Singular Traces and their Applications

ABSTRACTS

Battisti Ubertino: Weyls law on SG-calculus and on bisingular calculus

We describe the asymptotic expansion of the eigenvalue counting function of self-adjoint positive elliptic SG-operators, defined first on $\mathbb{R}^n$, and then on manifolds with cylindrical ends. Moreover, we show the relationship with the Wodzicki Residue. The model example of an elliptic SG-operator on $\mathbb{R}^n$ is $(1+|x|^2)(1-\Delta)$. Then, we consider bisingular operators, for example $P_1 \otimes P_2$, where P_1 and P_2 are pseudodifferential operators on the closed manifolds M_1 and M_2. We study the counting function of suitable elliptic operators in this class and we show an application to Dirichlet divisor problem. In both settings, in a particular case, we can determine not only the principal term but also the second term of the asymptotic expansion of the counting function. We finish with a slight modification of bisingular operator, that is, bisingular operators adapted to Shubin global calculus. We show that in this case the application to Dirichlet divisor problem becomes more transparent. These results are obtained in collaboration with S. Coriasco, T. Gramchev, S. Pilipović and L. Rodino.

Chacraborty Partha: Instances of equivalence of two approaches to Yang-Mills in NCG

There are two approaches to the Yang-Mills action functional in NCG. One due to Connes and Rieffel and the other due to Connes. We will show that suitably interpreted both these approaches coincide both in the cases of non-commutative torus and the quantum Heisenberg manifolds.

Englis Miroslav: Hankel operators and the Dixmier trace

We discuss the membership of (big) Hankel operators H_f on weighted Bergman spaces in the Dixmier class. For the unit disc and f holomorphic, $H_{\overline{f}}$ is in this class if and only if f' belongs to the Hardy one-space H^1. On the unit ball in $\mathbb{C}^n$, $n \geq 2$, there is an analogous result for any f smooth on the closed ball (not necessarily conjugate-holomorphic), which is reminiscent of the trace formula of Helton and Howe. The last result extends also to arbitrary smoothly bounded strictly pseudoconvex domains in $\mathbb{C}^n$, where the formula for the Dixmier trace turns out to involve the Levi form, thus exhibiting an interesting link with the geometry of the domain. The proofs involve analysis of pseudodifferential operators on the boundary of the domains. The case of the disc is joint work with Richard Rochberg, while the case of the ball and of pseudoconvex domains are joint with Kunyu Guo and Genkai Zhang.

Fathizadeh Farzad: Scalar curvature for the noncommutative two torus

I will give a local expression for the scalar curvature of the noncommutative two torus A_θ equipped with an arbitrary translation invariant complex structure and Weyl conformal factor. The metric information is encoded in the Dirac operator D of a spectral triple on this C^*-algebra. The local expression for the curvature is computed by evaluating the value of the (analytic continuation of the) spectral zeta function

$$\zeta_a(s) := \mathrm{Trace}\,(a|D|^{-s})$$

at s = 0 as a linear functional in $a \in A_\theta^\infty$. A new, purely noncommutative, feature is the appearance of the modular automorphism group from the theory of type III factors and quantum statistical mechanics in the final formula for the curvature. This formula coincides with the formula that was recently obtained independently by Connes and Moscovici in their recent paper. At the end, I will explain how this formula fits into our earlier work on the Gauss-Bonnet theorem for noncommutative two tori, which extends the Gauss-Bonnet theorem of Connes and Tretkoff to general conformal structures on A_θ. This is joint work with Masoud Khalkhali.

Guido Daniele: Spectral triples for the Sierpinski gasket (II)

We start by constructing a spectral triple on a quasi-circle and compute its pairing with K-theory. We then construct a two-parameter family of spectral triples for the Sierpinski gasket, and show that it gives back the metric, the integral w.r.t. Hausdorff measure, the pairing with K-theory, and the Kigami energy.

