Workshop on random trees and applications

Program and abstracts

CIRM, Luminy, Sept. 3-7, 2012

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
8.45 - 9.35	*Noy*	*Pfaffelhuber*	*Van der Hofstad*	*Goldschmidt*	*Le Gall*
9.35 - 10.25	*Miermont*	*Neininger*	*Guitter*	*Kortchemski*	*Berestycki*
10.25 - 10.50	**Coffee**	**Coffee**	**Coffee**	**Coffee**	**Coffee**
10.50 - 11.40	*Chassaing*	*Schweinsberg*	*Marckert*	*Delmas*	*Winkel*
11.40 - 12.30	*Mercier*	*Labbé*	*Curien*	*He*	*Bettinelli*
12.30 - 14	**Lunch**	**Lunch**	**Lunch**	**Lunch**	**Lunch**
14 - 14.50					*Li*
14.50 - 15.40					*Abraham*
15.40 - 16.30					*Hoscheit*
16.30 - 17	**Coffee**	**Coffee**		**Coffee**	
17 - 17.50	*Wakolbinger*	*Winter*		*Haas*	
17.50 - 18.40	*Grübel*	*Duquesne*		*Richard*	
18.40 - 19.30	*Broutin*	*Duhalde*		*Hénard*	

Abstracts of the talks (by alphabetic order)

Romain Abraham

β-coalescent in the pruned CRT

Considering a random binary tree with n randomly labelled leaves, we use a pruning procedure on this tree in order to construct a beta-coalescent process. We also use the continuous analogue of this construction, i.e. a pruning procedure on Aldous's continuum random tree, to construct a continuous state space process that has the same structure as the beta-coalescent process up to some time change. These two constructions enable us to obtain results on the coalescent process such as the asymptotics on the number of coalescent events or the law of the blocks involved in the last coalescent event.

Julien Berestycki

Branching Brownian motion with absorption : the critical case

Consider branching Brownian motion on the real line with absorption at zero, and where particles move according to independent Brownian motions with a critical drift to the origin. Kesten showed in 1978 that almost surely this process eventually dies out. In this talk, I will present new precise asymptotic results concerning the extinction time and the configuration of particles before extinction, as the position of the initial particle tends to infinity. This leads us to estimates on the probability that the process survives for an unusually long time. This improves considerably upon results of

Kesten (1978), and partially confirms nonrigorous predictions of Derrida and Simon (2007). This is joint work with N. Berestycki and J. Schweinsberg.

Jérémie Bettinelli

Increasing Forests and Quadrangulations via a Bijective Approach

In this talk, I will present bijections on planar maps allowing to increase some parameters. In particular, this allows to add a tree or an edge to a forest uniformly chosen among forest with a prescribed number of trees and edges, as well as to add a face to a uniform quadrangulation. We may also recover Tutte's "slicing formula" counting planar maps with faces having prescribed degrees, two at most being odd. The overall strategy consists in finding a path in the initial map, cutting along this path and gluing back after a shift. This modifies slightly the structure of the map and allows to tune the desired parameters.

Nicolas Broutin

The dual tree of a random lamination of the disk

In the recursive lamination of the disk, one tries to add chords one after an other at random by choosing two uniform extremities on the circle; a chord is kept and inserted if it does not intersect any of the previously inserted ones. Curien and Le Gall have proved that the set of chords converges in distribution to a limit triangulation of the disk encoded by a continuous process $\mathscr{M}$. Based on a new approach resembling ideas from the so-called contraction method, we prove that, when properly rescaled, the planar dual of the discrete lamination converges almost surely in the Gromov–Hausdorff sense to a limit real tree $\mathscr{T}$, which is encoded by $\mathscr{M}$. The limit $\mathscr{T}$ is a new object not coming from the excursion of a Lévy process, in particular its fractal dimension is lower than two.

Joint work with Henning Sulzbach (McGill).

Philippe Chassaing

The height of the Lyndon tree

We shall discuss the relation between the Lyndon tree, the binary search tree and the Yule process. As a consequence we give the first order asymptotic for the height of the Lyndon tree.

