The space of four end solutions to the Allen-Cahn equation in $\mathbb{R}^2$

Michal Kowalczyk

An entire solution of the Allen-Cahn equation $\Delta u = f(u)$, where f is an odd function and has exactly three zeros at ± 1 and 0, e.g. $f(u) = u(u^2 - 1)$, is called a $2k$ end solution if its nodal set is asymptotic to $2k$ half lines, and if along each of these half lines the function u looks (up to a multiplication by -1) like the one dimensional, odd, heteroclinic solution H, of $H'' = f(H)$. In this talk I will discuss a special class of this solutions with just four ends. A special example is the saddle solutions U whose nodal lines are precisely the straight lines $y = \pm x$. I will describe completely connected components of the moduli space of four end solutions. Finally I will discuss a uniqueness result which gives a complete classification of these solutions. It says that all four end solutions are continuous deformations of the saddle solution.

This is a joint work with Frank Pacard and Yong Liu.

A remark on natural constraints in variational methods and an application to superlinear Schrödinger systems

Benedetta Noris, INdAM-COFUND Marie Curie Fellow

Given a smooth functional J defined on a Hilbert space H, a natural constraint for J is a submanifold N of H with the property that any critical point of J constrained to N is a free critical point. A well known example is the Nehari manifold, i.e. the set $\{u \neq 0 : \; J'(u)u = 0\}$. In this talk we investigate sufficient conditions for a manifold to be a natural constraint for a given functional. We provide a unified approach to different natural constraints known in the literature, such as the Birkhoff-Hestenes natural isoperimetric conditions and the Nehari manifold. As an application, we prove multiplicity of solutions to the following class of superlinear Schrödinger systems

$$-\Delta u_i = \mu_i u_i^3 + u_i \sum_{j \neq i} \beta_{ij} u_j^2, \qquad u_i > 0, \; u_i \in H_0^1(\Omega), \qquad i = 1, \ldots, k,$$

$\mu_i > 0$, $\beta_{ij} = \beta_{ji} < 0$, on singularly perturbed domains $\Omega \subset \mathbb{R}^N$, $N = 2, 3$, thus extending some results obtained by Dancer and Beyon in the case of a single equation. This is a joint work with G. Verzini.

Results on sign changing solutions of some semilinear parabolic and elliptic problems

Filomena Pacella

We consider elliptic and parabolic semilinear problems of Lane Emden type in some symmetric domains in the plane. When the exponent of the nonlinearity is large we show the existence of a sign changing solution with two nodal regions and an interior nodal line. Some related questions will be discussed.

The results presented have been obtained in collaboration with Francesca De Marchis and Isabella Ianni.

Segregated and synchronized vector solutions for nonlinear Schrödinger systems

Shuangjie Peng

In this talk, we consider the following nonlinear Schrödinger system in $\mathbb{R}^3$

$$\begin{cases} -\Delta u + P(|x|)u = \mu|u|^2u + \beta v^2 u, & x \in \mathbb{R}^3, \\ -\Delta v + Q(|x|)v = \nu|v|^2v + \beta u^2 v, & x \in \mathbb{R}^3, \end{cases}$$

where $P(r)$ and $Q(r)$ are positive radial potentials, $\mu > 0, \nu > 0$ and $\beta \in \mathbb{R}$ a coupling constant. This type of system arises in particular in models in Bose-Einstein condensates theory. We examine the effect of the nonlinear coupling to the solution structure.In the repulsive case we construct an unbounded sequence of non-radial positive vector solutions of segregated type and in the attractive case we construct an unbounded sequence of non-radial positive vector solutions of synchronized type. Depending upon the system being repulsive or attractive our results exhibit distinct characteristic features of vector solutions.

This is a joint work with Professor Zhi-Qiang Wang.

Steady states with unbounded mass of the Keller-Segel system

Angela Pistoia

We consider the boundary value problem

$$\begin{cases} -\Delta u + u = \lambda e^u, & \text{in } B_{r_0} \\ \partial_\nu u = 0 & \text{on } \partial B_{r_0} \end{cases}$$

where B_{r_0} is the ball of radius r_0 in $\mathbb{R}^N$, $N \geq 2$, $\lambda > 0$ and ν is the outer normal derivative at ∂B_{r_0}. This problem is equivalent to the stationary Keller-Segel system from chemotaxis.