Ionescu Marius: Pseudo-differential operators on fractals

We define and study pseudo-differential operators on a class of fractals that include the post-critically finite self-similar sets and Sierpinski carpets. Using the sub-Gaussian estimates of the heat operator we prove that our operators have kernels that decay and, in the constant coefficient case, are smooth off the diagonal. Our analysis can be extended to product of fractals. While our results are applicable to a larger class of metric measure spaces with Laplacian, we use them to study elliptic, hypoelliptic, and quasi-elliptic operators on p.c.f. fractals, answering a few open questions posed in a series of recent papers. We extend our class of operators to include the so called Hörmander hypoelliptic operators and we initiate the study of wavefront sets and microlocal analysis on p.c.f. fractals. This talk is based on joint work with Robert Strichartz and Luke Rogers.

Isola Tommaso: Spectral triples for the Sierpinski gasket (I)

We start by constructing a spectral triple on a quasi-circle and compute its pairing with K-theory. We then construct a two-parameter family of spectral triples for the Sierpinski gasket, and show that it gives back the metric, the integral w.r.t. Hausdorff measure, the pairing with K-theory, and the Kigami energy.

Khalkhali Masoud: TBA

Kellendonk Johannes: Spectral triples from stationary Bratteli diagrams and substitution tilings

When Alain Connes spoke about Penrose tilings in his book, he spoke about their non-commutative topology, but not about their non-commutative geometry in the sense of the generalisation of Riemannian geometry given by spectral triples. Indeed, it seems difficult to define a spectral triple for the full tiling algebra, but recently progress has been made in describing spectral triples for a maximal abelian sub-algebra of it, that is, the algebra of continuous functions over the discrete tiling space. However this does not encode the dynamics defined by the translation action on the tiling. We are thus interested in spectral triples on the mapping torus of that action, which is also the continuous tiling space. To keep matters simple we consider a special class of tilings, called substitution tilings, which enjoy a strong form of selfsimilarity and can be described by stationary Bratteli diagrams (as did Alain Connes for the Penrose tilings). We are interested in particular in the zeta-function and the Laplace-operator defined by such a spectral triple. The latter is expected to be the generator of a Markov semi-group of an atomic diffusion mechanism in a quasi-crystal described by the tiling. It turns out that the question after the correct domain of the Laplacian is important and we will use dynamical information to specify this domain.

Levy Cyril: SPECTRAL TRIPLES AND MANIFOLDS WITH BOUNDARY

In this talk, we will see how one can construct spectral triples adapted to manifolds with boundary through boundary pseudodiffferential calculus (Boutet de Monvel calculus). The corresponding Chamseddine-Connes spectral action on manifolds with boundary (with or without torsion) will also be discussed.

Lord Steven: CONNES' TRACE THEOREM IS TRUE FOR ANY TRACE

We show in this talk that, if P is a classical pseudo-differential operator of order $-d$ and $\mathrm{Res}_W(P)$ is its Wodzicki residue, then

$$\tau(P) = \frac{1}{d(2\pi)^d}\mathrm{Res}_W(P)$$

for every trace on $\mathcal{L}_{1,\infty}$ such that $\tau\left(\mathrm{diag}\{n^{-1}\}\right) = 1$. Thus Connes' Trace Theorem is true for every trace not just the Dixmier traces. This unique trace property for classical operators is a much stronger result than the original. The technique of the proof is due to Kalton and it is very different from the original proof of Connes' Trace Theorem. It involves the commutator subspace of $\mathcal{L}_{1,\infty}$ and a new class of operators introduced by Kalton which extend pseudo-differential operators and have a residue map that extends the Wodzicki residue. The machinery results in a general trace theorem, which we describe, that has Connes' Trace Theorem as a corollary. We indicate two other corollaries that are of interest. First, if M_f is the multiplication operator for a square integrable function f and Δ is the Laplace-Beltrami operator, then

$$\tau(M_f(1-\Delta)^{-d/2}) = \frac{\mathrm{Vol}\,\mathbb{S}^{d-1}}{d(2\pi)^d}\int f(t)dt$$

for every trace on $\mathcal{L}_{1,\infty}$ such that $\tau\left(\mathrm{diag}\{n^{-1}\}\right) = 1$. Thus integration is recovered by any trace. Second, there is a (non-classical) pseudo-differential operator Q where

$$\mathrm{Tr}_\omega(Q) = \omega(\sin(\log(\log n^{1/d})))$$

and hence the value $\mathrm{Tr}_\omega(Q)$ depends on the generalised limit ω. Thus classical pseudo-differential operators have a totally unique trace while non-classical pseudo-differential operators are generally not measurable in the sense of Connes.

