QP33

33rd International Conference on Quantum Probability and Related Topics

CIRM, Luminy, France
1-5 October 2012

Luigi Accardi: The quantum decomposition of random variables without moments. Joint work with Habib Rebei, Anis Riahi

The quantum decomposition of a classical random variable is one of the deep results of quantum probability: it shows that **any classical random variable or stochastic process with moment of all orders has a built in non commutative structure which is intrinsic and canonical, and not artificially put by hands**.

In this sense one can say that the point of view of quantum probability is complementary to that of non commutative geometry: the latter postulates that coordinates (random variables) are non commutative; the former keeps commutativity of the coordinates but uses a state or weight, often intrinsically associated to a manifold, to deduce an intrinsic (microscopic) non commutative structure in which the commutation relations are deduced from the commutativity of the coordinates.

Up to now the quantum decomposition is deduced from:

– the theory of interacting Fock spaces

– Jacobi's tri–diagonal relation for orthogonal polynomials.

Therefore it requires the existence of moments of any order and cannot be applied to random variables without this property.

For such random variables even the statement of the problem to find an **analogue of the quantum decomposition** was not clear because for about fifteen years nobody had any idea of how such an analogue could look like.

In the talk we describe the quantum decomposition canonically associated to the Levy–Khintchin triple of any infinitely divisible classical random variable.

Surprisingly this decomposition is similar to the known one, but some essential technical differences arise. In particular, when no moments exist, a new type of renormalization is required, different from the ones used for the higher powers of white noise.

The basic tool used is the Araki–Woods–Parthasarathy–Schmidt Fock space characterization of the Kolmogorov representation of classical Levy–Khintchin triples in terms of 1–cocycles (helices) in Hilbert spaces.

Irina Arefeva: Holographic description of entropy production in quark-gluon plasma formation. The quark-gluon plasma is a state of matter formed from deconfined quarks, antiquarks, and gluons at high temperature. This strong coupled state of matter is produced in heavy ions collisions at the Relativistic Heavy Ion Collider and at the Large Hadron Collider. It is difficult to describe thermalization in strong coupled systems. Recently, a new method of dealing with strongly coupled system, based on the dual holographic description by using classical gravity equations in Anti de Sitter (AdS) space, has been actively explored. In the talk applications of this gauge/gravity duality to entropy production in quark-gluon plasma formation will be discussed. In the dual description thermalization of a quasi-conformal 4-dimensional quantum system is described by black hole formation in 5-dimensional AdS space and the entropy production can be estimated by the area of the trapped surface which presages the black hole formation. We present estimations of area of the trapped surface under collisions of shock waves in AdS5. We compare this estimation of entropy with multiplicity of particles production in heavy ions collisions.

Octavio Arizmendi: Convergence and Superconvergence in Non-Commutative Probability: Acombinatorial approach. In the first part of the talk we will explain the analogous of 4th moment theorems of Nualart and Peccati (2005) and Kemp, Nourdin, Peccati and Speicher (2011) for infinitely divisible measures with respect to classical, free, boolean, monotone and q-convolutions. Namely, when restricted to infinitely divisible distribution, the convergence of the first four moments to the Gaussian (w.r.t each convolution) implies the convergence in law. The main tool used in provingthese results is the Jacobi parameters.

On the second part of the talk I will explain how to use cumulants to give an simple proof of an instance of the so-called superconvergence of normalized sums of free random variables. Namely, that the operator norm of normalized sums of bounded free random variables with mean 0 and variance 1, converge to 2. Moreover, our approach generalizes in a straightforward way to monotone and boolean independence.

Nobuhiro Asai: Orthogonal Polynomials Originated from the Boas-Buck Type Generating Functions. In this talk, we shall give a quick overview on orthogonal polynomials of one variable generated from the Boas-Buck type generating functions. The generating functions involve many of important subfamilies, which contain the Appell, classical Meixner-Sheer, free Meixner-Sheer, Brenke and other families. This presentation contains our recent results on the Brenke-Chihara problem [1].

References [1] Asai, N., Kubo, I., and Kuo, H.-H.; The Brenke type generating functions and explicit forms of MRM-triples by means of q-hypergeometric series.(2011, submitted)

Stéphane Attal: Repeated quantum interaction: old and new.

Wided Ayed: White noise approach to quantum stochastic calculus. We study bosonic and free white noise Heisenberg equations giving rise to flows which are $*$-automorphisms of the observable algebra, but not necessarily inner automorphisms. We prove that the regularization used to get normally ordered form of these white noise Heisenberg equations is equivalent to causal normal order and leads to Evans-Hudson flows. This gives in particular, the microscopic structure of the maps defining these bosonic and free white noise flows, in terms of the original free white noise derivations.