This is joint work with Lucas Mercier.

Nicolas Curien

Iterating Brownian motions, ad libitum

Let $B_1, B_2, \ldots$ be independent one-dimensional Brownian motions parametrized by the whole real line such that $B_i(0) = 0$ for every $i \geq 1$. We consider the nth iterated Brownian motion

$$W_n(t) = B_n(B_{n-1}(\cdots(B_2(B_1(t)))\cdots)).$$

Although the sequence of processes $(W_n)_{n\geq 1}$ does not converge in a functional sense, we prove that the finite-dimensional marginals converge. As a consequence, we deduce that the random occupation measures of W_n converge towards a random probability measure μ_∞. We then prove that μ_∞ almost surely has a continuous density which should be thought of as the local time process of the infinite iteration W_∞ of independent Brownian motions.

Joint work with T. Konstantopoulos

Jean-François Delmas

The forest associated with the record process on a Lévy tree

We perform a pruning procedure on a Lévy tree and instead of throwing away the removed sub-tree, we regraft it on a given branch (not related to the Lévy tree). We prove that the tree constructed by regrafting is distributed as the original Lévy tree, generalizing a result from Adarrio-Berry, Broutin and Holmgren where only Aldous's tree is considered. As a consequence, we obtain that the quantity which represents in some sense the number of cuts needed to isolate the root of the tree, is distributed as the height of a leaf picked at random in the Lévy tree.

Xan Duhalde

The packing measure of the range of Super Brownian Motion

We consider a Super Brownian Motion with (rather) general branching mechanism ψ. We show that the total occupation measure of the SBM is a packing measure restricted to the range, for the gauge function.

$$g(r) = \frac{\log\log\frac{1}{r}}{\varphi^{-1}\left((\frac{1}{r}\log\log\frac{1}{r})^2\right)}$$

where $\varphi = \psi' \circ \psi^{-1}$. This work generalizes a result proved by Thomas Duquesne for the Dawson Watanabe process. The proof of our theorem uses the Lévy Snake introduced by Duquesne and Le Gall. The main step of the proof will be the study of the lower tail of the mass for a typical ball in the range of the snake.

This is joint work with Thomas Duquesne (LPMA).

Thomas Duquesne

Exceptionally smalls balls in stable trees

The γ-stable trees are random measured compact metric spaces that appear as the scaling limit of Galton-Watson trees whose offspring distribution lies in a γ-stable domain, $\gamma \in (1, 2]$. They form a specific class of Lévy trees (introduced by Le Gall and Le Jan in 1998) and the Brownian case $\gamma = 2$ corresponds to Aldous Continuum Random Tree (CRT). In this paper, we study fine properties of the mass measure, that is the natural measure on γ-stable trees: the mass measure appears as an exact packing measure and in the Brownian case as an exact Hausdorff measure. We first discuss the minimum of the mass measure of balls with radius r and we show that this quantity is of order $r^{\frac{\gamma}{\gamma-1}}(\log 1/r)^{-\frac{1}{\gamma-1}}$. We think that no similar result holds true for the maximum of the mass measure of balls with radius r, except in the Brownian case: when $\gamma = 2$, we prove that this quantity is of order $r^2 \log 1/r$.

Joint work with Guanying Wang.

Christina Goldschmidt

Behaviour near the extinction time in self-similar fragmentation chains

Suppose we have a collection of blocks, which gradually split apart as time goes on. Each block waits an exponential amount of time with parameter given by its size to some power alpha, independently of the other blocks. Every block then splits randomly, but according to the same distribution. In this talk, I will focus on the case where alpha is negative, which means that smaller blocks split faster than larger ones. This gives rise to the phenomenon of loss of mass, whereby the smaller blocks split faster and faster until they are reduced to “dust”. Indeed, it turns out that the whole state is reduced to dust in a finite time, almost surely (we call this the extinction time). A natural question is then: how do the block sizes behave as the process approaches its extinction time? The answer turns out to involve a somewhat unusual "spine" decomposition for the fragmentation, and Markov renewal theory.