Loreaux Jireh: SUBIDEALS

Techniques developed in the last decade generalize to arbitrary ideals the 1983 Fong-Radjavi determination of which principal ideals of compact operators are also B(H) ideals. This involves generalizing a notion of soft ideals used by Kaftal-Weiss (and studied earlier by Mityagin and Pietsch who used different terminology and in the context of Banach spaces). This generalization leads to a natural and simple characterization of all principal ideals (and all finitely generated ideals) inside an arbitrary B(H) ideal. Modest generalizations of these are presented as evidence of a completely general conjecture.

Mickelsson Jouko: CHARACTERISTIC CLASSES OF CERTAIN INFINITE RANK VECTOR BUNDLES

In many cases in computations in index theory the (families) index can be expressed formally as Chern character of an infinite-rank vector bundle. However, the curvature form takes values in an infinite-dimensional Lie al- gebra consisting of non trace-class operators and therefore the naive Chern character is ill-defined. I will discuss a renormalization of the curvature form based on a method in quantum field theory, previously applied to the computation of hamiltonian anomalies and construction of gauge current algebra in Yang-Mills theory in higher than two space-time dimensions.

Paycha Sylvie: LOCAL TRACES VERSUS GLOBAL TRACES; THE CANONICAL TRACE VERSUS THE NONCOMMUTATIVE RESIDUE

The canonical trace introduced by Kontsevich and Vishik is the unique "tracial" extension of the ordinary trace to non-integer order classical pseudodifferential operators. In contrast to the non-commutative residue, it does not define a trace on the whole algebra of classical pseudodifferential operators and it is non-local. However, it plays an essential role when regularising traces at integer order operators along holomorphic families of classical pseudo-differential operators; in spite of their name regularised traces are not tracial. Their lack of traciality and other discrepancies can be measured in terms of defect formulae. We shall show how the canonical trace and the noncommutative residue compete in defect formulae for regularised traces, the former arising as the global part, the latter as the local part of the regularised trace. We shall discuss a possible extension of the canonical trace to the framework of non-commutative geometry via spectral triples.

Pietsch Albrecht: TRACES ON OPERATOR IDEALS AND RELATED LINEAR FORMS ON SEQUENCE IDEALS

The Calkin theorem provides a one-to-one correspondence between

- all operators ideals $\mathfrak{A}(H)$ over the separable Hilbert space H

and

- all symmetric sequence ideals $\mathfrak{a}(\mathbb{N})$ over $\mathbb{N} := \{1, 2, \dots\}$.

Our main idea is to replace $\mathfrak{a}(\mathbb{N})$ by the ideal $\mathfrak{z}(\mathbb{N}_0)$ that consists of all sequences (α_h) over $\mathbb{N}_0 := \{0, 1, 2, \dots\}$ for which

$$(\alpha_0, \alpha_1, \alpha_1, \dots, \overbrace{\alpha_h, \dots, \alpha_h}^{2^h \text{ terms}}, \dots) \in \mathfrak{a}(\mathbb{N}).$$

This new kind of sequence ideals is characterized by two properties:
(1) For $(\alpha_h) \in \mathfrak{z}(\mathbb{N}_0)$ there exists $(\beta_h) \in \mathfrak{z}(\mathbb{N}_0)$ such that $|\alpha_h| \leq \beta_h$ and $\beta_0 \geq \beta_1 \geq \dots \geq 0$.