Abdessatar Barhoumi: QWN-Convolution operators with application to differential equations. In this talk we introduce a quantum white noise (`QWN`) convolution calculus over a nuclear algebra of operators. Then we discuss new solutions of some linear and non-linear differential equations.

Alexander Belton: An algebraic construction of quantum flows with unbounded generators. (Joint work with Stephen Wills, University College Cork)

We will explain how to construct *-homomorphic quantum stochastic Feller cocycles for certain unbounded stochastic generators, and so obtain dilations of various strongly continuous quantum dynamical semigroups on C* algebras. The construction is possible provided the generator satisfies an invariance property for some dense subalgebra A_0 of the C* algebra A and obeys the necessary structure relations; the iterates of the generator, when applied to a generating set for A_0, must satisfy a growth condition. Furthermore, either the subalgebra A_0 is generated by isometries and A is universal, or A_0 contains its square roots. Some examples obtained by applying the general theory will be presented.

B V Rajarama Bhat: Bures distance for completelypositive maps. D. Bures had defined a metric on states of C*-algebras using GNS representations of states. This notion has been extended to completely positive maps by D. Kretschmann, D. Schlingemann and R.F. Werner. We present a Hilbert von Neumann Module version of this theory. This is a joint work with K. Sumesh.

Philippe Biane: A concave version of free entropy. Free entropy is a quantity introduced by Voiculescu in free probability, in order to measure how well a set of noncommutative variables can be approximated in distribution by random matrices. We give a new definition of this quantity, and use it to solve the additivity problem for free entropy.

Andreas Boukas: Contractions and central extensions of quqntum white noise Lie algebras. (joint work with Luigi Accardi)

We show that the Renormalized Powers of Quantum White Noise Lie algebra $RPQWN_*$, with the convolution type renormalization $\delta^n(t-s) = \delta(s)\,\delta(t-s)$ of the $n \geq 2$ powers of the Dirac delta function, can be obtained through a contraction of the Renormalized Powers of Quantum White Noise Lie algebra $RPQWN_c$, with Ivanov's renormalization $\delta^n(t) = c^{n-1}\,\delta(t)$, $c > 0$. Using Ivanov's renormalization, we also obtain a Lie algebra $W_\infty(c)$ which contains the w_∞ of Bakas and the Witt algebra as contractions. Motivated by the W_∞ algebra of Pope, Romans and Shen, we show that $W_\infty(c)$ can also be centrally extended in a non-trivial fashion. In the case of the Witt subalgebra of W_∞, the central extension coincides with that of the Virasoro algebra.

Raffaella Carbone: Environment induced decoherence for quantum Markov semigroups. We study the mathematical definition of environment induced decoherence (according to the definition introduced by Blanchard and Olkiewicz) for markovian evolutions. In particular, we describe a necessary and sufficient condition for decoherence of Markov semigroups acting on matrix algebras. This condition is related to the spectral analysis of the generator L of the semigroup and is easily stated: the evolution displays decoherence if and only if the maximal algebra N on which the semigroup is *-automorphic contains all the eigenvalues of L associated with eigenvectors with null real part. Moreover, this condition is surely verified when the semigroup admits a faithful invariant state. (This is a joint work with E. Sasso and V. Umanit)

Ricardo Castro Santis: Approximation of quantum dynamics for Quantum Markov Semigroups. This talk presents the evolution of a quantum system interacting with an external field. The evolution is described by an Hadson-Parthasarathy equation with coefficients unbounded. We consider only system observables, therefore the study is centered on the "reduced dynamic" of the system. The evolution of this dynamic does not meet the semigroup properties, but we can approximate this for quantum Markov semigroups ordered temporarily. In this work will also present the generardor (time dependent) of the "reduced dynamics" and a backward differential equation for her.

Fabio Cipriani: Noncommutative Potential Theory

Vito Crismale: A De Finetti theorem on the CAR algebra The symmetric states on a quasi local C*-algebra on the infinite set of indices J are those invariant under the action of the group of the permutations moving only a finite, but arbitrary, number of elements of J. The celebrated De Finetti Theorem describes the structure of the symmetric states (i.e. exchangeable probability measures) in classical probability. In the present talk we show an extension of De Finetti Theorem to the case of the CAR algebra, that is for physical systems describing Fermions. Namely, we show that the compact convex set of such states is a Choquet simplex, whose extremal (i.e. ergodic w.r.t. the action of the group of permutations previously described) are precisely the product states in the sense of Araki-Moriya. In order to do that, we present some ergodic properties naturally enjoyed by the symmetric states which have a self-containing interest. The talk is based on a joint work with Francesco Fidaleo (Department of Mathematics, University of Tor Vergata, Roma).

