Rencontre RAS au CIRM, du 17 au 21 septembre 2012

Riccardo Adami: ***Stability of ground states for NLS on simples graphs.***

The introduction of NLS on graphs is motivated by several physical reasons, e.g., the study of propagation of waves in ramified structures in nonlinear media and the analysis of the dynamics of the Bose-Einstein condensates in the presence of impurities. The mathematical problem can be formulated as a system of as many NLS as edges in the graph, coupled by suitable self-adjoint matching conditions at the vertices. The analysis is only at its beginning and mainly concerns the structure of the family of the ground states; however, some results show the occurence of phenomena that are unexpected and far from the corresponding results for the standard NLS. Among them, the appearance of symmetry-breaking bifurcation, and the absence of ground states in the case of free (i.e. Kirchhoff's) infinite graphs. We give examples of such phenomena on simple star graphs with delta and delta-prime vertex conditions.

Clotilde Fermanian: ***A nonlinear Landau-Zener formula***

In this talk, we present a scattering result about a system of two coupled ordinary differential equations. This system appears as an envelope equation in Bose–Einstein Condensation and can be viewed as a nonlinear extension of the model studied by Landau and Zener in the 20's which is at the root of results about eigenvalue crossings. We study the large time behavior of the solutions and show the existence of a nonlinear scattering operator. We compute the first non-trivial term of its asymptotic expansion as the strength of the nonlinearity goes to zero. These results have been obtained in collaboration with Rémi Carles.

Philippe Gravejat: ***Hartree-Fock models for the vacuum polarization.***

We review recent results about the derivation and the analysis of two Hartree-Fock-type models for the vacuum polarization, the reduced Bogoliubov-Dirac-Fock model and the Pauli-Villars regulated model. In both cases, we pay particular attention to the variational construction of a self-consistent polarized vacuum. Concerning the reduced Bogoliubov-Dirac-Fock model, we also investigate the physical agreement between our non-perturbative construction and the perturbative description provided by Quantum Electrodynamics.

Benoît Grébert: ***Resonnant normal form and applications to PDEs.***

I will present some nonlinear Schrödinger equation on the circle that exhibit resonances between the Fourier modes. In particular I will consider the following coupled cubic Schrödinger equations

$$\begin{cases} i\partial_t u + \partial_x^2 u = \varepsilon^2 |v|^2 u, \quad (t,x) \in \mathbb{R} \times S^1, \\ i\partial_t v + \partial_x^2 v = \varepsilon^2 |u|^2 v\,. \end{cases}$$

We prove that there exists a beating effect, i.e. an energy exchange between different modes. This construction may be transported to the linear timedependent Schrödinger equation: we build solutions such that their Sobolev norms grow logarithmically.

Juan Huang: ***The energy-critical nonlinear Schrodinger equations with subcritical perturbations.***

In this talk, we combine variational methods and harmonic analysis to discuss the Cauchy problem of the focusing nonlinear Schrödinger equation. We study the global well-posedness, finite time blowup and asymptotic behavior of this problem. By Hamiltonian property, we establish two types of invariant evolution flows. Then using one flow and the stability of classical energy-critical nonlinear Schrödinger equation, we find the solution exists globally and scattering will occur. From the other flow, we get a precision blowup criterion of this problem for positive energy initial data.

Oana Ivanovici: ***Dispersive estimates for the wave equation inside convex domains.***

We consider a model case for a strictly convex domain of dimension $d \geqslant 2$ with smooth boundary and we describe dispersion for the wave equation with Dirichlet boundary conditions. More specifically, we obtain the optimal fixed time decay rate for the smoothed out Green function: a $t^{1/4}$ loss occurs with respect to the boundary less case, due to repeated occurrences of swallowtail type singularities in the wave front set. This is a joint work with Gilles Lebeau and Fabrice Planchon.

Christian Klein: ***Semiclassical limit of nonlinear Schrödinger equations and Painlevé transcendents.***

The semiclassical limit of $1+1$-dimensional nonlinear Schrödinger (NLS) equations is studied close to the singularity of the corresponding solution of the semiclassical equations. By performing a multiscales analysis, we conjecture that the solution to the NLS equations near the critical point is given in terms of special solutions to ordinary differential equations from the Painlev hierarchies. For focusing NLS equations these are the tritronque solutions of the Painlevé I equation, for defocusing NLS equations a special solution to the second equation in the Painlevé I hierarchy. We give strong numerical evidence in support of this conjecture. A numerical study of a similar situation for the Davey-Stewartson II system is presented.

