Théorie analytique des nombres : en l'honneur d'Etienne Fouvry
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V. Blomer

p-adic analytic twists and strong subconvexity

For a fixed newform f and a Dirichlet character χ of conductor $q = p^n$ we prove a Weyl-type subconvexity bound for the twisted L-function $L(f \otimes \chi, 1/2 + it)$ as n tends to infinity. It is obtained by exhibiting strong cancellation between the Hecke eigenvalues of f and the values of χ, which act as twists by exponentials with a p-adically analytic phase. Among the tools, we develop a general result on p-adic approximation by rationals (a p-adic counterpart to Farey dissection) and a p-adic version of van der Corput's method for exponential sums. This is joint work with D. Milicevic.

T. Browning

A polynomial sieve

We generalise the square sieve and use it to show that almost all integers have at most one representation as a sum of two values of a given integral polynomial of degree at least 3.

J. Brüdern

Correlation estimates for sums of three cubes

Let $r(n)$ be the number of representations of the natural number n as the sum of three positive cubes. Moreover, let $L_1, \dots, L_{2r}$ denote linear forms in r variables $x_1, \dots, x_r$ with integer coefficients. Subject to the condition that any r of the forms L_j are linearly independent, an estimate will be provided for the sum

$$\sum_{|x_j| \le B} r(L_1(\mathbf{x})) \cdots r(L_{2r}(\mathbf{x}))$$

that exceeds the best possible one by a factor $\sqrt{B}$. Variants, as well as applications to linear equations in sums of three cubes, will also be presented. This talk is describes joint work with Trevor Wooley.

A. Cojocaru

TBA

J-L. Colliot-Thélène

Points entiers sur les familles d'espaces homogènes

Dans un travail avec David Harari, nous donnons des conditions suffisantes d'existence et de densité pour les points entiers de variétés affines fibrées sur la droite affine en espaces homogènes de groupes semisimples. Un cas particulier (obtenu avec F. Xu) est celui des équations diophantiennes $ax^2 + by^2 + cz^2 = p(t)$ (à résoudre en entiers x, y, z, t) avec a, b, c entiers non nuls non tous de même signe et $p(t)$ un polynôme à coefficients entiers.

C. Dartyge

Le problème de Tchébychev pour le douzième polynôme cyclotomique.

On note $P^+(n)$ le plus grand facteur premier de l'entier n et $\Phi_{12}(n) = n^4 - n^2 + 1$. Nous montrons qu'il existe $c > 0$ tel que pour X assez grand on ait :

$$P^+\Big(\prod_{X<n\le 2X} \Phi_{12}(n)\Big) \ge X^{1+c}.$$

La valeur $c = 10^{-47016}$ est admissible.

C. David

Groups of points of abelian surfaces over finite fields

Let A be an abelian surface over the finite field $\mathbb{F}_q$. The rational points on A over $\mathbb{F}_q$ form an abelian group $A(\mathbb{F}_q) \simeq \mathbb{Z}/n_1\mathbb{Z} \times \mathbb{Z}/n_1n_2\mathbb{Z} \times \mathbb{Z}/n_1n_2n_3\mathbb{Z} \times \mathbb{Z}/n_1n_2n_3n_4\mathbb{Z}$. We are interested in determining which groups actually occur as the group of points on an abelian surface over $\mathbb{F}_q$. A characterization of these groups for abelian varieties of any dimension in terms of the characteristic polynomial of the Frobenius endomorphism was found recently by Rybakov. Using this characterization for the case of abelian surfaces, we show that "very split" groups (i.e. when n_1, n_2 are large compared to n_3, n_4) are less likely to occur, by relating this question to the uniform distribution modulo one of $n_2^{1/2} n_3 n_4^{1/2}$. This is compatible with the general philosophy of the Cohen-Lenstra heuristics, which predict that random abelian groups naturally occur with probability inversely proportional to the size of their automorphism groups.

This is joint work with D. Garton, Z. Scherr, A. Shankar, E. Smith and L. Thompson.

D. Fiorilli

Montgomery-type Conjectures and L-functions at the central point.