(2) $\mathfrak{z}(\mathbb{N}_0)$ is invariant under the shift operator

$$S_+ : (\alpha_0, \alpha_1, \alpha_2, \dots) \mapsto (0, \alpha_0, \alpha_1, \dots).$$

There are canonical isomorphisms between the linear spaces of

- all traces on $\mathfrak{A}(H)$
- all symmetric linear forms on $\mathfrak{a}(\mathbb{N})$,

and

- all $\frac{1}{2}S_+$-invariant linear forms on $\mathfrak{z}(\mathbb{N}_0)$.

In this way, the theory of linear forms on ideals in a *non-commutative* ring that are invariant under the members of a *non-commutative* group is reduced to the theory of linear forms on ideals of a *commutative* ring that are invariant under a *single* operator. Should some time be left, the lecture will conclude with a historical survey.

Potapov Denis: MEASURES FROM DIXMIER TRACES

The talk will discuss the recovery of the Lebesgue integration from the Dixmier traces. In particular, it will show that a claim in the monograph [J.M. Gracia-Bonda, J.C. Vrilly, H. Figueroa, "Elements of Noncommutative Geometry", Birkhuser Adv. Texts, Birkhuser, Boston, 2001], that the equality on C^∞-functions between the Lebesgue integral and an operator-theoretic expression involving a Dixmier trace (obtained from Connes Trace Theorem) can be extended to any integrable function, is false. The talk is based on joint work with S.Lord and F.Sukochev.

Schrohe Elmar: REGULARITY OF THE ETA FUNCTION FOR BOUNDARY VALUE PROBLEMS

On a compact manifold X with boundary we consider the realization $B = P_T$ of an elliptic boundary problem, consisting of a differential operator P of positive order and a differential boundary condition T. We assume that B is parameter-elliptic in small sectors around two rays in the complex plane, say $\arg\lambda = \phi$ and $\arg\lambda = \theta$. Associated to the cuts along the rays one can then define two zeta functions ζ_ϕ and ζ_θ for B. Both extend to meromorphic functions on the plane; the origin is a regular point. In the boundaryless case, a deep result, going back to Atiyah–Patodi–Singer, Gilkey and Wodzicki, shows that $\zeta_\phi(0) = \zeta_\theta(0)$. The question is whether this also holds for boundary value problems. Interest in this problem arose from the following observation: When B is self-adjoint, and one ray lies in the upper half plane and one in the lower, then $\zeta_\phi(0) - \zeta_\theta(0)$ equals – up to a constant – the residue of the eta function at the origin. The fact that the zeta values coincide thus implies the regularity of the eta function. In the boundaryless case this is crucial for the Atiyah-Patodi-Singer index theorem. In the case of boundary value problems progress was made in 2008 by Grubb and Gaarde. Following the approach taken by Wodzicki, they related the difference of the values at the origin to the associated sectorial projection $\Pi_{\theta,\phi}(B)$ defined by

$$\Pi_{\theta,\phi}u = \frac{i}{2\pi}\int_{\Gamma_{\theta,\phi}} \lambda^{-1}B(B-\lambda)^{-1}u\,d\lambda, \quad u \in \mathrm{dom}(B).$$

Here $\Gamma_{\theta,\phi}$ is a contour which runs on the first ray from infinity to $r_0e^{i\phi}$ for some $r_0 > 0$, then clockwise about the origin on the circle of radius r_0 to $r_0e^{i\theta}$ and back to infinity along the second ray. Grubb and Gaarde showed that whenever $\Pi_{\theta,\phi}$ is of order and class zero in Boutet de Monvel's calculus, its noncommutative residue (in the sense of Fedosov-Golse-Leichtnam-Schrohe (FGLS)) coincides with the difference of the zeta functions in zero. Moreover, Gaarde proved that the noncommutative residue then is zero, so that the values of the zeta functions agree. In most cases, however, the sectorial projection will not belong to the Boutet de Monvel algebra, and the above methods are not applicable. We solve the problem by constructing an operator algebra which extends Boutet de Monvel's. It is large enough to contain the sectorial projections of the above boundary value problems, yet its K-theory naturally coincides with that of Boutet de Monvel's algebra. Moreover, we show that it carries a noncommutative residue which extends that of FGLS and vanishes on projections. Hence the values of the zeta functions will coincide and, if B is self-adjoint, its eta function will be regular in zero. (Joint work with H. Gimperlein.)