Bata Krishna Das: Quantum stochastic analysis in operator space and unitary random walk approximation. The abstract characterisation of matrix space tensor product transform quantum stochastic mapping process in to the following form:

$$t \mapsto k_t \in L(\mathcal{E}; CB(\langle\mathcal{F}|; CB(\mathsf{V}_1; \mathsf{V}_2))). \tag{1}$$

By replacing $CB(\mathsf{V}_1; \mathsf{V}_2)$ by an abstract operator space V, we extended the standard quantum stochastic analysis and obtained a consistent theory unifying the theory of operator processes, mapping processes and convolution processes through different choices of operator spaces.

In this talk, I will show how this theory unifies different notion of processes, and provides new results for convolution cocycles. Operator space quantum stochastic cocycles will be approximated by quantum random walks, which refines and generalises the random walk approximation result of Belton. Using this result a class of non- Markov regular unitary cocycles will be approximated by unitary random walks. This class includes all the unitary cocycles which implements Lévy processes on a separable CQG algebra.

This is joint work with Martin Lindsay.

Nizar Demni: Spectral distribution of the free Jacobi process. The free Jacobi process is an inifinite dimensional version of the matrix-variate complex Jacobi process. It is defined as the radial part of the compression of a free unitary Brownian motion in a von Neumann algebra by two projections that are in free relation with this Brownian motion. In this work, we give an explicit description of the spectral distribution of this process in the compressed von Neumann algebra when the compression is realized by means of a single projection of rank 1/2. It turns out that this distribution fits the 'cosine' of the unitary Brownian motion, up to a deterministic time scale and up to a dilation. We give two proofs of this result: the first one is analytic and relies on an explicit solution of a non linear pde. The second proof uses combinatorics and has the merit (relatively to the first proof) to extend to arbitrary ranks. We shall also give recent results on the spectral distribution of the free Jacobi process when the compression is realized by means of two projections. This is a joint work with T. Hmidi and T. Hamdi.

Julien Deschamps: Classical Action of Quantum Baths and Complex Obtuse Random Variables. We consider the model of a quantum environment acting on a quantum system via quantum repeated interactions. We characterize some interactions where the environment acts as a classical noise. We show that this situation is closely related to obtuse random variables characterized by a 3-tensor which admits certain symmetries. These tensors allows for instance to write the multiplication operator by these variables. Then we obtain a representation of the classical unitary actions of the environment in term of complex obtuse variables. Moreover, the interaction with the environment can be described in this context as an explicit random walk in the unitary group.

Santanu Dey: Outgoing Cuntz scattering system for a coisometric lifting. Using a coisometric lifting of a tuple, anoutgoing Cuntz scattering system is developed. A transferfunction is introduced for the system.

Stephen Dias-Barreto: Disordered fermion systems on a lattice. We study Fermion systems on a lattice with random interactions through their dynamics and the associated KMS states. These systems require a more complex approach compared with the standard spin systems on a lattice, on account of the difference in commutation rules for the local algebras for disjoint regions, between these two systems. It is for this reason that some of the known formulations and proofs in the case of the spin lattice systems with random interactions do not automatically go over to the case of disordered Fermion lattice systems. We extend to the disordered CAR algebra some standard results concerning the spectral properties exhibited by temperature states of disordered quantum spin systems. We investigate the Arveson spectrum, known to physicists as the set of Bohr frequencies. Further we also establish its connection with the Connes and Borchers spectra, and with the associated invariants for such W-dynamical systems which determine the type of von Neumann algebras generated by a temperature state. We prove that all such spectra are independent of the disorder. As a natural application, we show that a temperature state can generate only a type III von Neumann algebra (with the type III_0 component excluded). In the case of the pure thermodynamic phase, the associated von Neumann algebra is of type III_λ for some $\lambda \in (0, 1]$, independent of the disorder.

All such results are in accordance with the principle of self-averaging which affirms that thephysically relevant quantities do not depend on the disorder.