Probabilistic arguments as well as numerical data suggests that for large moduli, the error term in the prime number theorem for arithmetic progressions is much smaller than what GRH predicts. Based on such arguments Montgomery formulated a conjecture which fits the numerical data and which implies several well believed conjectures for primes in arithmetic progressions, such as the Elliot-Halberstam Conjecture and the equidistribution of primes up to x modulo q as soon as x exceeds $q^{1+\epsilon}$. In formulating Montgomery's Conjecture, one should assume that Dirichlet L-functions do not vanish at the central point. We will show how to reformulate the conjecture without this assumption, and show how the modified conjecture implies that almost all Dirichlet L-functions do not vanish at the central point.

We will then show that these arguments can be modified to other families of L-functions, and will focus on families of elliptic curve L-functions. In work in progress, our results are that a conjecture analogous to Montgomery's implies that the average analytic rank of the curves in the family is bounded above by 1/2, and in some cases we can show that exactly half of the curves have algebraic rank 0, and the remaining half have algebraic rank 1.

É. Fouvry

Quelques cas d'orthogonalité fortes.

Nous présenterons des exemples récents, issus de l'arithmétique, de suites complexes (a_n) et (b_n) telles que la somme $\sum_{n\leq N} a_n b_n$ soit bien plus petite que la borne triviale.

J. Friedlander

On a theorem of Fouvry-Iwaniec and some of its children

We discuss some developments, old and new, in the distribution of primes.

A. Harper

The typical maximum size of the Riemann zeta function

Recently, Fyodorov and Keating made a very precise conjecture about the maximum of the zeta function in a "typical" very short interval on the critical line. The conjecture is based on a random matrix model, but even this model cannot be analysed rigorously, and various heuristics from statistical mechanics are invoked to complete their analysis.

I will give a different approach to Fyodorov and Keating's conjecture, in which the probabilistic analysis can be performed rigorously. One key point is to "get rid of" the influence of the zeros of the zeta function. This notion is also highly important in much other work studying the zeta function on the critical line.

H. Helfgott

Vers la conjecture ternaire de Goldbach

La conjecture ternaire de Goldbach (1742) affirme que tout nombre impair plus grand que 5 est la somme de trois nombres premiers. D'après les pionniers (Hardy et Littlewood), Vinogradov prouva (1937) que tout nombre impair plus grand qu'une constante C satisfait la conjecture. Dans les trois quatrièmes de siècle suivants, il y a eu une succession de résultats réduisant C, mais seulement à des niveaux beaucoup trop grands pour qu'une vérification mécanique jusqu'à C soit possible ($C > 10^{1300}$). (Par ailleurs, les travaux de Ramaré et Tao ont prouvé les problèmes correspondants avec six et cinq nombres premiers en place de trois.)

Mes travaux récents prouvent la conjecture.

H. Iwaniec

Long mollifiers

This will be a survey of joint work in progress with B. Conrey.

F. Jouve

Bezoutians and hypergeometric groups

Let k be a field of characteristic not 2. To a pair of polynomials (p, q) in $k[T]$ a well known construction due to Bézout associates a symmetric matrix of size $\max(\deg(p), \deg(q))$ whose determinant equals the resultant of p and q. We study a twisted version of Bézout's construction we call the skew bezoutian. If q is reciprocal, p is anti-reciprocal and $(p, q) = 1$ the skew bezoutian construction produces an explicit non-degenerate quadratic form together with isometries A and B with respective characteristic polynomial p and q. These isometries generate a hypergeometric group in the sense of Beukers–Heckman. In the case where k is the field of rational numbers, the quadratic form is hyperbolic, and p, q are products of cyclotomics, recent work of Fuchs–Meiri–Sarnak shows that certain families of hypergeometric groups are examples of infinite families of thin monodromy groups (i.e. groups that are of infinite index in the group of integral points of their Zariski closure). This is joint work with F. Rodriguez-Villegas.

