Schedule for the conference
“Vortices and solitons in classical and quantum fluids”

Monday, March 26th

9h30 : Carlo F. Barenghi, University of Newcastle
Vortices and turbulence with the Gross-Pitaevskii equation I
“I shall introduce the concept of vortex solution of the Gross-Pitaevskii equation. Then I shall describe vortex structures of increasing complexity : vortex rings, vortex perturbed by Kelvin waves, vortex knots, vortex reconnections, turbulent tangles of vortices. Finally I shall summarise the most recent turbulence results, paying particular attention to the observed similarities between quantum turbulence and ordinary turbulence.”

10h15 : ***Coffee break***

10h30 : Carlo F. Barenghi, University of Newcastle
Vortices and turbulence with the Gross-Pitaevskii equation II

11h15 : ***Coffee break***

11h30 : David Chiron, Université de Nice-Sophia-Antipolis
Travelling waves for the nonlinear Schrödinger equations with nonzero condition at infinity : existence, stability
“In this talk, we shall study the travelling waves for the nonlinear Schrödinger equations with a nonzero condition at infinity, like the Gross-Pitaevskii equation. The first part will be devoted to existence results in dimension *d* _ 2 via constrained minimization approaches (joint work with M. Maris), which provides some orbital stability results. We also investigate the transonic limit. In the second part, we focus on the one dimensional case and determine the complete energy-momentum diagrams for some model cases of nonlinearities. We conclude with general results on stability, including the case of the kink.”

17h30 : Luis Vega, Universidad del Pais Vasco (Bilbao)
Stability of the selfsimilar dynamics of a vortex filament

“I shall present some recent results in collaboration with V. Banica about selfsimilar solutions of the vortex filament equation, also known as the binormal flow (BF) or the localized induction equation (LIE). Our main result is the stability of the selfsimilar dynamics of small pertubations of a given selfsimilar solution. The proof relies on finding precise asymptotics in space and time for the tangent and the normal vectors of the perturbations.
A main ingredient in the proof is the control of the evolution of weighted norms for a cubic 1-d Schrödinger equation, connected to the binormal flow by Hasimoto’s transform.”

18h30 : Oana Pocovnicu, Imperial College London
Global well-posedness of the Gross-Pitaevskii equation on R^4
“In this talk we consider the Gross-Pitaevskii equation on R^4 with non-vanishing boundary condition at spatial infinity. By viewing it as a perturbation of the energy-critical Schrödinger equation, we prove that it is globally well-posed in its energy space. We address the same question in the context of the cubic-quintic nonlinear Schrödinger equation with non-vanishing boundary condition on R^3. An extra difficulty is added by the fact that the energy is not sign definite. The talk presents a joint work with R. Killip, T. Oh, and M. Visan.”

Tuesday, March 27th

9h30 : Stéphane le Dizès, Aix-Marseille Université
Waves on vortices : Landau damping versus radiative growth I
"In the first part of the talk, I will analyse the characteristics of the linear waves living on a vortex in an incompressible inviscid homogeneous fluid. I will show that a large axial wavenumber asymptotic analysis can be used to provide information on their spatial structure and dispersion relation. The stabilizing role of critical point singularities will be discussed and analysed in this framework. Asymptotic results will be illustrated and compared to numerical results for a family of vortices ranging from the Rankine vortex (disk of uniform vorticity) to the Lamb-Oseen vortex (gaussian vorticity profile)."

10h15 : *Coffee break*

10h30 : Stéphane le Dizès, Aix-Marseille Université
Waves on vortices : Landau damping versus radiative growth II
"In the second part of the talk, I will consider the waves on similar vortices but in a fluid uniformly stratified in the direction of the vortex axis. I will show that stratification is a source of instability. Using the large axial wavenumber asymptotic analysis, I will show that the instability mechanism is associated with the radiative character of the waves. Connections with similar instability in shallow water or in a compressible fluid will be made. Experimental evidence of the radiative instability will be also provided."

11h15 : *Coffee break*

11h30 : Valeria Banica, Université d'Évry
Some special dynamics for nearly parallel vortex filaments
"We consider the Schrödinger system with Newton-type interactions that was derived by R. Klein, A. Majda and K. Damodaran to modelize the dynamics of *N* nearly parallel vortex filaments in a 3-dimensional homogeneous incompressible fluid. We prove local existence in a suitable energy space. Large time existence results are obtained for particular configurations of four nearly parallel filaments and for a class of configurations of *N* nearly parallel filaments for any *N* _ 2. We also show the existence of travelling wave type dynamics. Finally we describe configurations leading to collision. This is a joint work with Evelyne Miot."

17h30 : Raphaël Danchin, Université Paris Est Créteil
Madelung, Gross-Pitaevskii, quantum and Korteweg fluids
"In this talk, we shall survey various aspects of the hydrodynamic formulation of the nonlinear Schrödinger equation obtained via the Madelung transform. We shall point out some relationships with models of fluid mechanics : quantum fluids, compressible Euler equations and capillary fluids of Korteweg type. This is a joint work with R. Carles and J.-C. Saut."

