CATS4

Structures catégoriques supérieures en géométrie algébrique

2-7 Juillet 2012

1. **Oren Ben-Bassat:** *Sub-varieties and Descent.*

   Let X be a variety and Z be a sub-variety. Can one glue vector bundles on X-Z with vector bundles on some small neighborhood of Z? We survey two recent results on the process of gluing a vector bundle on the complement of a sub-variety with a vector bundle on some 'small' neighborhood of the sub-variety. This is joint work. The first with M. Temkin and available at arxiv.org/abs/1201.4227 and is about gluing categories of coherent sheaves over the category of coherent sheaves on a Berkovich analytic space. The second with J. Block and available at arxiv.org/abs/1201.6118 is about gluing dg enhancements of the derived category of coherent sheaves.

2. **Clemens Berger:** *On the homotopy theory of enriched categories*

   We give sufficient conditions for the existence of a Quillen model structure on small categories enriched in a given monoidal model category. This yields a unified treatment for the known model structures on simplicial, topological, dg- and spectral categories. Our proof is mainly based on a fundamental property of cofibrant enriched categories on two objects, the so-called Interval Cofibrancy Theorem.

3. **Julie Bergner:** *Comparing models for* (∞, n)*-categories.*

   With many definitions being given for (∞, n)-categories, one criterion to check is whether they can be thought of as categories enriched in $(\infty, n-1)$-categories. In joint work with Charles Rezk, we are establishing a chain of Quillen equivalences from the model structure for Θ_n-spaces and the model structure for categories enriched in Θ_{n-1}-spaces. This comparison also gives insight into how to compare to other known models.

4. **Jonathan Block:** *The Baum-Connes conjecture vs Beilinson Bernstein.*

   We explore connections between the Baum-Connes conjecture in operator K-theory and the geometric representation theory of reductive Lie groups. We shall recast the topological K-theory approach to the Weyl character formula, due basically to Atiyah and Bott, in the language of Kasparov's KK-theory. Then we shall show how our KK-theoretic Weyl character formula can be carried over to noncompact groups, where it is related to both the Baum-Connes conjecture and to Beilinson Bernstein's localization theory. A crucial tool is Lurie's Barr-Beck theorem.

5. **Mikhail Bondarko:** *Weight structures and various filtrations for cohomology.*

   There are several important constructions that describe certain functorial (cohomological) invariants in non- canonical (or 'not very much functorial') terms. For example, various cohomology of objects and complexes over an abelian category is often defined in terms of injective or projective (hyper)resolutions; Deligne's weights for singular cohomology (and for the corresponding mixed Hodge structures) are defined in terms of nice compactifications and smooth hyperresolutions of varieties; Atiyah-Hirzebruch spectral sequences are defined in terms of cellular filtration of spectra (which can be considered as objects of the stable homotopy category). It turns out that all of these examples can be 'explained' and studied via weight structures for triangulated categories. Weight structures are defined somewhat similarly to t-structures and are deeply related with them; they are also closely related with stupid truncations of complexes. The speaker will describe several examples of weight structures (and of related constructions); the most interesting of them are 'motivic'.

6. **Damien Calaque:** *Classical TFTs and symplectic structures on mapping spaces with boundary conditions.*

   In this talk I will present an extension of a recent result of Pantev-Toen-Vaquie-Vezzosi on the construction of shifted symplectic structures on derived mapping stacks. The main new ingredient is the presence of boundary conditions. I will then explain how this is related to the project of constructing various extended TFTs with values in an appropriate category of lagrangian correspondences.

7. **Denis-Charles Cisinski:** *Dendroidal sets and simplicial operads I*

   The category of dendroidal sets is an extension of that of simplicial sets, suitable for defining nerves of operads rather than just of categories. In part I, we will introduce this category of dendroidal sets, and prove that it carries the structure of a monoidal Quillen model category. The Quillen structure extends the Joyal structure on simplicial sets, while the monoidal structure is reminiscent of the Boardman-Vogt tensor product between operads. The fibrant objects in this model category provide a model for the notion of infinity-operad. In part II, we will show that there is Quillen equivalence between this model category and the category of simplicial (or topological) coloured operads. The proof of this equivalence uses a dendroidal version of the Rezk theory of complete Segal spaces, which (if time) we will also explain.