Sedaev Aleksandr: The notion of Connes' measurable elements and stabilizing subsets in Marcinkiewicz spaces

Let $\psi(t),\ 0 \le t < \infty$, be an increasing concave continuous function, $\psi(0) = 0$, and let $M(\psi)$ designate the corresponding Marcinkiewicz space of measurable functions on $\mathbb{R}_+$. In particular for $\psi(t) = \ln(t+1)$ we have the well known Dixmier space $M_{1,\infty} = M(\ln(1+t))$. If ψ satisfies the condition

$$\lim_{t\to\nu} \frac{\psi(2t)}{\psi(t)} = 1, \tag{1}$$

where $\nu = 0$ or ∞, then the Marcinkiewicz space $M(\psi)$ have sufficiently large set $\mathbf{D}(\nu)$ of the so called singular (fully) symmetric functionals (SSF). That is the cone of positive linear functionals $F \in M(\psi)^*$ such that for every positive $x, y \in M(\psi)$ the condition

$$\int_0^t x^*(s)ds \le \int_0^t y^*(s)ds \text{ for all } t > 0$$

implies $F(x) \le F(y)$ and $F(x) = 0$ for all $x \in M(\psi) \cap L_\mu$ where $\mu = \infty$ if $\nu = 0$ and $\mu = 1$ if $\nu = \infty$. Here and below x^* is the permutation of $|x|$ in the decreasing order. It is well known that such functionals are the commutative counterparts of the Dixmier Singular Traces. In particular every $F \in \mathbf{D}(\nu)$ has the form

$$F(x) = \omega(a(x,t)),\ x \in M(\psi), \tag{2}$$

where

$$a(x,t) = \frac{1}{\psi(t)} \int_0^t (x_+^* - x_-^*)(s)ds \in C_b(\mathbb{R}_+)$$

and ω is a dilation invariant generalized limit at the point ν defined on the space $C_b(\mathbb{R}_+)$ consisting of bounded continues functions. In what follows we restrict our consideration (for simplicity) to the case $\nu = \infty$. If γ is an arbitrary generalized limit at ∞ we can take

$$\omega(f) = \gamma(M(f)), \quad \text{where } M(f)(t) = \frac{1}{\ln(t)} \int_1^t f(s) d\ln s,\ f \in C_b(\mathbb{R}_+), \tag{3}$$

then an SSF F, defined by (2) belongs to the proper subset $\mathbf{CD}(\infty) \subset \mathbf{D}(\infty)$. Such SSF are called Connes-Dixmier functionals (traces). We also consider subsets $\mathbf{H}(\infty), \mathbf{P}(\infty)$ where the operator M in (3) is replaced by

$$H(f)(t) = \frac{1}{t}\int_0^t f(s)ds, \text{ or by } P(f) = \frac{1}{\ln\ln(t+1)} \int_e^{t+1} f(s) d\ln\ln s,\ t > 0,$$

respectively. In the case of the space $M_{1,\infty}$ we have other subsets of $\mathbf{D}(\infty)$ defined by the famous ζ-formula or the heat kernel formula. *Let $S \subset \mathbf{D}(\nu)$. Following A. Connes an element $x \in M(\psi)$ is called S-measurable if $F(x)$ does not depend on the choice of $F \in S$. An element $x \in M(\psi)$ is called tauberian* *or stable at ν iff the limit* $\lim_{t\to\nu} a(x,t)$ *does exist. The set* $M \subset M(\psi)$ *we call* S*-stabilizing if every element* $x \in M$ *is* S*-measurable iff it is tauberian at* ν*.* The problem describing S-measurable elements and S-stabilizing subsets (at least for some sets S) turned out to be interesting and nontrivial. The main objective of the report is to show a partial solution for the problem. Following G.G. Lorentz we describe S-measurable elements in terms of minimal sublinear functional p_S, majorizing all $F \in S$, $\|F\| \leq 1$. We define (and investigate properties of) maximal and nontrivial symmetric subspace $V_\psi(\nu) \subset M(\psi)$ which is $\mathbf{D}(\nu)$-stabilizing. If $\psi = \ln(1+t)$ then $V_\psi(\infty)$ is $\mathbf{H}(\infty)$-stabilizing and is not $\mathbf{P}(\infty)$-stabilizing. It is not clear what do we have for $S = \mathbf{CD}(\infty)$ or for S given by the ζ-function formula.