Demosthenes Ellinas: Algebraic Random walks for operator valued measures and symmetric functions. Hopf algebras of functions and operators are utilized to develop a mathematical scheme for building generalized diffusion equations, algebraic Markov chains and quantum master equations, by means of algebraic random walks. The main construction treats exemplary cases of elementary Hopf algebras, such as circle, line, plane, sphere, nite groups. It investigates pairs of covariance, formed by a translation operator and its associated operator valued measures (OVM); such OVMs are related to Wigner distribution functions, or to positive OVM (POVM) related to position/momentum distributions of canonical quantum systems, and to analogous POVMs related to distributions associated to nite dimensional Hilbert spaces (spin quantum systems). The construction utilizes (completely) positive trace preserving maps (CPTP) of density operators. The appropriately defined (weak) asymptotic limit of the action of such maps, is shown to lead to quantum master equations of Lindblad type, describing information loss. Using the standard ARW formulation we introduce ARWs for co-and Hopfalgebraic structures in the ring of symmetric functions. The construction proceeds by dualization dierent types of pairs of outer multiplication and outer coproduct, inner multiplication and inner coproduct, and symmetric function plethysm multiplication and plethystic coproduct.If coordinates are interpreted as occupancies of walker(s) at dierent locations, these walks introduce translations, dilations and combinatorial inations of height coordinates, respectively.

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Franco Fagnola: Entropy production for quantum Markov semigroups. An invariant state of a quantum Markov semigroup is anequilibrium state if it satisfies a quantum detailed balance condition. In this talk we introduce theentropy production for a faithful normal invariant state of a quantum Markov semigroup on B(h) asnumerical index measuring how far is the invariant state from equilibrium, extending the classical probabilistic concept. The entropy production is defined as derivative of the relative entropy of theone-step forward and backward evolution, it turns out to be zero if and only if a standard quantumdetailed balance condition holds and can be computed explicitly essentially for all quantumMarkov semigroups commuting with the modular group of the invariant state. Non-commutative issues and new features are discussed and illustrated by examples. This talk is based on joint works with L. Accardi, R. Quezada, R.Rebolledo and V.Umanità.

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Philip Feinsilver: Multivariate Krawtchouk polynomials and a spectral theorem for symmetric tensors. The construction of multivariate Krawtchouk polynomials using symmetric tensors yields families of polynomials orthogonal with respect to associated multinomial distributions. After presenting this, we prove a spectral theorem for corresponding quantum observables, including quantum random walks. Examples are given where the conditions required for orthogonality appear naturally. As an application, the standard 3-term recurrence is interpreted as level one of a linearization formula and the extension to arbitrary order appears directly.

Francesco Fidelao: The nonconventional ergodic theorem.

Remus Floricel: Asymptotic properties of quasi-shift endomorphisms of von Neumann algebras. A quasi-shift is a unital normal *-endomorphism acting on a vonNeumann algebra, of which tail and fixed-point algebras coincide. Theclass of quasi-shift endomorphisms may be regarded as the noncommutative analogue of the class of steady Markov chains, i.e., Markov chains ofwhich tail and Poisson boundaries coincide. Our purpose, in this presentation, is to discuss several asymptotic characterizations of quasi-shifts associated with representations of Cuntzalgebras.

Wolfgang Freudenberg: On a Quantum Statistical Model of the Recognition Process. One of the main activities of the brain is the recognition of signals. Signals could be described in terms of a classical probabilistic model. But some well-known aspects of the behaviour of the brain are in contradiction to classical models. For that reason we will consider a quantum-like model based on suitably chosen Fock spaces. Incoming signals as well as the information stored in the memory are described by quantum states on these spaces. The recognition process based on a comparison of an incoming signal with signals chosen from the memory will be described by appropriately chosen quantum channels.

Matteo Gregoratti: Quantum measurements in continuous time, non Markovian evolutionsand feedback. We present a non Markovian version of quantum trajectory theory, based on the the stochastic Schroedinger equation withstochastic coefficients. In this framework we can model the nonMarkovian evolution a two-level atom stimulated by a laser, in the case of imperfections in the stimulating laser, and of a feedback loop basedon the detection of the fluorescence light. Indeed, realisticdescriptions of a feedback loop have to include delay and thus need a non Markovian theory of measurements in continuous time and measurementbased feedback. In particular, chosen a specific feedback, we explicitlycompute the homodyne spectrum of the fluorescence light in order to control its fluorescence light (squeezing). Let us stress the change ofpoint of view with respect to the usual applications of control theory. Here the "system" is the two-level atom, but we do not want to controlits state. Our aim is to control the properties of the emitted light; moreover, we want to control the spectrum, which is not a property at asingle time, but involves a long interval of times (a Fourier transformof the autocorrelation function of the observed output is needed).