This is joint work with Bénédicte Haas (Paris-Dauphine).

Rudolf Grübel

Search trees: metric aspects and strong limit theorems

We consider random binary trees that appear as the output of certain standard algorithms for sorting and searching if the input is random. We introduce the subtree size metric on search trees and show that the resulting metric spaces converge with probability 1. This is then used to obtain almost sure convergence for various tree functionals, together with representations of the respective limit random variables as functions of the limit tree.

Emmanuel Guitter

Loop models on random maps via nested loops: application to the O(n) and Potts models

I will consider a large class of loop models on random maps, incorporating both loop bending-energy and domain-symmetry-breaking weights. The standard O(n) and Potts models are particular examples in this class. I will explain how to use a bijective decomposition of the configurations at hand to transform these models into much simpler models of random maps with controlled face degrees. This connection translates into a set of functional equations for the "resolvents" of these models, which may be solved by generalizing standard analytic techniques. Very explicit results are obtained for the location, in the phase diagram of these models, of their so-called non-generic critical variety.

This talk is based on some joint work with G. Borot and J. Bouttier.

Bénédicte Haas

The stable trees are nested

We will see that we can construct simultaneously all the stable trees as a nested family. More precisely if $1 < \beta < \beta' \leq 2$ we prove that hidden inside any β-stable tree we can find a version of a β'-stable tree rescaled by an independent Mittag-Leffler type distribution. This tree can be explicitly constructed by a pruning procedure of the underlying stable tree or by a modification of the fragmentation associated with it. Our proofs are based on a recursive construction due to Marchal which is proved to converge almost surely towards a stable tree.

Based on a joint work with Nicolas Curien.

Hui He

Pruning of CRT-subtrees

By combining pruning procedures on Lévy CRTs developed in Duquesne and Winkel (2007) and Abraham and Delmas (2012), we obtain Galton-Watson real tree-valued processes (subtree processes) from Lévy CRTs. By studying the subtree processes we give an alternative proof of the convergence of subtrees to the Lévy tree in Duquesne and Winkel (2007) and show that the law of limit tree satisfies the Girsanov transformation for Lévy CRTs. Several properties of the subtree processes are also given.

This is a joint work with R. Abraham and J.-F. Delmas.

Olivier Hénard

The excursions of the Q-process

We confine a regenerative process either in the real time scale either in the local time scale, after Knight [Brownian local times and taboo processes, Trans. Amer. Math. Soc., 1969]. This gives in general two different answers, and we give a construction relating the corresponding paths. Our study is based on the lengths of the excursions from a fixed point, which shows an interesting connection with a subordinator conditioned to reach a high level at a random time.

Joint work with Stephan Gufler, Goethe-Universität, Frankfurt.

Patrick Hoscheit

A central limit theorem for the convergence of separation times in Brownian trees

In this talk, we will be looking at the Aldous-Pitman fragmentation of the Brownian CRT. In particular, we will consider the time of separation from the root, when averaged over the leaves of the tree. Much is known about this random variable, for instance its annealed distribution (when averaging over Brownian trees, it is Rayleigh distributed) as well as its quenched moments. We will show how this random variable is related to record processes on the CRT, as well as to cutting problems on discrete trees, following the work of Abraham and Delmas (arXiv:1107.3657) and Bertoin and Miermont (arXiv:1201.4081). In particular, the average separation time can be obtained as an almost sure limit of the rescaled number of cuts required to isolate the root in a random discrete subtree of the CRT. We will be interested in the scale of the fluctuations around this a.s. limit. In particular, we obtained an (annealed) Central Limit Theorem describing these fluctuations. If time permits, we shall go into a summary of the methods involved.

Igor Kortchemski

Galton-Watson trees conditioned to be large

The behavior of Galton-Watson trees whose offspring distribution is critical and has finite variance, conditioned on having a fixed large size, has drawn a lot of attention. We will be interested in what happens outside of this typical framework. More precisely, what can be said if the conditioning on having a fixed size is replaced by the conditioning on having a fixed number of leaves? What happens if the offspring distribution is not critical?