18h30 : Mariana Haragus, Université de Franche-Comté (Besançon)
Transverse spectral stability of periodic waves for the Kadomtsev-Petviashvili equation
"The Kadomtsev-Petviashvili (KP) equation possesses a four-parameter family of one-dimensional periodic traveling waves. We study the spectral stability of the waves with small amplitude with respect to two-dimensional perturbations which are either periodic in the direction of propagation, with the same period as the one-dimensional traveling wave, or non-periodic (localized or bounded). We focus on the so-called KP-I equation (positive dispersion case), for which we show that these periodic waves are unstable with respect to both types of perturbations. Finally, we briefly discuss the KP-II equation, for which we show that these periodic waves are spectrally stable with respect to perturbations which are periodic in the direction of propagation, and have long wavelengths in the transverse direction."

Wednesday, March 28th

9h30 : Dmitry Pelinovsky, Mc Master University
Bifurcations of asymmetric vortices in symmetric harmonic traps
"We show that the rotating symmetric vortices located at the center of a two-dimensional harmonic potential undertake a pitchfork bifurcation with radial symmetry. This bifurcation leads to the family of vortices, which precess constantly along an orbit enclosing the center of symmetry. The radius of the orbit depends on the precessional frequency. We show that both symmetric and asymmetric vortices are spectrally and orbitally stable with respect to small time-dependent perturbations for large precessional frequency. At the same time, although the symmetric vortex is a local minimizer of energy in the parameter region where the asymmetric vortex exists, the latter corresponds to a saddle point of energy. This is a joint work with P. Kevrekidis (University of Massachusetts)."

10h30 : *Coffee break*

10h45 : Nicola Visciglia, Universita di Pisa
Existence of maximizers and profile decomposition associated with Sobolev-Strichartz inequalities
"The profile decomposition associated with a given linear propagator is a basic tool to study the long-time behavior of solutions to the corresponding nonlinear problems. We shall present a profile decomposition of bounded sequences in Sobolev spaces H_s in a general framework, in particular we allow also the case of systems. We shall start the presentation by the simpler problem of the existence of maximizers for Sobolev-Strichartz inequalities. The results are in collaboration with L. Fanelli and L. Vega."

11h45 : Scipio Cuccagna, Universita di Trieste
On stability of standing waves of the nonlinear Dirac equation
"We discuss a joint paper with N. Boussaid. The variational methods used to study stability of standing waves of equations like the NLS, based as they are on coercivity of certain functionals, do not appear easy to extend to the Dirac equation, where energy is strongly indefinite. A possible alternative which we explore here, is the use of the linear dispersion of the continuous modes, which follows from work by Erdogan et al. One needs to prove that the discrete modes loose energy. We are able to prove it through a form of the nonlinear Fermi Golden Rule (FGR). This is a natural extension of work previously developed for the NLS. But energy indefiniteness creates some trouble because the quadratic form in the FGR is not always obviously positive."

Thursday, March 29th

9h30 : Renzo L. Ricca, Universita di Milano-Bicocca
Tackling structural complexity of vortex dynamics I
"We review some of the author's most recent results in classical topological fluid mechanics, with particular emphasis on relations between dynamical aspects of ideal vortex motion and structural complexity. For this we recall some basic results in geometric and topological dynamics and present recent developments based on applications of knot theory.
First we consider the case of an isolated vortex filament under Euler equations. In this context it is well known that quantities such as kinetic energy, helicity, linear and angular momenta are conserved. We shall discuss how to interpret these quantities in terms of geometric and topological information, discussing their role in relation to linking numbers, writhing and total twist and, by introducing diagram projections, weighted areas. We shall give examples to demonstrate how dynamical information can be extracted by using geometric and topological information even in presence of dissipation and change of topology [1, 2]. Particular integrable dynamics, such as those governed by localized induction equations (LIAs), give rise to even stronger conservation laws that can be all interpreted in terms of global geometric functionals.
We shall then consider a tangle of vortex filaments in space. Measures provided by average crossing number, linking numbers and knot polynomials can be profitably employed to

establish relations between the tangle structural complexity and its energy and helicity. We shall demonstrate how to relate average crossing number to total length and kinetic energy [3]. Finally, we shall present some new results based on the derivation of a Jones polynomial from the helicity of vortex knots and links to demonstrate that new, more powerful invariants are readily available to tackle finer aspects of structural complexity of vortex dynamics [4]."

References :

[1] Ricca, R.L. (2008) Momenta of a vortex tangle by structural complexity analysis. Physica D 237, 2223-2227.

[2] Ricca, R.L. (2009) Structural complexity and dynamical systems. In Lectures on Topological Fluid Mechanics (ed. R.L. Ricca), pp. 169-188. Springer-CIME Lecture Notes in Mathematics 1973. Springer-Verlag.

[3] Ricca, R.L. (2009) New developments in topological fluid mechanics. Nuovo Cimento C 32, 185-192.