Skander Hachicha: Generic Quantum Markov Semigroups With Instantaeous states. We construct a generic quantum Markov semigroups with instantaneous states exploiting the invariance of the diagonal algebra and the explicit form of the action of the pre-generator on off-diagonal matrix elements. We discuss the generic hydrogen type atoms as an exemple.

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Toshihide Hara: On the Theoretical Relationship between Information Dynamics, Kolmogorov-Sinai Entropy and Lyapunov Exponent. There exist several criteria to describe the chaotic behavior of a dynamical system. These criteria are used in case by case. In this talk, we discuss the relations of those three; entropic chaos degree, Kolmogorov-Shinai entropy and Lyapunov exponent. (joint work with Takeo Kamizawa, Masanori Ohya and Keiko Sato)

Robin Hillier: Noncommutative geometry of conformal nets. We provide an introduction to the theory of conformal quantum field nets – certain systems of von Neumann algebras with deep geometric-modular structure. The aim of the talk is then to show how those nets can be efficiently described in terms of noncommutative geometry, namely spectral triples, cyclic cocycles, equivariant K-theory classes and index formulae, and what new information is obtained this way. We conclude with a few interesting relations to quantum probability, in particular noncommutative continuous Bernoulli shifts and white noises.

Robin Hudson: The mystery of time reversal in quantum stochastic calculus. We investigate the role of time reversal in quantum stochastic calculus. It is related to the antipode S in the sticky shuffle Hopf algebra whose multiplication describes the product of iterated stochastic integrals. But $S^2 \neq id$ whereas, surely, doubly reversed time is the original

Satoshi Iriyama: On a new quantum search algorithm and its complexity. (Joint work with M.Ohya)

For a given function f and y, a search problem is to find one solution x such that $f(x) = y$, and one of the special problems is to find an inverse function of f. Originally a search problem has been discussed by Levin and Solomonoff. A quantum algorithm of search problem was proposed by Grover in 1996, and its computational complexityis a square root of the number of elements in the domain. In this talk, we introduce a new quantum algorithm for a search problem, and discuss its computational complexity.

Young Yi Kim: Convolution Products in White Noise Theory. In this talk, we introduce general definition of convolution product of test white noise functionals which is extended as convolution product of generalized white noise functionals. Then by applying the kernel theorem we study convolution products of white noise operators as quantum extension of the (classical) convolution product of generalized white noise functionals. Finally, we study a quantum-classical correspondence. This talk is based on a joint work with U.C. Ji and Y.J. Park.

Anna Kula: Symmetries of Lévy processes on compact quantum groups. We discuss the symmetry properties of generators of Levy processes on a compact quantum group, which allow to apply the quantum potential theory to the non-tracial case. This is a joint work with U. Franz and F. Cipriani.

Yuh-Jia Lee: Calculus of Generalized Brownian of Functionals, Revisited. Being motivated by the Laurent Schwartz's Theory of Distribution, in this talk, we reformulate Schwartz's theory on Gaussian space and then we extend the Schwartz's distribution on Gaussian space to infinite dimensions. The generalized Brownian functional is defined by its linear form instead of its inverse S-transform. The construction of generalized Brownian functional via renormalization and its regularization sequences will also be discussed. It is shown that the theory also recover the Hida calculus.

Martin Lindsay: Lévy processes on compact quantum groups & stochastic flows on noncommutative manifolds. A structure theory of Lévy processes on compact quantum groups will be described. This extends the bounded-generator case developed with Skalski, and the original algebraic theory due to Schürmann and coworkers. By restriction to classical compact groups, all the classical Lévy processes are realised. If time permits, a characterisation will be given of the class of isometric quantum stochastic flows on an 'admissible' spectral triple of finite compact type.

The key ingredients are: the representation theory of compact quantum groups, in particular the Peter-Weyl theory; quantum stochastic cocycles, and their generation via quantum stochastic differential equations in the sense of Hudson, Evans and Parthasarathy; and quantum isometry groups of noncommutative manifolds in the sense of Goswami.

This is joint work with Biswarup Das. It was supported by the UKIERI Research Collaboration Network *Quantum Probability, Noncommutative Geometry & Quantum Information*.