Semenov Evgeniy: EXTREME POINTS OF THE SET OF BANACH LIMITS

A linear functional $B \in l_\infty^*$ is called a Banach limit if

(i) $B \geq 0$, i. e. $Bx \geq 0$ for $x \geq 0$ and $B1 = 1$.

(ii) $B(Tx) = B(x)$ for all $x \in l_\infty$, where T is a shift operator, i. e.

$$T(x_1, x_2, \ldots) = (x_2, x_3, \ldots).$$

The existence of Banach limits was proven by S. Banach in his book. By Krein-Milman theorem, we have

$$\mathfrak{B} = \overline{conv}\, ext(\mathfrak{B}),$$

where the closure is taken in the $\sigma(l_\infty^*, l_\infty)$-topology.

Theorem 0.1. *Let* $B_k \in ext(\mathfrak{B})$*,* $k \geq 0$*. If* $B_k \neq B_l$ *for every* $k \neq l$*, then*

$$\left\| \sum_{k=0}^{\infty} c_k B_k \right\|_{l_\infty^*} = \sum_{k=0}^{\infty} |c_k|, \tag{0.1}$$

for every $c = (c_k)_{k\geq 0} \in l_1$*.*

The following question arises naturally: does any sequence $(B_i)_{i\geq 1} \subset \mathfrak{B}$ contains a subsequence $(B_{k_i})_{i\geq 1}$ converging in the $\sigma(l_\infty^*, l_\infty)$-topology? Theorem 0.2 below answers this question in negative.

Corollary 0.2. *Let* $(B_i) \subset ext(\mathfrak{B})$*,* $B_i \neq B_j$ *whenever* $i \neq j$*. The sequence* $(B_i)_{i\geq 1}$ *does not contain a* $\sigma(l_\infty^*, l_\infty)$*-weakly converging subsequence.*

Theorem 0.3. *Let* $B_k \in ext(\mathfrak{B})$*,* $k \geq 1$*. For every* $\tilde{B}$ *from the* $\sigma(l_\infty^*, l_\infty)$*-closed convex hull of the set* $\{B_k\}_{k\geq 1}$ *and every Cesáro invariant Banach limit* B *(*$B = BC$ *where* C *is the Cesáro operator) we have* $\|B - \tilde{B}\|_{l_\infty^*} = 2$*. In particular,* B *does not belong to the* $\sigma(l_\infty^*, l_\infty)$*-closed convex hull of the set* $\{B_k\}_{k\geq 1}$*.*

The set $A \subset l_\infty$ is called the set of uniqueness if the fact that two Banach limits B_1 and B_2 coincide on A implies that $B_1 = B_2$. Denote by $2^{\mathbb{N}}$ the set of all sequences such that $x_k = 0$ or 1 only. Let

$$U := \{x : x \in 2^{\mathbb{N}}, q(x) = 0, p(x) = 1\},$$

where

$$q(x) = \lim_{n\to\infty} \inf_{m\in\mathbb{N}} \frac{1}{n} \sum_{k=m+1}^{m+n} x_k, \qquad p(x) = \lim_{n\to\infty} \sup_{m\in\mathbb{N}} \frac{1}{n} \sum_{k=m+1}^{m+n} x_k.$$

Denote by V the set of all sequences $x = (x_1, x_2, ...)$ such that

$$x_i = \begin{cases} 1, \ n_{2k} \le i < n_{2k+1} \\ 0, \ n_{2k-1} \le i < n_{2k} \end{cases}, \quad k \in \mathbb{N},$$

where $n_k \in \mathbb{N}$ and $\lim_{k\to\infty}(n_{k+1} - n_k) = \infty$. It is clear that $V \subset U$.