[4] Liu, X. and Ricca, R.L. A Jones'polynomial for fluid knots from helicity. Sub judice.

10h15 : ***Coffee break***

10h30 : Renzo L. Ricca, Universita di Milano-Bicocca
Tackling structural complexity of vortex dynamics II

11h15 : ***Coffee break***

11h30 : Mathias Kurzke, University of Bonn
The parabolic Ginzburg-Landau equations in the hydrodynamic limit

17h30 : Robert L. Jerrard, University of Toronto
On the stability of binormal curvature flow and Schrödinger maps
"I will present recent results on stability estimates in very weak norms for binormal curvature flow, and application to 1-d Schrödinger maps. This is joint work with Didier Smets."

18h30 : Francesca Maggioni, Universita di Bergamo
Velocity, energy and helicity of vortex knots and unknots
"In this talk we examine the effect of several geometric and topological aspects on the dynamics and energetics of vortex torus knots and unknots.

The knots are given by small-amplitude torus knot solutions [1] to the Localized Induction Approximation (LIA) law. Vortex evolution is thus studied in the context of the Euler equations by direct numerical integration of the Biot-Savart law. Earlier stability results on vortex knots and unknots [2] are here extended [3]-[4], and the velocity, helicity and kinetic energy of different vortex knots and unknots are presented for comparison.

Vortex complexity is parametrized by the winding number w given by the ratio of the number of meridian wraps to that of longitudinal wraps. We find that for $w < 1$ vortex knots and toroidal coils move faster and carry more energy than a reference vortex ring of same size and circulation, whereas for $w > 1$ knots and poloidal coils have approximately same speed and energy of the reference vortex ring.

Kinetic helicity is dominated by writhe contributions and increases with knot complexity. The stabilizing effect of the Biot-Savart law for all knots and unknots tested is also confirmed.

Our results provide information on relationships between geometry, topology and dynamics of complex vortex systems and apply to quantized vortices in superfluid 4He."

References :

[1] Ricca, R.L. (1993) Torus knots and polynomial invariants for a class of soliton equations. Chaos 3, 83-91. [1995 Erratum. Chaos 5, 346.]

[2] Ricca, R.L., Samuels, D.C. & Barenghi, C.F. (1999) Evolution of vortex knots. J. Fluid. Mech. 391, 29-44.

[3] Maggioni, F., Alamri, S.Z., Barenghi, C.F. & Ricca, R.L. (2009) Kinetic energy of vortex knots and unknots. Il Nuovo Cimento C, 32(1), 133-142.

[4] Maggioni, F., Alamri, S., Barenghi, C.F. & Ricca R.L. (2010) Velocity, energy and helicity of vortex knots and unknots. Phys. Rev. E., 82(2), 026309-026317.

Friday, March 30th

9h30 : Catherine Sulem, University of Toronto
Coupling between internal and surface waves in a two-layers fluid I
"Internal waves occur within a fluid that is stratified by temperature or salinity variation. They are commonly generated in the oceans, and large amplitude, long wavelength nonlinear waves can be produced in the interface and propagate over large distances.
In some physically realistic situations, the visible signature of internal waves on the surface of the ocean is a band of roughness, sometimes referred to as a "rip" which propagates at the same velocity as the internal wave, followed after its passage, by the "mill pond" effect, the complete calmness of the sea.
We propose an asymptotic analysis of the coupling between the interface and the free surface of a two layers fluid in a scaling regime chosen to capture these observations. In particular, we describe the rip region of the free surface as being generated by the resonant coupling between internal solitons and the free-surface wave mode. We also give an explanation of the mill pond effect as the result of a strong reflection coefficient for free-surface waves in the modulational regime, in a frame of reference moving with the internal soliton. This talk is based on joint work with Walter Craig and Philippe Guyenne."

10h15 : ***Coffee break***

10h30 : Catherine Sulem, University of Toronto
Coupling between internal and surface waves in a two-layers fluid II

11h15 : ***Coffee break***

11h30 : Thierry Gallay, Université Joseph Fourier (Grenoble)
Interaction of vortices in viscous planar flows
"It is a well established fact that vortex interactions play a crucial role in the time evolution of viscous planar flows. In particular, numerical simulations of two-dimensional freely decaying turbulence reveal a fascinating coarsening dynamics, which is related to the inverse energy cascade and appears to be essentially driven by vortex mergers. Although strongly nonlinear couplings such as vortex mergers are extremely hard to investigate mathematically, it is possible to obtain a rigorous description of vortex interactions in the perturbative regime where the size of the vortex cores is much smaller than the distance between the vortex centers. This is the case, for instance, if the initial flow is a superposition of point vortices, and if the Navier-Stokes evolution is considered in the vanishing viscosity limit.
In this way, we obtain a simple and rigorous derivation of the Helmholtz-Kirchhoff point vortex system, together with an accurate description of the deformations of the vortex profiles due to mutual interactions. In the particular case of a single vortex pair, we conjecture that the slightly viscous solution can be nicely approximated by a particular solution of Euler's equation which is stationary in a uniformly translating or rotating frame."