

Tropical hypersurfaces : In this case the universal inequality turns out to be an equality in the smooth case (joint work with B. Bertrand and L. Lopez de Medrano).

19h30- : SPECIAL DINNER

FRIDAY

9h10-10h : Michel COSTE (Université de Rennes 1)

Title : Singularités de robots parallèles

Abstract : Je montrerai en quoi l'étude des singularités est importante pour la résolution des problèmes cinématiques des robots parallèles. Je traiterai quelques exemples : singularités asymptotiques de robots parallèles plans, rationalité de l'ensemble des configurations singulières d'une plateforme de Stewart.

10h-10h30 : COFFEE BREAK

10h30-11h20 : Bernard TEISSIER (Intitut Mathématique Jussieu)

Title : Filtrations et gradué associé en géométrie des singularités

Abstract : Un survol de différents exemples de filtrations géométriquement utiles dans l'étude des singularités.

11h30-12h20 Krzysztof KURDYKA (Université de Savoie)

Title: Reaching generalized critical values of a polynomial

Joint work with Z. Jelonek

Abstract: Let $f : \mathbb{K}^n \to \mathbb{K}$ be a polynomial, $\mathbb{K} = \mathbb{R}$, \mathbb{C} . We give an algorithm to compute the set of generalized critical values. The algorithm uses a finite dimensional space of rational arcs along which we can reach all generalized critical values of f.

12h30-13h30 : LUNCH

14h30-15h10 : Olivier LE GAL (Université de Savoie)

Title: Lojasiewicz inequality on non compact domains and vanishing components.

Joint work with Dinh Si Tiep and Krzysztof Kurdyka

Abstract : We present an improvement of a Lojasiewicz inequality on non compact domains given by Dinh Si Tiep, Ha Huy Vui and Nguyen Thi Thao, and apply it to localize certain vanishing components of polynomial fibrations.

15h20 : END of the CONFERENCE

LIST OF THE POSTERS

Newton filtrations and non-degenerate ideals in the ring of polynomials, Jorge Alberto Coripaco Huarcaya, ICMC, Sao-Carlos.

Topology of simple singularities of ruled surfaces in \mathbb{R}^p , Grazielle Feliciani Barbosa, Universidade Federal de Sao-Carlos.

On the topology of real analytic maps, Nivaldo de Góes Grulha Júnior, ICMC, Sao-Carlos.

On the boundary of the Milnor fibre of real singularities, Aurelio Menegon Neto, ICMC, Sao-Carlos.

Corank 2 map germs from \mathbb{R}^2 to \mathbb{R}^2 , Juan Antonio Moya Pérez, Universitat de Valencia.

Cobordism on maps between \mathbb{Z}_2 -Witt spaces, Eliris Rizziolli, UNESP, Rio-Claro.

Affine properties of surfaces in \mathbb{R}^4 : asymptotic lines, Luis Florial Espinoza Sánchez, ICMC, Sao-Carlos.

Some Pratical informations :

1) Lunch is at 12:30 and Dinner is at 19:30. Please be on time !

2) Payment is on Tuesday.

If you came with your wife/husband, please do not forget to pay her/his stay. If you have any question or doubt about this, ask Claudio or Nicolas.

3) Wednesday afternoon is free. If the weather is good, we will organize a walk in the Calanques.

4) If you intend to have dinner outside on Wednesday evening, please inform the organizers.

5) There is a special dinner on Thursday evening. If you want to participate but are not sure you have registered, contact the organizers as soon as possible.

6) There is a poster exhibition.

Please have a look at the posters and contact the authors for dicussions.

Topol	logy and Geom	letry of singula	ar spaces - CIH	RM – 29-10/ 2-	11-2012
	Monday	Tuesday	Wednesday	Thursday	Friday
Morning					
9h1010h	A. du Plessis	JP. Brasselet	H. Brodersen	S. Koike	M. Coste
10h10h30	Coffee	Coffee	Coffee	Coffee	Coffee
10h3011h20	Lê Dung Tràng	A. Parusinski	C.T.C. Wall	J. Damon	B. Teissier
11h3012h20	H. Hamm	C. McCrory	L. Wilson	M. Shiota	K. Kurdyka
12h3013h30	Lunch	Lunch	Lunch	Lunch	Lunch
13h3015h30	POSTERS	POSTERS		POSTERS	14h30-15h10 O. Legal
Afternoon					
15h3016h10	V. Goryunov	G. Comte	FREE	E. Bierstone	
16h1016h40	Coffee	Coffee	Coffee	Coffee	Coffee
16h4017h20	D. Siersma	V. Grandjean	FREE	M. Pflaum	
17h3018h10	U. Ludwig	P. Orro	FREE	A. Galligo	
18h2019h00	A. Nowel	G. Valette	FREE	JJ. Risler	
19h30-20h30	Dinner	Dinner	Dinner	Dinner	Dinner