Dynamics and PDE's

November 12–16, 2012

Abstracts

Thomas Alazard, Ecole Normale Supérieure, CNRS

On the Cauchy problem for gravity water-waves

Marie-Claude Arnaud, Université d'Avignon

Variation on a theorem due to Birkhoff: invariant manifolds for conservative twisting dynamics

Abstract: a famous theorem due to George Birkhoff asserts that every simple loop that is invariant by a symplectic twist map of the 2-dimensional annulus and non-homotopic to a point is the graph of a continuous map. We will deal about possible versions of this theorem for higher dimensional dynamics.

Artur Avila, IMPA and CNRS

TBA

Viviane Baladi, University of Copenhagen

Natural boundary for the susceptibility function of generic piecewise expanding unimodal maps

Abstract: (joint with S. Marmi and D. Sauzin) We consider the susceptibility function $\Psi(z)$ of a piecewise expanding unimodal interval map. Combining previous results (deduced from spectral properties of Ruelle transfer operators) with recent work of Breuer-Simon (based on techniques from the spectral theory of Jacobi matrices and a classical paper of Agmon), we show that density of the postcritical orbit (a generic condition) implies that $\Psi(z)$ has a strong natural boundary on the unit circle. The Breuer-Simon method provides uncountably many candidates for the outer functions of $\Psi(z)$, associated to precritical orbits. If the perturbation is horizontal, a generic condition (Birkhoff typicality of the postcritical orbit) implies that the nontangential limit of the $\Psi(z)$ as z tends to 1 exists and coincides with the derivative of the acim with respect to the map (linear response formula).

Dario Bambusi, Università degli studi di Milano

Asymptotic stability of breathers in some Hamiltonian networks of weakly coupled oscillators

Abstract: consider a Hamiltonian chain of weakly coupled anharmonic oscillators. It is well known that if the coupling is weak enough then the system admits families of periodic solutions exponentially localized in space (breathers). I will give a survey on the theory

leading to the existence of breathers and present a result of asymptotic stability in energy space of such solutions. Asymptotic stability is possible notwithstanding the Hamiltonian nature of the system, due to the presence of dispersion. The proof is based on two steps: first we use canonical perturbation theory to put the system in a suitable normal form in a neighborhood of the breather, second we use dispersion in order to prove asymptotic stability.

Michael Benedicks, Royal Institute of Technology

Problems on Evolution equations and coupled map lattices

Patrick Bernard, Université Paris Dauphine

Arnold's diffusion, from the a priori unstable to the apriori stable case

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Massimiliano Berti, Universita degli Studi di Napoli "Federico II"

KAM theory for quasi-linear and fully nonlnear KdV equations

Abstract: We prove the existence of quasi-periodic, small amplitude, solutions for quasi-linear and fully nonlinear forced perturbations of KdV equations, including Hamiltonian quasi-linear KdV as a particular case. For reversible nonlinearities we also prove the linear stability of the solutions.

Claire Chavaudret, Université de Nice Sophia-Antipolis

Reducibility of quasi-periodic cocycles under a Brjuno-Rssmann arithmetical condition

Abstract: Linear quasi-periodic systems with coefficients in $SL(2,\mathbb{R})$ and with many frequencies are shown to be reducible under an arithmetical condition on the frequency and on the rotation number which is weaker than the diophantine condition and similar to the usual Brjuno condition. This gives an extension of a theorem by Eliasson. (joint work with S. Marmi)

Walter Craig, McMaster University

Vortex filament interactions and Hamiltonian PDEs

Abstract: Mathematicians working in the intersection of the areas of PDEs and Hamiltonian dynamical systems have developed techniques for the phase space analysis of the dynamics of a number of model nonlinear Hamiltonian PDEs. In this talk I will describe an application of these ideas to a problem in fluid dynamics concerning the interaction of two near-parallel vortex filaments in three dimensions. In addition, I will discuss generalizations of this problem, and will use the opportunity to speculate about further applications of the techniques of Hamiltonian PDEs to other nonlinear systems that occur in physics in the form of nonlinear evolution problems.

David Damanik, Rice University

The spectrum of quasi-periodic Schrödinger operators in the perturbative regime

Abstract: We discuss continuum Schrödinger operators on the line with small quasi-periodic potentials. Through a multi-scale analysis scheme based on the Schur complement formula, we derive a description of the spectrum in terms of the Fourier coefficients of the potential. As a result, we obtain a two-way relation between the size of the Fourier coefficients and the lengths of the gaps of the spectrum. This is joint work with Michael Goldstein.

Albert Fathi, Ecole Normale Supérieure de Lyon

Lyapunov forms

Abstract: Pursuing a work that was done with Pierre Pageault on Lyapunov functions, for a dynamical system, we study the problem of existence closed 1-forms who's integral along orbits is non-positive, thus generalizing some work of Schwartzman–compare also with the work of Farber, Kappeler, Latschev, and Zehnder.

