Anosov systems

ABSTRACTS

Masayuki Asaoka : Local rigidity of homogeneous actions of parabolic subgroups of rank-one Lie groups

Let P be a Borel subgroup of $\mathrm{SO}_+(n,1)$, Γ a cocompact and torsion-free lattice of $\mathrm{SO}_+(n,1)$. By M_Γ, we denote the quotient space $\Gamma\backslash \mathrm{SO}_+(n,1)$. We define a locally free action ρ_Γ of P on M_Γ by $\rho_\Gamma^h(\Gamma g) = \Gamma(gh)$. When $n = 2$, the pair $(\mathrm{SO}_+(2,1), P)$ is naturally isomorphic to $(\mathrm{PSL}_2(\mathbb{R}), \mathrm{GA})$, where GA is the identity component of the group of upper triangular matrices in $\mathrm{PSL}_2(\mathbb{R})$. In this case, possible perturbation of ρ_Γ is thoroughly studied by Ghys (area-preserving case, 1979, 1985) and the author (general case, 2012). In fact, perturbations of ρ_Γ is parametrized by $H^1(M_\Gamma) \times \mathrm{Teich}(\Gamma\backslash\mathbb{H}^2)$, where $\mathrm{Teich}(\Gamma\backslash\mathbb{H}^2)$ be the Teichmüller space of a hyperbolic manifold $\Gamma\backslash\mathbb{H}^2$.

In this talk, we discuss deformation of ρ_Γ for $n \geq 3$. By the Mostow rigidity, 'the Teichmüller space' of $\Gamma\backslash\mathbb{H}^n$ is trivial. So, one may expect that perturbations of ρ_Γ are parametrized by $H^1(M_\Gamma)$, which is non-trivial in many cases. However, the main result of this talk asserts that ρ_Γ has no non-trivial perturbation.

Theorem 1. *The action ρ_Γ is locally rigid i.e., for any P-action sufficiently close to ρ_Γ, there exists a diffeomorphism H of $\Gamma\backslash SO_+(n,1)$ such that $\rho^h = H \circ \rho_\Gamma^h \circ H^{-1}$ for any $h \in P$.*

Proof is done by showing that the Anosov flow induced by ρ is smoothly conjugate to the flow associated by ρ_Γ (it is just the geodesic flow on $\Gamma\backslash\mathbb{H}^n$). Main ingredient of the proof is de Llave's rigidity theorem on conformal Anosov flows.

Thierry Barbot : Anosov representations

In this talk, I will try to review several geometric or dynamical situations in which Anosov representations and their properties appear as central ingredients.

Thomas Barthelmé : Knot theory of $\mathbb{R}$-covered Anosov flows : homotopy versus isotopy of closed orbits

$\mathbb{R}$-covered Anosov flows are Anosov flows on 3-manifolds such that their stable and unstable foliations are well-behaved (their leaf spaces are homeomorphic to $\mathbb{R}$). Geodesic flows of negatively curved manifolds and suspensions of Anosov diffeomorphisms are examples of $\mathbb{R}$-covered Anosov flows, but there are many other examples in all kinds of 3-manifolds. When the manifold is hyperbolic, S. Fenley showed that every free homotopy class of a closed orbit contains infinitely many closed orbits. Hence a free homotopy class of a closed orbit gives a family of knots in an hyperbolic manifold. We study whether these knots are equivalent or not (Work in common with Sergio Fenley).

Yves Benoist : Random Cat Map

I will explain that part of the chaotic behavior for the iteration of the cat map on the torus disappears when one deals with random iterates of cat maps. This is joint work with Jean-Francois Quint.

Christian Bonatti : Marco Brunella's work on Anosov flow

Before his "holomorphic foliations" period Marco Brunella had several works on Anosov flows. I will present a paper of him where he start a classification of non-transitive Anosov flows on 3-manifolds by considering transverse tori. We wil see that his work opened the door for the study of transitive and non-transitive Anosov flows that we are now proceeding in a join program with Francois Béguin and Bin Yu.

Mickaël Crampon : On the geodesic flow of Hilbert metrics

Joint work with Ludovic Marquis.

Pierre Dehornoy : Linking of periodic orbits in geodesic flows

A flow on a rational homology sphere is left-handed if it has many periodic orbits and if every pair of periodic orbits have negative linking number. This topological property implies that every collection of periodic orbits bounds a Birkhoff section. Birkhoff's construction of sections for the geodesic flow on a surface and the study of the geodesic flow on the modular surface by Ghys suggest a general statement : the geodesic flow on every 2-orbifold with no rational homology is left-handed. We will discuss this question and give a positive answer in several cases where the orbifold has negative curvature.

Sergio Fenley : Structure and examples of pseudo-Anosov flows in graph manifolds and Seifert fibered pieces

We describe an interaction of a pseudo-Anosov flow with possible Seifert fibered pieces in the torus decomposition of the underlying manifold : if the fiber is associated to a periodic orbit of the flow, we produce a standard form for the flow in the piece which is a neighborhood of finitely many weakly embedded Birkhoff annuli. A Birkhoff annulus is an annulus so that each boundary component is a closed orbit of the flow and the flow is transverse to the interior of the annulus. Using collections of Birkhoff annuli as a skeleton for some flows, we then produce a very large class of new examples of pseudo-Anosov flows in graph manifolds. This is joint work with Thierry Barbot.