Wladyslaw A. Majewski: On the structure of positive maps. A natural and intrinsic characterization of the structure of positive maps is given. This seems to be a partial answer to an old open problem studied both in Quantum Information and Operator Algebras. Our arguments are based on the concept of exposed points and links between tensor products and mapping spaces. The detailed analysis of low dimensional case as well illustrative examples will be presented. In particular, the nature of the first example of non-decomposable map (the Choi map) will be explained.

Oliver Margetts: Quasifree cocycles. The quantum stochastic calculus associated with a quasifree representation of the CCR algebra is developed. This extends previous work, by covering the cases of infinite dimensional and squeezed noise. Using this we show that every strongly continuous operator cocycle adapted to this filtration is the solution of a QSDE.

Pierre Martinetti: Kantorovich metric in noncommutative geometry. We shall give an overview of the metric aspect of noncommutative geometry, emphasizing how Connes' spectral distance formula allows to extend to the quantum setting the Monge-Kantorovich metric between probability distributions. Several applications to physics will be discussed, from the recently discoveredHiggs field to the Moyal plane.

Takashi Matsuoka: Mutual entropy and Quantum correlation. We discuss the correlations on classical and quantum system from the information theoretical points of view. There exists an essential difference between such two types of correlation. How can we understand the difference? In this talk we will try to answer this question through the analysis of mutual entropy on quantum composite systems.

Carlos Manuel Mora: Ehrenfest-type theorems for open quantum systems. This talk is based on a joint work with Franco Fagnola, Politecnico di Milano, where we study open quantum systems with state space $L^2\left(\mathbb{R}^d, \mathbb{C}\right)$. First, we develop basic properties of quantum systems in Gorini-Kossakowski-Sudarshan-Linblad form with Hamiltonian

$$H(t) = -\alpha\Delta + i\sum_{j=1}^{d}\left(A^j(t,\cdot)\partial_j + \partial_j A^j(t,\cdot)\right) + V(t,\cdot)$$

and interaction operators

$$L_\ell(t) = \begin{cases} \sum_{j=1}^{d} \sigma_{\ell j}(t,\cdot)\,\partial_j + \eta_\ell(t,\cdot), & \text{if } 1 \le \ell \le m \\ 0, & \text{if } \ell > m \end{cases} .$$

Here $t \geq 0$, $m \in \mathbb{N}$, α is a non-negative real constant, ∂_j denotes the partial derivative with respect to the j^{th}-coordinate, $V, A^j : [0,+\infty[\times\mathbb{R}^d \to \mathbb{R}$ and $\sigma_{\ell j}$, $\eta_\ell : [0,+\infty[\times\mathbb{R}^d \to \mathbb{C}$ are measurable smooth functions. Then, we establish that the mean-value of the observable A at time t satisfies, roughly speaking, the Ehrenfest-type equation

$$\begin{aligned} &\frac{d}{dt}tr\left(\rho_t A\right) \qquad (2)\\ &= tr\left(\rho_t\left(-i\left[A, H(t)\right] + \frac{1}{2}L_\ell(t)^*\left[A, L_\ell(t)\right] + \frac{1}{2}\left[L_\ell(t)^*, A\right]L_\ell(t)\right)\right), \end{aligned}$$

where A is relatively bounded with respect to $-\Delta + |x|^2$, ρ_t denotes the reduced density operator at time t, and $[\cdot,\cdot]$, resp. $tr(\cdot)$, stands for the commutator between two operators, resp. the trace operation. Finally, using (2), together with its stochastic version, we study the dynamics of physical systems such as fluctuating ion traps and quantum measurement processes of position.

Supported in part by FONDECYT Grant 1110787 and BASAL Grants PFB-03 and FBO-16.

Nobuaki Obata: Distance k-graphs of direct product graphs and their asymptotic spectral distributions. Let $G = (V, E)$ be a graph and $G^N = (V^N, E^N)$ the direct product of N copies of G. The distance k-graph is a graph $G^{(N,k)} = (V^N, E^{(N,k)})$, where $x = (\xi_1, \ldots, \xi_N), y = (\eta_1, \ldots, \eta_N) \in V^N$ are adjacent if and only if $\partial_{G^N}(x, y) = k$. We are interested in the asymptotic spectral distribution of $G^{(N,k)}$ as $N \to \infty$. It is shown that the limit distribution is described by $H_k(g)$, where H_k is the Hermitian polynomial and g is a Gaussian variable. Moreover, for $G = K_2$ (complete graph with two vertices), i.e., G^N is a hypercube, given suitable weights on edges, the limit distribution is described by $H_k^q(g_q)$, where H_k^q is the q-Hermitian polynomial and g_q is a q-Gaussian variable. The presentation is based on the joint work with Hun Hee Lee (Chungbuk national University) and Yuji Hibino (Saga University).