We show the existence of a solution concentrating at the boundary of the ball as λ goes to zero.

The result is obtained in collaboration with Giusi Vaira.

Sign-changing blow-up for scalar curvature type equations

Frederic Robert

Given (M, g) a compact Riemannian manifold of dimension $n \geq 3$, we are interested in the existence of blowing-up sign-changing families $(u_\epsilon)_{\epsilon>0} \in C^{2,\theta}(M)$, $\theta \in (0,1)$, of solutions to

$$\Delta_g u_\epsilon + h u_\epsilon = |u_\epsilon|^{\frac{4}{n-2}-\epsilon} u_\epsilon \text{ in } M\,,$$

where $\Delta_g := -\mathrm{div}_g(\nabla)$ and $h \in C^{0,\theta}(M)$ is a potential. Assuming the existence of a nondegenerate solution to the limiting equation (which is a generic assumption), we prove that such families exist in two main cases: in small dimension $n \in \{3, 4, 5, 6\}$ for any potential h or in dimension $3 \leq n \leq 9$ when $h \equiv \frac{n-2}{4(n-1)}\mathrm{Scal}_g$. These examples yield a complete panorama of the compactness/noncompactness of critical elliptic equations of scalar curvature type on compact manifolds. The changing of the sign is necessary due to the compactness results of Druet and Khuri–Marques–Schoen.

This is joint work with Jerome Vetois (Nice).

Remarks on the Trudinger-Moser inequality

Bernhard Ruf

The Trudinger-Moser (TM) inequality concerns a limiting case of the well-known Sobolev embeddings. The TM-inequality has many similarities to the Sobolev inequalities, but there are also some important differences. In particular, unlike the critical Sobolev case, no group invariance nor Pohozaev type identity are known. We show that these can be recovered by choosing a suitable target space.

Nash-Moser without regularity

Eric Séré

This is joint work with Ivar Ekeland. The Nash-Moser theorem allows to solve a functional equation $F(u) = 0$ in a "scale" of Banach spaces, assuming that $F(0)$ is small and that near 0 the differential DF has a right inverse which loses derivatives. The classical proof uses a Newton iteration scheme, which converges when F is of class C^2. In contrast, we only assume that F is continuous and has a Gâteau first differential, which is right-invertible with loss of derivatives. We solve the functional equation thanks to an iteration scheme using Ekeland's variational principle at each step. As an application, we prove the existence of periodic solutions of nonlinear wave equations, under diophantine conditions and mild regularity assumptions on the nonlinearity.

Multi-bump positive solutions for a nonlinear elliptic problem in expanding tubular domains

Kazunaga Tanaka

The role of Liouville type systems in the study of non-topological vortices in Chern-Simons theory

Gabriella Tarantello

We give a survey on the role of Liouville type problems in the study of non-topological vortex configurations in (selfdual) Chern-Simons theory. In particular we shall discuss a class of Liouville type systems with singular sources, as derived from a non-abelian Chern-Simons model recently introduced in the physical literature in connection with the delicate issue of matter confinement. We present some recent results and discuss many of the still open questions.

Positive and sign-changing solutions for the Brezis-Nirenberg problem

Giusi Vaira

Uniform Hölder bounds for strongly competing systems involving the square root of the laplacian.

Gianmaria Verzini, Politecnico di Milano

For a class of nonlinear competition-diffusion systems involving the square root of the Laplacian, including the fractional Gross–Pitaevskii system

$$(-\Delta)^{1/2}u_i = \lambda_i u_i + \omega_i u_i^3 - \beta u_i \sum_{j\neq i} a_{ij}u_j^2, \qquad i = 1, \ldots, k,$$

where $u_i \in H^{1/2}(\mathbb{R}^N)$, we prove that L^∞–boundedness implies $C^{0,\alpha}$–boundedness for every $\alpha \in [0, 1/2)$, uniformly as $\beta \to +\infty$. Moreover we prove that the limiting profile is $C^{0,1/2}$. To study such problem, we exploit the classical local realization of $(-\Delta)^{1/2}$ as a Dirichlet-to-Neumann map in $\mathbb{R}^{N+1}_+$, and we are lead to consider a system where the competition between the different densities is localized on the boundary. This is a joint work with Susanna Terracini (Università di Torino) and Alessandro Zilio (Politecnico di Milano).