Cyril Labbé

The Eve property and the genealogy of continuous-state branching processes

Consider a population whose size varies according to a continutous-state branching process (CSBP). We will introduce the Eve property that asserts the existence of an individual, called the Eve, from which descends asymptotically all the individuals when t gets close to the lifetime of the CSBP. We will see that this property does not hold for all CSBPs, but when it does, it has many consequences. In particular, it allows to define a coupling between the flow of subordinators (introduced by Bertoin and Le Gall) and the lookdown representation (Donnelly and Kurtz).

Jean-François Le Gall

The Brownian map: A universal limit for large random planar maps

Planar maps are graphs embedded in the plane, considered up to continuous deformation. They have been studied extensively in combinatorics, and they have also significant geometrical applications. Particular cases of planar maps are p-angulations, where each face (meaning each component of the complement of edges) has exactly p edges in its boundary. Random planar maps have been used in theoretical physics, where they serve as models of random geometry. Our goal is to discuss the convergence in distribution of rescaled random planar maps viewed as random metric spaces. More precisely, we consider a random planar map $M(n)$ which is uniformly distributed over the set of all p-angulations with n vertices. We equip the set of vertices of $M(n)$ with the graph distance rescaled by the factor n to the power -1/4. Both in the case $p = 3$ and when $p > 3$ is even, we prove that the resulting random metric spaces converge as n tends to infinity to a universal object called the Brownian map (the case $p = 4$ has been obtained independently by Miermont). This convergence holds in the sense of the Gromov-Hausdorff distance between compact metric spaces. In the particular case of triangulations ($p = 3$), this result solves an open problem stated by Schramm in his 2006 ICM paper. As a key tool, we use bijections between planar maps and various classes of labeled trees.

Zenghu Li

Tree-valued processes and path-valued processes

A family of continuous-state branching processes are constructed as the solution flow of a jump-type stochastic equation system driven by time-space noises. The family can be regarded as an inhomogeneous increasing path-valued branching Markov process. Two nonlocal branching immigration superprocesses can be defined from the flow. We identify explicitly the branching mechanisms of those processes. The models give different formulations of the tree-valued Markov processes of Aldous and Pitman (1998) and Abraham and Delmas (2010).

Jean-François Marckert

Compact convexes of the plane and probability theory

We revisit the connections between compact convexes of the plane and probability measures. The starting point is a bijection accredited to Gauss-Minkowski, between the set of probability measures μ on $[0, 2\pi]$ such that $\int_0^{2\pi} e^{ix} d\mu(x) = 0$ and compact convexes of the plane with length 1. We show that some natural operations on convexes − for example, the Minkowski sum − have natural translations in terms of operations on probability measures. Further applications are provided, as a new notion of convolution of convexes, and the proof that a polygonal curve associated with a sample of n random variables (satisfying $\int_0^{2\pi} e^{ix} d\mu(x) = 0$) converges to a convex associated with μ at speed $\sqrt{n}$, result much similar to the convergence of empirical process in statistics. In the end, we present some models of smooth random convexes and simulations.

This is a common work with David Renault (LaBRI).

Lucas Mercier

Dynamic Erdös-Rényi graph with forbidden degree and local mixing time

Initially, the process is an empty graph with n vertices. At each step, an edge chosen uniformly at random is added, and then, the edges adjacent to the vertices of degree k are removed. The question is whether a giant component appears (depending on the value of k).

In this process appears local limit results, and also stationnary measure for the Markov chain, therefore allows us to look at a notion of local mixing time.

Grégory Miermont

A panorama on random maps

The goal of this talk is to survey the recent research on random maps and their scaling limits. In passing, we will discuss some lines for the future research in this topic.

Ralph Neininger

A functional contraction method and random trees

The contraction method is a proof methodology to derive convergence in distribution for sequences of random variables with distributions that satisfy appropriate recurrences. The method originated from the probabilistic analysis of algorithms. It has been developed during the last 20 years for sequences in the real numbers, random vectors as well as random sequences in separable Hilbert spaces. We discuss an extension to Banach spaces with main focus on the function spaces C[0,1] and D[0,1] of continuous respectively cadlag functions. A crucial part is to derive implications of convergence in the Zolotarev metric for distributions on these spaces.