Patrick Foulon : New contact Anosov Flows

Geodesic flows of Riemannian or Finsler manifolds have been the only known contact Anosov flows. We show that even in dimension 3 the world of contact Anosov flow is vastly larger via a surgery construction near a transverse Legendrian knot that encompasses both the Handel–Thurston and Goodman surgeries and that produces flows that are not topologically orbit equivalent to any algebraic flow. This includes examples on many hyperbolic 3-manifolds, any of which have a remarkable array of dynamical and geometric properties. It is a joint work with B. Hasselblatt.

François Guéritaud : Timelike geodesic foliations in Lorentz spacetimes of constant curvature

A group action on AdS is the same as a pair of representations into $\mathrm{SL}(2,\mathbb{R})$, acting from the left and right respectively. Properness of this action corresponds in metric terms to the fact that one representation moves points of the hyperbolic plane less than the other, in the sense that there is a better-than-1-Lipschitz equivariant map from $\mathbb{H}^2$ to itself (Kassel). I will explain how to extract, from this information, a foliation of the quotient AdS manifold into timelike geodesics. An infinitesimal version of this construction, for affine actions on Minkowski space, analogously yields foliations of complete flat spacetimes, which teaches us something about their topology. Joint work with F. Kassel and J. Danciger.

Masahiko Kanai : Cross ratio and its relatives in geometry and dynamics

This is a short survey on cross ratio and its 'relatives' such as Schwarzian derivative, geodesic currents and paraKaehler structures emphasizing applications to geometry and dynamics.

Miguel Paternain : Some applications of dynamics on loop space

We discuss some work in progress concerning some applications to conservative and hyperbolic systems of suitable dynamics on loop spaces.

Tali Pinsky : Knotted geodesics on Hecke triangles

Ghys discovered in 2006 that the periodic orbits of the modular flow have very peculiar properties as knots, when embedded into $\mathbb{S}^3$. For example, they are all prime, fibered, and have positive signature. This result was obtained using a "template" for the flow, which is a tool that reduces the original three dimensional flow to a much simpler two dimensional system, while preserving the topological properties of the orbits. In this talk I will explain what templates are, and a method for computing them for the geodesic flow on an infinite class of Hecke triangle groups, generalizing Ghys' results. We will then use the arising templates to show that as knots in the appropriate three-manifolds, all closed geodesics corresponding to any of these groups are prime.

Federico Rodriguez-Hertz : Arithmeticity and topology for actions of higher rank abelian groups

In a recent work with B. Kalinin and A. Katok we showed that : Theorem : an ergodic invariant measure by a R^k action on a $2k+1$ dimensional manifold M, $k \geq 2$ with positive entropy and Lyapunov exponents in general position is absolutely continuous with respect to Lesbesgue measure.

The only examples of actions of this type that are known are suspensions of linear actions on tori and some constructions starting with such linear actions blowing up some compact orbits and following with some gluing procedure or taking some finite quotients.

In a joint work with A. Katok we are able to handle the discrete actions and to prove that from a measure theoretic point of view the linear examples are the only possible examples. Moreover we can show the measurable conjugacy extends to a continuous map from an open invariant subset U of the manifold M to some open invariant subset V of the torus which is necessarily the complement of a finite set. Moreover U is homeomorphic to V.

During the talk we shall describe the examples and we will try to show the ideas of the proof of this result and discuss corollaries and related problems both in the discrete and continuous time case.

Maria Alejandra Rodriguez-Hertz : Partially hyperbolic sets with positive measure

Every proper partially hyperbolic set with positive measure and center dimension one, has at least two periodic points with a strong heteroclinic intersection. This implies that C^r-generically, for any $r \geq 1$, a diffeomorphism does not contain such sets.

Slobodan Simic : Towards a Hoelder geometry of invariant distributions

Hoelder continuous distributions are ubiquitous in Anosov and more generally partially hyperbolic dynamical systems. They naturally define certain geometric structures whose properties are both dynamically significant and interesting in their own right, as generalizations of their smooth analogs. On one side of the spectrum of such properties we have integrability, on the other, accessibility. Both are well understood for smooth distributions and only partially understood for merely continuous distributions. We will present some

ideas and results, as well as open questions, related to these issues for Hoelder continuous (invariant or not) distributions.

Régis Varão Filho : Absolute continuity for diffeomorphisms with non-compact center leaves

We study the measure-theoretical properties of center foliations of volume preserving partially hyperbolic diffeomorphisms with one-dimensional center direction. Recent work of Avila, Viana, Wilkinson dealt with situations where the center leaves are compact or can be compactified in a suitable way. Using different techniques we focus on the non-compact case and obtain very different conclusions. For instance, in our context the disintegration of volume may be neither atomic nor Lebesgue. Such examples are found even among Anosov diffeomorphisms. Moreover, even an Anosov may have absolutely continuous center foliation without being C^1-conjugate to its linearization.

This is a joint work with M. Viana and A. Tahzibi.