Masanori Ohya: Mathematical formulation of adaptive dynamics and non-Kolmogorov probability theory. There exist several phenomena (systems) breaking the classical probability laws. Such systems are not Kolmogorovian so that they are contextualdependent adaptive systems. In this talk, we present mathematical foundation of adaptive dynamics,and we introduce a new formula to compute the probability in those systems by using the adaptive dynamics and the lifting. (joint work with Keito Sato)

Dimitri Petritis: Random walks on directed graphs.

Roberto Quezada: A cycle representation and entropy production for circulant quantum Markov semigroups. After introducing a notion of entropy production for quantum Markov semigroups (qms), we study stationary states of a family of GKSL generators whose restriction to invariant subspaces coincides with the action of a circulant irreducible matrix. We compute explicitly the entropy production for these class of qms and prove that zero entropy production holds if and only if the qms satisfies a quantum detailed balance condition.

Joint work with Jorge Bolanos

Kay Schwieger: Convergence of diagonal quantum couplings. In this talk we provide a Quantum Coupling Inequality, similar to the well-known inequality of the classical coupling method. Moreover, we present the construction of the diagonal coupling associated to a tensor dilation and, finally, state conditions for this coupling to converge.

Kalyan Sinha: A Homomorphism theorem for Quantum Stochastic flows and Trotter Product for Processes. A theorem for testing the homomorphism property of flows is proven , in a spirit similar to that for the unitarity of solutions of H-P equations . This is then used to prove the strong convergence of Trotter-like product of stochastic flows.

Adam Skalski: Hopf images, idempotent states and matrix models of quantum groups. A Hopf image of a given representation of a Hopf algebra A is the largest Hopf quotient of A through which the representation factorises in a natural way. This notion was introduced by T. Banica and J. Bichon in 2010. If the Hopf image is equal to A, the considered representation is called inner faithful. The question of the existence of an inner faithful finite-dimensional representation of a given Hopf algebra is a natural counterpart of the investigation of linearity of a given discrete group G. Hence Hopf algebras admitting such representations are called inner linear. In this talk, we will recall the theory developed by Banica and Bichon, present a new approach to Hopf images of compact quantum groups via the theory of idempotent states developed mainly by U. Franz and A. Skalski, formulate some open problems related to inner linearity, and discuss connections to various notions of matrix models of quantum groups. (Joint work with Teodor Banica and Uwe Franz.)

Michael Skeide: Characterization of CPH-Maps — What might they be good for? (Joint work with Kappil Sumesh.)

If τ is a linear map from a C^*-algebra B to a C^*-algebra C, a **τ-map** is a map T from a Hilbert B-module E to a Hilbert C-module F such that

$$\langle T(x), T(x') \rangle \;=\; \tau(\langle x, x' \rangle).$$

Such maps are well-known in the case when τ is a homomorphism. The next step is to look a the case when τ is a CP-map. (This was proposed by Asadi and examined in a very special version with τ unital,$C = B(G)$, $F = B(G, H)$, and yet another strong strong extra condition.) Since then, such maps have drawn some attention. Some people started calling them CP-maps between Hilbert modules. But still nobody seem to have a clue what they actually might be good for.

In this talk we characterize τ-maps in full generality — and without knowing τ! We give a number of equivalent conditions. Two of them are intrinsic and allow to say if a map T is a τ-map for some CP-map τ, by just looking at T and checking some simple conditions. A third condition in terms of extension of T to a map between the linking algebras of E and of F shows that CP-map is **not** the right name for such maps T. We shall explain why we are going to call them **CPH-maps**.

Finally, we explain that CPH-maps occur in a context that generalizes the situation of (weak) dilation of CP-maps, so-called **CPH-dilations**. We are not sure, if this, finally, can answer the question what CPH-maps might be good for. But, at least, we believe that there are some obvious connections with CPD-kernels and with Morita equivalence that, finally, might **justify** the effort to analyze CPH-maps.