J. Klüners

TBA

D. Koukoulopoulos

On the distribution of the maximum of character sums

Given a Dirichlet character χ modulo q, we set $M(\chi) = \max_{x\geq 1} |\sum_{n\leq x} \chi(n)|$. Pólya and Vinogradov showed that $M(\chi) \ll \sqrt{q}\log q$, and Montgomery and Vaughan improved this estimate to $M(\chi) \ll \sqrt{q}\log\log q$ under the assumption of the Generalized Riemann Hypothesis. On the other hand, Paley showed that this last upper bound is attained for infinitely many characters χ. However, such extremal examples are believed to be rare. In order to study the frequency with which large values of $M(\chi)$ occur, we write $P(\alpha)$ for the proportion of characters modulo q for which $M(\chi) > \alpha\sqrt{q}$, where α is a number between 1 and $\log\log q$. We will present new results on $P(\alpha)$ that improve upon previous results of Montgomery and Vaughan and of Bober and Goldmakher. This is joint work with J. Bober, L. Goldmakher and A. Granville.

K. Matomäki

A short interval sieving problem with applications

For a positive constant θ and a subset $\mathcal{P}$ of the primes, I shall discuss lower bounds for the quantity $\mathcal{A}(x) = |\{n \in [x, x+x^\theta] : p \mid n \implies p \in \mathcal{P}\}|$ as well as for some closely related quantities. In the most interesting case when θ is small, I will show good lower bounds for $\mathcal{A}(x)$ for almost all x or for a certain proportion of x, depending on what one knows about the set $\mathcal{P}$.

A large part of the talk is devoted to applications of the sieving results ; among other things they imply that, for any $\varepsilon > 0$, the sequence $(\lambda_f(n))_{n\leq x}$ of Hecke eigenvalues of a holomorphic Hecke cusp form changes sign at least $x^{1-\varepsilon}$ times.

L. Matthiesen

Norm forms as products of linear polynomials

Using methods from additive combinatorics we show that the Hasse principle and weak approximation hold for certain varieties defined by systems of linear equations involving norm forms. This allows us to show that the Brauer-Manin obstruction controls weak approximation on varieties of the shape $N_K(x_1, ..., x_n) = P(t)$, where $P(t)$ is a product of linear polynomials all defined over $\mathbb{Q}$ and $K/\mathbb{Q}$ is an arbitrary extension of degree n. This is joint work with Tim Browning.

A. Perelli

Twists and resonance of L-functions.

This is a survey of recent work joint with J. Kaczorowski about the analytic properties of certain nonlinear twists of a rather general class of L-functions. Applications to the resonance problem are also discussed.

L. Pierce

On a conjecture of Serre

A conjecture of Serre concerns the number of rational points of bounded height on a finite cover of projective space. This talk will describe joint work with Roger Heath-Brown that verifies Serre's conjecture in the case of smooth cyclic covers, and even improves on it in sufficiently high dimensions. The key is to prove an upper bound for the number of perfect power values of a polynomial, via a combination of the power sieve, the q-analogue of van der Corput's method, and a version of Poisson summation with arithmetic restrictions.

G. Ricotta

Fourier coefficients of $GL(3)$ automorphic forms in arithmetic progressions

We prove that the Fourier coefficients of $GL(3)$ Hecke-Maass cusp forms in arithmetic progression modulo a prime number p have a Gaussian limit distribution as p goes to infinity. This is a joint work with Emmanuel Kowalski.

E. Royer

Star products on quasimodular forms.

Rankin-Cohen brackets have been proved (Cohen-Manin and Zagier, Yao) to give the algebra of modular forms a formal deformation (Eholzer product). In a joint work with François Dumas, we construct formal deformations for quasimodular forms after having built and classified all the Poisson structures on their algebra.

T. Wooley

Strongly diagonal behaviour via efficient congruencing

An even moment of a Weyl sum is said to exhibit strongly diagonal behaviour if its magnitude misses the diagonal (square-root) barrier by an epsilon. In recent work joint with Kevin Ford we vastly extend the range of moments for which one is able to obtain strongly diagonal behaviour in the context of Vinogradov's mean value theorem. In this talk we will report on the work joint with Ford, and we will also explore what can be said for exponential sums over (potentially) sparse polynomials. The latter conclusions follow by means of a generalisation of the efficient congruencing method recently developed by the speaker which is applicable to non-translation-dilation invariant systems.

J. Wu

Un problème de Linnik pour les formes modulaires

Dans cet exposé, nous présenterons quelques résultats récents concernant un problème de Linnik pour les formes modulaires, obtenus en collaboration avec E. Kowalski, Y.-K. Lau, J.-Y. Liu, K. Soundararajan.