Theorem 0.4. *The set U is the set of uniqueness.*

The kernel of every Banach limit has a non-empty intersection with the set V. Moreover, the following Theorem holds.

Theorem 0.5. *Let $B_k \in \mathfrak{B}$, $k \in \mathbb{N}$. There exists $x \in V$ such that $B_k x = 0$ for all $k \in \mathbb{N}$.*

Joint work with F. A. Sukochev and A. S.Usachev. This work was supported by RFBR.

Usachev Alexender: Generalized limits with additional invariance properties and their applications

Let l_∞ be the Banach space of all bounded sequences $x = (x_1, x_2, ...)$ with the uniform norm. Let $\mathcal{M}_{1,\infty}$, be an ideal of compact operators on an infinite-dimensional Hilbert space H with logarithmic divergence of the partial sums of their singular values. Originally, J. Dixmier constructed the positive singular trace Tr_ω on $M_{1,\infty}$, using dilation invariant generalized limit ω on l_∞. There are several natural subclasses of Dixmier traces which are useful in noncommutative geometry. Initially, it was observed by A. Connes, that for any generalised limit γ on l_∞ a functional $\omega := \gamma \circ M$ is dilation invariant. Here M is a logarithmic Cesáro operator. The class of all Dixmier traces Tr_ω defined by such ω's is termed Connes-Dixmier traces. Subsequently, various important formulae of noncommutative geometry have been frequently established for yet a smaller subset $\mathcal{D}_M$ of Connes-Dixmier traces, generated by M-invariant generalized limit ω, that is when $\omega = \omega \circ M$. In this talk we show that the three classes of Dixmier traces, Connes-Dixmier traces and $\mathcal{D}_M$

pairwise distinct. For each subclass (say $\mathcal{B}$) of singular traces we consider a set of $\mathcal{B}$-measurable operators, that is on which all traces from B coincide. It is known that for positive operators the sets of Dixmier and Connes-Dixmier measurable elements are coincide (in spite of the fact that classes of traces are different). We also present the fact that there exists a positive $\mathcal{D}_M$-measurable operator, which is not Dixmier measurable. This talk based on the joint work with F. Sukochev and D. Zanin.

Weiss Gary: Brief introductions to: I. An infinite dimensional Schur-Horn theorem and majorization theory II. A codimension conjecture for traces III. History and some recent progress on single commutators

I. A short history and an open question will be given on Schur-Horn theorems connecting majorization to a characterization of diagonals of an operator in arbitrary bases and to stochastic matrices. The following theorem extends to infinite dimensions the classical finite-dimensional Schur-Horn Theorem: For two nonincreasing nonsummable sequences ξ and η that converge to 0, there exists a positive compact operator A with eigenvalue list η and diagonal sequence ξ if and only if $\sum_{j=1}^{n} \xi_j \leq \sum_{j=1}^{n} \eta_j$ for every n if and only if $\xi = Q\eta$ for some orthostochastic matrix Q. When ξ and η are summable, requiring additionally equality of their infinite series obtains the same conclusion. This extends work by Arveson and Kadison, Gohberg and Markus, and others. Our proof depends on the construction and analysis of an infinite product of T-transform matrices and a lim inf generalization of the condition of summability of a positive sequence. Further results on majorization for infinite sequences providing "intermediate" sequences generalize known results from the finite case. Majorization properties and invariance under various classes of stochastic matrices are then used to characterize arithmetic mean closed operator ideals and arithmetic mean at infinity closed operator ideals studied by Dykema, Figiel, W and Wodzicki, and Kaftal and W. Joint work with V. Kaftal.