A new variational approach to some quasilinear elliptic problems

Zhi-Qiang Wang

In this talk, we present recent results on a new variational framework for a class of quasilinear elliptic equations. The new approach allows us to deal with lack of smoothness and compactness and to make use of minimax arguments to obtain multiplicity results.

Poincaré inequality and a relative isoperimetric inequality

Michel Willem

We consider various aspects of the Poincaré inequality in the space of functions of bounded variations. In particular we describe the relations between the optimal Poincaré inequality and some relative isoperimetric inequalities.

Fully Bubbling solutions for the SU(3) Toda System of Mean Field Type on a Torus

Shusen Yan

In this talk, the following SU(3) Toda system on a torus Ω will be discussed:

$$\begin{cases} -\Delta u_1 = 2\rho_1\left(\frac{h_1(x)e^{u_1}}{\int_\Omega h_1(x)e^{u_1}} - \frac{1}{|\Omega|}\right) - \rho_2\left(\frac{h_2(x)e^{u_2}}{\int_\Omega h_2(x)e^{u_2}} - \frac{1}{|\Omega|}\right), \\ -\Delta u_2 = 2\rho_2\left(\frac{h_2(x)e^{u_2}}{\int_\Omega h_2(x)e^{u_2}} - \frac{1}{|\Omega|}\right) - \rho_1\left(\frac{h_1(x)e^{u_1}}{\int_\Omega h_1(x)e^{u_1}} - \frac{1}{|\Omega|}\right), \\ u_1 \text{ and } u_2 \text{ are doubly periodic on } \partial\Omega, \end{cases} \tag{1}$$

where $h_i(x) \in C^3(\Omega)$, $h_i(x) > 0$, $i = 1, 2$, and Ω is a parallelogram in $\mathbb{R}^2$. The result is that if q is a critical point of h_1 and h_2, satisfying certain conditions, (1) has a fully bubbling solution at q for some $\rho_i \to 8\pi$, $i = 1, 2$.

This is a joint work with C. S. Lin.

Singular limit problems related to the Onofri inequality

Jean Dolbeault

This lecture will be devoted to a brief overview of recent developments related with the Onofri inequality.

In dimension two, the Onofri inequality written in the euclidean space can be seen as a limiting case in a family of Gagliardo-Nirenberg inequalities. This can be generalized to higher dimensions. With different weights, the Onofri inequality can be related with Caffarelli-Kohn-Nirenberg inequalities in two space dimensions and provides some interesting insights into symmetry breaking issues.

By duality the usual Onofri inequality can be related to the logarithmic Hardy-Littlewood-Sobolev inequality. This can also be done using a flow of fast diffusion type. When dealing with the two-dimensional Keller-Segel model, the sharp constant in the logarithmic Hardy-Littlewood-Sobolev inequality determines the threshold between global existence of solutions and blow-up in finite time; the critical parameter is the mass. In the subcritical range, the sharp asymptotic behaviour of the solutions can be determined in a framework corresponding to a subcritical family of logarithmic Hardy-Littlewood-Sobolev inequalities, which by duality provide a new family of Onofri type inequalities. The sharp cases in the usual Onofri and logarithmic Hardy-Littlewood-Sobolev inequalities are then recovered by taking a singular limit.

Sequences of bump solutions for an interface nonlinear Schrödinger equation

David Ruiz

In this talk we consider the problem:

$$-\Delta u + V(x)u = f(u)$$

for certain superlinear terms $f(u)$. Here $V(x)$ is an interface potential, that is, $V(x)$ is equal to $V_1(x)$ if $x_1 > 0$ and $V_2(x)$ for $x_1 < 0$, where $V_i(x)$ are periodic positive potentials. This problem arises in the study of a quantum system located at the interface of two different media.

By using variational methods and a localization argument, we prove the existence of sequences of bump solutions located at large values of x_1.

This is joint work with Hans-Jürgen Freisinger, from Karlsruhe.