As an application a functional limit theorem is presented which captures cost measures of partial match queries in random quadtrees on a fine scale. Partial match is a fundamental operation on multivariate data; seminal analysis has been done, starting in the 1980s, by Philippe Flajolet and coauthors. Our functional limit theorem settles a couple of questions that have been left open in earlier studies.

This talk is based on joint work with Henning Sulzbach, see arXiv:1202.1370, and joint work with Nicolas Broutin and Henning Sulzbach, see arXiv:1202.1342.

Marc Noy

On the diameter of random planar graphs

We show that the diameter $\mathrm{diam}(G_n)$ of a random labelled connected planar graph with n vertices is asymptotically almost surely of order $n^{1/4}$, in the sense that there exists a constant $c > 0$ such that

$$P(\mathrm{diam}(G_n) \in (n^{1/4-\varepsilon}, n^{1/4+\varepsilon})) \geq 1 - \varepsilon(-n^{c\varepsilon})$$

for ε small enough and $n \geq n_0(\varepsilon)$. We prove similar statements for 2-connected and 3-connected planar graphs and maps.

The starting point in our research is the pioneering work by Chassaing and Schaeffer (2004) on the radius $r(Q_n)$ of random quadrangulations with n vertices, where they show that $r(Q_n)$ rescaled by $n^{1/4}$ converges as $n \to \infty$ to an explicit continuous distribution related to the Brownian snake.

This is joint work with Guillaume Chapuy, Éric Fusy and Omer Giménez.

Peter Pfaffelhuber

Path-properties of the tree-valued Fleming-Viot process

The Fleming-Viot superprocess arises in mathematical population genetics by the evolution of type frequencies in a large-population limit. Recently, a tree-valued counterpart of this process was constructed where the tree encodes genealogical relationships of individuals within the population

(Greven, Pfaffelhuber, Winter. Tree-valued resampling dynamics. Martingale problems and applications, [Arxiv math.PR/0806.2224], in press, Prob. Theo. Rel. Fields, 2012; Depperschmidt, Greven, Pfaffelhuber. Tree-valued Fleming-Viot dynamics with mutation and selection, [Arxiv math.PR/1101.0759], in press, Ann. Appl. Probab., 2012). It is ergodic and has the Kingman coalescent as a stationary state. In this talk, we will examine path-properties of this process. Most importantly, we will lift known results about the almost sure behavior of the Kingman coalescent to almost sure behavior along paths of the tree-valued Fleming-Viot process.

This is joint work with Andrej Depperschmidt and Andreas Greven.

Mathieu Richard

Lévy process conditioned on having a large height process

We consider a spectrally positive Lévy process X (*i.e.* without negative jumps) that starts from $x > 0$ and that does not drift to $+\infty$. We first give a new definition of a Lévy process conditioned to stay positive: the process X is conditioned to reach arbitrarily high heights in the sense of its associated *height process* (cf. Duquesne & Le Gall 02). The law $P_x^\star$ of the conditioned process is defined via a h-transform using the *exploration process* of Duquesne and Le Gall. When X has finite variation paths, *i.e.* is a compensated compound Poisson process, the study is simple because the process can be viewed as the contour process of a random chronological tree called *splitting tree* (cf. Geiger&Kersting 97) and then to condition X to reach arbitrarily high heights is equivalent to condition the tree to reach an arbitrarily high number of generations. Second, we will state some properties of X under $P_x^\star$ (convergence, path decomposition, ...) and in the finite variation case, give the link with conditioned splitting trees.