Stefan Le Coz: ***Multi-solitons of NLS.***

We consider a nonlinear Schrödinger equation with a general nonlinearity. In space dimension 2 or higher, this equation admits solitons (standing/traveling waves) with a fixed profile which is not a ground state. These types of profiles are called excited states. Due to instability, excited solitons are singular objects for the dynamics of NLS. Nevertheless, we will show in this talk how to exhibit solutions of NLS behaving in large time like a sum of excited solitons with high relative speeds.

Benoît Pausader : ***On scattering for the quintic defocusing nonlinear Schrödinger equation on*** $\mathbb{R} \times \mathbb{T}^2$**.**

The simplest asymptotic dynamics for dispersive equations is scattering. A natural question is to try to understand which geometric aspects of the domain make such a behavior possible. This is still rather poorly understood, and the case of the quintic semilinear Schrödinger equation posed on $\mathbb{R} \times \mathbb{T}^2$ appears as an interesting example to test some assumptions (the equation is in some sense critical for mass and energy; even the evolution of rough linear solutions is interesting and not quite completely understood). We show that the large energy scattering on that space boils down to a global existence issue for a new equation (known as vector Schrödinger equation) posed on $\mathbb{R}$.

Fabrice Planchon: ***On the cubic NLS on 3D compact domains.***

We prove global well-posedness for the cubic NLS on compact 3D domains with smooth data (H^s for $s > 1$) and Dirichlet boundary conditions. The proof relies on a suitable Cauchy theory at a regularity level very close to $s = 1$, a Brezis-Gallouet-Yudovitch type of argument (which, in our context, will produce a triple exponential upper bound for the growth of Sobolev norms) and bilinear estimates on domains which are of independent interest.

Roman Schubert: ***Quantum normal forms and transition state theory: Microlocal analysis of chemical reactions.***

We consider the situation where after the Born-Oppenheimer approximation the dynamics of a molecule with N atoms is described by a semiclassical Schrödinger operator with a small parameter given by the ratio of the electron mass to the mass of the nuclei, and with a potential energy $V(x)$ of the nuclei in the average field of the electrons in their ground state. For the dynamics of such a system a particularly important role is played by the critical points of $V(x)$. Local minima correspond to stable configurations and a chemical reaction is a transition between two such minima, inbetween two minima the system has to cross a saddle point of $V(x)$. The saddle point is called the transition state and we are interested in describing the dynamics near the transition state. To this end we use quantum normal form theory to approximate the Hamilton operator by a simpler one whose dynamics we can solve explicitly. This allows to compute a local S-matrix, resonances and reaction probabilities. We developed explicit algorithms to compute these quantities and we illustrate the results with some examples.

Didier Smets: ***Stability and asymptotic stability for solitons of the 1D Gross-Pitaevskii equation.***

In a joint work with Fabrice Bethuel (UPMC) and Philippe Gravejat (Ecole Polytechnique), we study the dynamical stability of solitons of the one dimensional Gross-Pitaevskii equation. Thanks to a new monotonicity formula, we prove not only the orbital stability of a single or a train of solitons, but also the asymptotic stability in the energy space, locally in space, of any soliton of non zero speed. As far as we are aware, this is the first explicit example, without any a priori spectral assumption, of asymptotic stability of a soliton for a NLS type equation.

Benjamin Texier: ***From Newton to Boltzmann: the case of short-range potentials.***

We fill in all details in the proof of Lanford's theorem. This provides a rigorous derivation of the Boltzmann equation as the mesoscopic limit of systems of Newtonian particles interacting via a short-range potential, as the number of particles N goes to infinity and the characteristic size of the particles ε simultaneously goes to 0, in the Boltzmann-Grad scaling $N\varepsilon^{d-1} \equiv 1$. The case of localized elastic interactions, i.e., hard spheres, is a corollary of the proof. The time of validity of the convergence is a fraction of the mean free time between two collisions, due to a limitation of the time on which one can prove the existence of the BBGKY and Boltzmann hierarchies. Our proof relies on the important contributions of King, Cercignani, Illner and Pulvirenti, and Cercignani, Gerasimenko and Petrina.

Laurent Thomann: ***Dynamics of Klein-Gordon near an homoclinic orbit.***

We consider a Klein-Gordon equation on a Riemannian surface which has an homoclinic orbit to the origin, and we study the dynamics close to it. Thanks to a strategy of Groves-Schneider, we show that there are many solutions which stay close to this homocline during all times. We point out that the solutions we construct are not small. This is a joint work with Benoît Grébert and Tiphaine Jézéquel.