Igor Volovich: Black hole formation paradox and stochastic field theory. Arbitrary real numbers are unobservable. Therefore the widely used modeling of physical phenomena by using differential equations, which was introduced by Newton, does not have an immediate physical meaning. It was suggested in [1,2] that the physical meaning should be attributed not to individual trajectory in the phase space but only to probability distribution function. Even for the single particle the fundamental dynamical equation in the proposed "functional" stochastic approach is not the Newton equation but the Liouville equation or the Fokker-Planck - Kolmogorov equation. The Newton equation in functional mechanics appears as an approximate equation for the expected values ofthe position and momentum. Applications of this non-Newtonian mechanics and field theory to the black hole formation paradox will be discussed. There is an astronomical evidence that many galaxies, including the Milky Way, contain supermassive black holes at their centers. However, a paradox occurs that for the formation of a black hole an infinite time isrequired as can be seen by an external observer, and that is incontradiction with the finite time of existing of the Universe. Weargue that to solve the paradox and also to gain insight into the time irreversibility problem one has to develop a new approach to classical and quantum mechanics based on the idea of stochastic fluctuations ofall physical parameters including spacetime manifolds. In the functional approach to field theory and general relaivity onedeals with stochastic geometry of spacetime manifolds which is different from quantum gravity. Probability of formation of a black hole for the external observer in finite time during collapse is estimated. Anextension of the method of the stochastic limit [3] to the case ofstochastic field theory will be also discussed.

References

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2 M. Ohya, I. Volovich, Mathematical foundations of quantum information and computation and its applications to nano- and bio-systems, Springer, Dordrecht, 2011.

3 L. Accardi, Lu Yun Gang, I. Volovich, Quantum theory and its stochastic limit, Springer-Verlag, Berlin, 2002.

Wilhelm von Waldenfels: New results about the amplified oscillator. The quantum stochastic differential equation canbe solved in a closed, normal ordered form. Fromthere one establishes an apriori estimate for the solution. This allows the proof of theunitarity and the establishment of the Heisenbergequations. From there a better estimate can be obtained, which enables the calculation of the Hamiltonianof the corresponding one-parameter group. Amplification, due to the process, will be shown in some examples.

Noboru Watanabe: On Complexities of Quantum Transmission Processes.

Janusz Wysoczanski: Orthogonal polynomials related to anyon statistics. Let T be a locally compact Polish space (i.e. separable, completely metrizable topological space), with a non-atomic Radon measure σ. Let $D \subset T \times T$ be the diagonal:

$$D := \{(t,t) \in T^2 | t \in T\}$$

Let also $A \subset T \times T$ be symmetric subset, i.e. if $(s,t) \in A$ then $(t,s) \in A$, containing the diagonal $D \subset A$. We consider a hermitian kernel (on the complement of A):

$$Q : (T \times T \setminus A) \to S^1 := \{z \in \mathbb{C} : |z| = 1\}, \quad Q(s,t) = \overline{Q(t,s)}.$$

The basic example is the anyonic case: for $T = \mathbb{R}$ or $T = \mathbb{R}^+$ and $A = D$ and fixed $|q| = 1$ we put

$$Q(s,t) := \begin{cases} q & \text{if} \quad s < t \\ \overline{q} & \text{if} \quad t < s \end{cases}$$

One defines the Q-deformed (i.e. Q-symmetric) Fock space (for $\mathcal{H} := L^2(T,\sigma)$) andthe related Q-creation,and Q-neutral operators. Rigorous meaning is given to the Q-creation, Q-annihilation and Q-neutral operators at points:

$$\partial_t^\dagger := a^\dagger(\delta_t), \quad \partial_t := a(\delta_t), \quad \partial_t^\dagger \partial_t := a^0(\delta_t).$$

These operators satisfy Q-deformed commutation relations:

$$\begin{aligned} \partial_s \partial_t^\dagger &= \delta(s,t) + Q(s,t)\partial_t^\dagger \partial_s \\ \partial_s \partial_t &= Q(t,s)\partial_t \partial_s \\ \partial_s^\dagger \partial_t^\dagger &= Q(t,s)\partial_t^\dagger \partial_s^\dagger \end{aligned}$$

For $\lambda > 0$ one defines $\omega(t) := \partial_t^\dagger + \lambda \partial_t^\dagger \partial_t + \partial_t$ (generalized Q-Gaussian field if $\lambda = 0$ and Q-Poisson (centered) field if $\lambda = 1$).

The talk will focus on the related Q-Hermite ($\lambda = 0$) and Q-Charlier ($\lambda = 0$) orthogonal polynomials. We obtain some recurrence relations for these polynomials and study associated Wick ordering. The combinatorics of marked partitions appears as a tool.

The talk is based on the paper:

Bożejko, M., Lytvynov, E. Wysoczański, J. *"Non-commutative Lévy processes for generalized (particularly anyon) statistics"*, Commun. Math. Phys. 313 (2012), 535–569.