Jason Schweinsberg

A tree-valued process associated with the Bolthausen-Sznitman coalescent

Consider a population of fixed size that evolves over time. At each time, the genealogical structure of the population can be described by a coalescent tree whose branches are traced back to the most recent common ancestor of the population. As time goes forward, the genealogy of the population evolves, leading to a tree-valued process known as an evolving coalescent. We will study the evolving coalescent for populations whose genealogy can be described by the Bolthausen-Sznitman coalescent. We obtain the limiting behavior of the evolution of the time back to the most recent common ancestor and the total length of the branches in the tree. By similar methods, we also obtain a new result concerning the number of blocks in the Bolthausen-Sznitman coalescent.

Remco van der Hofstad

First passage percolation on random graphs

We investigate shortest-weight problems on the configuration model, in which flow passes through the network minimizing the total weight along edges. In practice, one is both interested in the actual weight of the minimal weight path, which represents its cost, as well as the number of edges used or hopcount, as this is often a good measure of the delay observed in the network.

We assume that the edge weights are independent random variables, leading to first passage percolation on the configuration model. We then investigate the total weight and hopcount of the minimal weight path. We study how the minimal weight and hopcount depend on the structure of the edge weights as well as on the structure of the graph. Our proofs crucially rely on the connection between first passage percolation and continuous-time branching processes, which is due to the tree-like nature of the configuration model.

The above research is inspired by transport in real-world networks, such as the Internet. Measurements have shown fascinating features of the Internet, such as the 'small world phenomenon'. The small-world phenomenon states that typical distances in the network under consideration is small. Also, the degrees in the Internet are rather different from the degree structure in classical random graphs. Internet is a key example of a complex network, other examples being the Internet Movie Data Base, social networks, biological networks, the WWW, etc.

[This is joint work with Gerard Hooghiemstra, Shankar Bhamidi, Piet Van Mieghem, Henri van den Esker and Dmitri Znamenski.]

Anton Wakolbinger

Trickle-down processes and their boundaries

"What is the Doob-Martin boundary of the binary search tree-process?" We will explain and answer this question, and put it into a more general framework: initially, all vertices of a directed, acyclic graph are unoccupied, particles are fed in one-by-one at a distinguished source vertex, successive particles proceed along directed edges according to an appropriate stochastic mechanism, and each particle comes to rest once it encounters an unoccupied vertex. Examples include the binary and digital search tree processes, random recursive tree process, and Mallows tree processes. For these processes, we analyse and characterize the Doob-Martin boundary.

This is joint work with Steve Evans and Rudolf Gruebel (EJP vol. 17, 1-58, (2012)).

Matthias Winkel

Hereditary properties, Galton-Watson real trees and Lévy trees

Neveu studied leaf-length erasure of Galton-Watson trees, Geiger and Kauffmann the subtree spanned by vertices picked uniformly at random and Duquesne and Winkel the subtree spanned by leaves picked uniformly at random, each finding that the reduced tree is also a Galton-Watson tree. We observe that the offspring distributions that occur in the three examples are the same, and we introduce the notion of a hereditary property to offer a unified approach. The notion of leaf-length erasure has recently been exploited by Evans, Winter and co-authors in a context of real trees. We continue these developments and use results about hereditary properties to obtain strong convergence results of Galton-Watson real trees to Levy trees and characterisations and properties of the limits. We also have an invariance principle for Galton-Watson trees and decomposition results for Galton-Watson and Levy trees.

This is joint work with Thomas Duquesne.

Anita Winter

Evolving genealogies of catalytic interacting branching diffusions

We construct and study the evolution of genealogies for a spatial catalyst and reactant branching population. In this model particles branch critically and migrate on finite geographic space $\{1, ..., N\}$. We also consider the process arising as the McKean Vlasov limit ($N \to \infty$) which consists of a collection of independent catalytic branching processes with immigration and emigration.

We show existence of a genealogical diffusion limit of the particle model as well as of the McKean Vlasov process, and characterize the limit dynamics by a wellposed martingale problem in the space of marked ultra-metric (finite) measure spaces. This requires new techniques due to singularities in the reactant branching and migration rates reflecting the fact that the catalyst recurrently hits zero.

We show that the process is ergodic, and establish the mean field finite system scheme on the level of catalyst and reactant genealogies.

(This is joint work with Andreas Greven and Lea Popovic.)