Bassam Fayad, Université Paris 6, CNRS

Local rigidity for affine $\mathbb{Z}^k$ actions on the torus

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Jacek Graczyk, Université Paris-Sud, Orsay

Metric properties of mean wiggly continua

Abstract: We study lower and upper bounds of the Hausdorff dimension of continua in euclidean spaces which are wiggly at scales of positive density. The main technical ingredient is corona type construction of a Borel probabilistic measure with a superlinear scaling property at "wiggly points". The theory of mean wiggly continua leads to new geometric estimates of the Hausdorff dimension for compact sets. We will also discuss some applications of this theory in complex dynamics. This is a joint work with P. Jones and N. Mihalache.

Benoit Grébert, Université de Nantes

KAM for the Beam Equation on the torus

Abstract: we prove a KAM result for the non linear beam equation on the d-dimensional torus

$$u_{tt} + \Delta^2 u + mu = f(x,u)\ , \quad t \in, x \in \mathbb{T}^d$$

where $f(x,u) = u^3 + O(u^5)$. Roughly speaking we prove that, for generic m, most of the small amplitude, finite dimensional, invariant tori of the linear equation ($f = 0$) are preserved when turning on the nonlinearity. Moreover these tori are linearly stable. We plan to extend the result to the nonlinear wave equation.
This is a joint work with H. Eliasson and S. Kuksin.

Marcel Guardia, University of Maryland

Growth of Sobolev norms for the cubic defocusing NLS with and without a convolution potential

Abstract: Consider the cubic defocusing nonlinear Schrödinger equation with periodic boundary conditions and fix $s > 1$. Colliander, Keel, Staffilani, Tao and Takaoka (2010) proved the existence of solutions whose s-Sobolev norm grows in time by any given factor R. Refining their methods in several aspects and using Dynamical Systems techniques, jointly with V. Kaloshin we obtain solutions with s-Sobolev norm growing in polynomial time in R. These improvements allow also to show that the growth of Sobolev norms in polynomial time can also be attained by solutions of the cubic defocusing NLS with a convolution potential.

Kurt Johansson, Royal Institute of Technology

Dimer models and random matrix statistics

Abstract: Dimer models on certain bipartite graphs or equivalently random tilings of certain regions have many relations to random matrices. I will give an overview of this area and also describe some recent results on continuum scaling limits.

Thomas Kappeler, Universität Zürich

Large number of particles asymptotics of Toda lattices

Abstract: For periodic Toda lattices with a large number N of particles close to the equilibrium we obtain asymptotics of various conserved quantities which turn out to involve a pair of KdV equations. In particular, for N large, the Toda frequencies up to leading order are given by KdV frequencies. This is joint work with Dario Bambusi and Thierry Paul.

Anatole Katok, Penn State

*A*pplications of KAM method to rigidity of group actions; recent progress, difficulties and prospects

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Konstantin Khanin, University of Toronto

On renormalization and rigidity for circle maps with breaks

Abstract: We'll discuss hyperbolicity of renormalization for circle maps with breaks, and related rigidity results.

Raphael Krikorian, Université Paris 6

Density of reducible quasi-periodic cocycles on $\mathbb{T}^2 \times SU(2)$ in the smooth case.

Abstract: " KAM theory is an important tool to understand the local theory of reducibility of smooth quasi-periodic cocycles (with diophantine frequency vector $a \in \mathbb{T}^d$) $(a, A) : \mathbb{T}^d \times G \to \mathbb{T}^d \times G$, $(a, A) : (x, y) \mapsto (x + a, A(x)y)$ (here G is a Lie group and $A : \mathbb{T}^d \to G$

a smooth map). In that situation, given a diophantine, the set of A, close to constants, for which the cocycle (a, A) is reducible is (loosely speaking) a set of "full measure" (this was first proven in the case $G = SL(2, R)$ by Håkan Eliasson). To go one step further and prove global reducibility results (we now concentrate on the case when G is compact) one strategy is to use renormalization techniques (coupled with KAM); this allows to prove the following density result: given a in a set of full measure on $\mathbb{T}$, the set of A for which (a, A) is reducible is dense. The drawback of renormalization techniques is that they are (up to now) efficient only when one deals with one frequency systems. In this talk we shall concentrate on two-frequencies $SU(2)$-valued quasiperiodic cocycles ($d = 2$, $G = SU(2)$). Our aim is to present the following non-perturbative result: given any $(a, A) \in \mathbb{T}^2 \times C^\infty(\mathbb{T}^2, SU(2))$ one can perturb it in $\mathbb{T}^2 \times C^\infty(\mathbb{T}^2, SU(2))$ so that it becomes smoothly reducible.

Sergei Kuksin, Ecole Polytechnique, CNRS

On quantum averaging, KAM and diffusion

Abstract: The quantisation construction establishes a correspondence between a nonautonomous classical Hamiltonian of the form $|p|^2 + V(t, q)$ and a quantum system with the nonautonomous Hamiltonian operator $-h^2\Delta + V(t, x)$. I will discuss dynamical properties of the quantum system, dual to those of the classical system which are described by the KAM-related theories. Namely, by the proper KAM, the averaging, the Nekhoroshev stability, the diffusion and the capture in resonance. The talk is based on my work with Håkan and a joint paper with A.Neishtadt.