II. The trace space for a B(H)-ideal I can be thought of as the dual of the quotient of I by its commutator ideal (commutator space), I/[I,B(H)], and via a Hamel basis can be easily associated with this quotient itself. Evidence from numerous cases suggests that the algebraic dimension of this quotient is either 0, 1 or infinity. Some background on this will be discussed. Joint work with V. Kaftal.

III. A brief history of the study of single commutators will be given along with the answer to an open problem and the open question it implies: Which strictly positive compact operators are single commutators of compact operators? Joint with Sasmita Patnaik and related to work of Davidson, Marcoux and Radjavi.

Whittaker Mickael: KMS states for self-similar group actions

A self-similar action (G, X) consists of a group G along with a self-similar action of the group on the tree of finite words in the alphabet X. Heuristically, self-similarity is displayed when the action of the group repeats at all levels of the tree, in a similar fashion to fractals where patterns are repeated at all scales. A self-similar action gives rise to a Hilbert bimodule, discovered by V. Nekrashevych, to which a Toeplitz algebra and a Cuntz-Pimsner algebra can be associated. There is an action of the real line on the Toeplitz algebra and we completely describe the simplex KMS states associated with this action. There is a phase transition at inverse temperature $\log |X|$, which turns out to be the unique KMS state that passes through the quotient to the Cuntz-Pimsner algebra. We describe this unique KMS state spatially as a Gibbs state associated with a Dixmier trace. This is joint work with Marcelo Laca, Iain Raeburn, and Jacqui Ramagge.

Wodzicki Mariusz: Megatrace

The title says it all: a unifying construction for positive traces on operator ideals. All traces studied in "Vestigia vestiganda" can be derived from the megatrace.

Zanin Dmitriy: Singular traces in symmetrically normed operator ideals

Let us recall that ideal $\mathcal{I}$ of the algebra $B(H)$ is called symmetrically normed if

(i) $\mathcal{I}$ is equipped with a Banach norm $\|\cdot\|_{\mathcal{I}}$.
(ii) If $A \in \mathcal{I}$ and $B \in B(H)$, then

$$\|AB\|_{\mathcal{I}} \leq \|A\|_{\mathcal{I}}\|B\|, \quad \|BA\|_{\mathcal{I}} \leq \|A\|_{\mathcal{I}}\|B\|.$$

All nontrivial ideals of $B(H)$ consist of compact operators. **Bounded** functional $\varphi : \mathcal{I} \to \mathbb{C}$ is called trace if $\varphi(AB) = \varphi(BA)$ for all $A \in \mathcal{I}$ and for all $B \in B(H)$. The most important examples of traces are Dixmier traces on Marcinkiewicz ideals. Those traces have a crucial additional property — they respect Hardy-Littlewood preorder (see paragraph below). For every compact operator A, let $\{\mu(k,A)\}_{k\geq 0}$ be the sequence of its singular values. If

$$\sum_{k=0}^{n} \mu(k,B) \leq \sum_{k=0}^{n} \mu(k,A), \quad n \geq 0,$$

then we say that B is majorized by A (written $B \prec\prec A$). For every Dixmier trace φ, the condition $B \prec\prec A$ implies $\varphi(B) \leq \varphi(A)$ (for positive operators $A, B \in \mathcal{I}$). Our main result follows.

Theorem 0.6. *Let $\mathcal{I} \neq \mathcal{L}_1$ be a symmetrically normed operator ideal such that the norm on $\mathcal{I}$ is a Fatou norm. The following conditions are equivalent:*

(i) *There exists an operator $A \in \mathcal{I}$ such that*

$$\lim_{n\to\infty} \frac{1}{n} \|A^{\oplus n}\|_{\mathcal{I}} > 0.$$

(ii) *There exists a singular trace on $\mathcal{I}$.*

(iii) *There exists a singular trace on $\mathcal{I}$, which respects the Hardy-Littlewood preorder.*

(iv) *There exists a singular trace on $\mathcal{I}$, which does not respect the Hardy-Littlewood preorder.*