Rafael de la Llave, Georgia Institute of Technology.

Quasi-periodic solutions for some ill-posed Hamiltonian evolution equations

Abstract: We prove an a-posteriori KAM theorem which applies to some ill-posed Hamiltonian equations. We show that given an approximate solution of an invariance equation which also satisfies some non-degeneracy conditions, there is a true solution nearby. Furthermore, the solution is "whiskered" in the sense that it has stable and unstable directions. We do not assume that the equation defines an evolution equation. Some examples are the Boussinesq equation (and system) and the elliptic equations in cylindrical domains. This is joint work with Y. Sire. Related work with E. Fontich and Y. Sire.

Stefano Marmi, Scuola Normale Superiore

There is only one KAM curve

Abstratct: We consider the standard family of area-preserving twist maps of the annulus and the corresponding KAM curves. Addressing a question raised by Kolmogorov, we show that, instead of viewing these invariant curves as separate objects, each of which having its own Diophantine frequency, one can encode them in a single function of the frequency which is naturally defined in a complex domain containing the real Diophantine frequencies and which is monogenic in the sense of Borel; this implies a remarkable property of quasi-analyticity, a form of uniqueness of the monogenic continuation, although real frequencies

constitute a natural boundary for the analytic continuation from the Weierstrass point of view because of the density of the resonances.)

Carlos Matheus, Université Paris 13

A coding-free approach to the Lyapunov exponents of Teichmuller curves

Abstract: The study of the Lyapunov exponents of the so-called Kontsevich-Zorich cocycle is a fruitful suject due to its several applications to the deviations of ergodic averages of interval exchange transformations, translation flows, and the rate of diffusion of trajectories in the so-called Ehrenfest wind-tree model of Lorentz gases, for example.

In the literature, the qualitative properties of Lyapunov exponents (such as simplicity of Lyapunov spectra) are normally obtained with the aid of codings (countable Markov partitions adapted to the probability at hand). However, the construction of codings is a delicate technical issue in general, and thus it is desirable to know whether it is possible to say something meaningful about Lyapunov exponents without relying on particular codings.

In this talk, we will discuss a joint work with Alex Eskin where we point out that a profound theorem of H. Furstenberg allows to implement a "coding-free" approach for the Lyapunov exponents of the Kontsevich-Zorich cocycle in the particular case of very well-behaved orbits known as Teichmuller curves.

Gueorgui Popov, Université de Nantes

Isospectal Deformations KAM tori and Spectral Rigidity

Abstract: We consider a family of Laplace-Beltrami operators corresponding to a smooth deformation of Riemannian metrics on a compact manifold with or without boundary. We suppose that the initial metric is either completely integrable or close to a non-degenerate completely integrable metric (KAM system). If the deformation is isospectral we prove that the values of the corresponding Mather's α-function given by the average action on the KAM tori is constant along the deformation. As an application we obtain infinitesimal rigidity of Liouville billiard tables. The proof is based on a construction of quasi-modes associated with KAM tori.

Jörg Schmeling, Lund University

Multifractal analysis of some multiple ergodic average

Abstract: Let (X, T) be a topological dynamical system where T is a continuous map on a compact metric space X. Fürstenberg had initiated the study of the *multiple ergodic average*:

$$\frac{1}{n}\sum_{k=1}^{n} f_1(T^k x) f_2(T^{2k} x) \cdots f_s(T^{sk} x) \tag{1}$$

where $f_1, \cdots, f_s$ are s continuous functions on X with $s \geq 2$ when he proved the existence

of arithmetic sequences of arbitrary length amongst sets of integers with positive density. Later on, the research of such a kind of average has attributed a lot of attentions.

We study the multiple ergodic averages

$$\frac{1}{n}\sum_{k=1}^{n}\varphi(x_k, x_{kq}, \cdots, x_{kq^{\ell-1}})$$

on the symbolic space $\Sigma_m = \{0, 1, \cdots, m-1\}^{\mathbb{N}^*}$ where $m \geq 2, \ell \geq 2, q \geq 2$ are integers. We give a complete solution to the problem of multifractal analysis of the limit of the multiple ergodic averages.

This is joint work with Ai-Hua FAN, and Meng WU.

Jacopo De Simoi, II Università di Roma - Tor Vergata

High energy dynamics of some piecewise smooth Fermi-Ulam models

Abstract: We find a normal form which describes the high energy dynamics of a class of piecewise smooth Fermi-Ulam ping pong models. Depending on the value of a single real parameter, the dynamics can be either hyperbolic or elliptic. In the first case we prove that the set of orbits undergoing Fermi acceleration has zero measure but full Hausdorff dimension. We also show that for almost every orbit, energy eventually falls below a fixed threshold. In the second case we prove that, generically, we have stable periodic orbits for arbitrarily high energies, and that the set of Fermi accelerating orbits may have infinite measure. This is a joint work with D. Dolgopyat

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