8. **Tobias Dyckerhoff:** *Higher Segal Spaces (Part I).*

9. **David Favero:** *Variation of Geometric Invariant Theory for Derived Categories*

   Given a quasi-projective algebraic variety, X, with the action of a linear algebraic group, G, there are various (birational) incarnations of the quotient X/G coming from a choice of a G-equivariant ample line bundle. As we vary this choice, there is a semi-orthogonal relationship between the derived categories of the resulting quotients, A and B. Furthermore, if (X,w) is a Landau-Ginzburg model, and w is a G-invariant section of a line bundle on X, then the same holds for "coherent sheaves on" (A,w) and (B,w) (categories of matrix factorizations/categories of singularities/stable derived categories). As special cases, one can produce full exceptional collections of sheaves on moduli spaces of n-points on a rational curve, and reproduce a theorem of Orlov relating categories of coherent sheaves for complete intersections in projective space to the graded category of singularities of the cone, a theorem of Herbst and Walcher demonstrating an equivalence of derived categories between "neighboring" Calabi-Yau complete intersections in toric varieties, and two theorems of Kawamata; one concerning behavior of derived categories of algebraic varieties under simple toroidal flips, the other stating that the derived category of coherent sheaves on any smooth toric variety has a full exceptional collection (in the projective case).

10. **Denis Gaitsgory:** *Singular support of coherent sheaves.*

11. **Dominic Joyce:** *D-manifolds and derived differential geometry.*

    I will describe a new class of geometric objects I call "d-manifolds". D-manifolds are a kind of "derived" smooth manifold, where "derived" is in the sense of the derived algebraic geometry of Jacob Lurie, Bertrand Toen, etc. The closest thing to them in the literature is the "derived manifolds" of David Spivak (Duke Math. J. 153 (2010), 55-128). But d-manifolds are rather simpler than Spivak's derived manifolds – d-manifolds form a 2-category which is constructed using fairly basic techniques from algebraic geometry, but derived manifolds form a simplicial category which uses advanced ideas like homotopy sheaves and Bousfeld localization.

    Manifolds are examples of d-manifolds – that is, the category of manifolds embeds as a full subcategory of the 2- category of d-manifolds – but d-manifolds also include many spaces one would regard classically as singular or obstructed. A d-manifold has a virtual dimension, an integer, which may be negative. Almost all the main ideas of differential geometry have analogues for d-manifolds – submersions, immersions, embeddings, submanifolds, orientations, transverse fibre products, and so on – but the derived versions are often stronger. For example, the intersection of two submanifolds in a manifold exists as a manifold if the intersection is transverse, but it always exists as a d-manifold. There are also good notions of d-manifolds with boundary and d-manifolds with corners, and orbifold versions of all this, d-orbifolds.

    D-manifolds and d-orbifolds have well-behaved virtual classes or virtual chains. There are truncation functors from geometric structures currently used to define virtual classes to d-manifolds and d-orbifolds. For instance, any moduli space of solutions of a smooth nonlinear elliptic p.d.e. on a compact manifold is a d-manifold. In algebraic geometry, a C-scheme with a perfect obstruction theory can be made into a d-manifold. In symplectic geometry, Kuranishi spaces and polyfold structures on moduli spaces of J-holomorphic curves induce d-orbifold structures.

12. **Dimitry Kaledin:** *Hochschild-Witt homology.*

For any DG category over a finite field, we define a homology theory that in the case of algebraic varieties reduces to the de Rham-Witt forms, thus giving a non-commutative generalization of cristalline cohomology. Our theory has all the additional sructures known for cristalline cohomology, and it has a localization property for short exact sequences of DG categories, similar to Hochschild and cyclic homology.

13. **Mikhail Kapranov:** *Higher Segal Spaces (Part II).*

14. **Ludmil Katzarkov:** *Stability Hodge Structures and applicatons.*

    In this talk we will introduce a new categorical notion - Stability Hodge Structures. We will consider applications to classical questions in Algebraic Geometry.

15. **Wendy Lowen:** *On compact generation of deformed schemes.*

    We discuss a theorem which allows to prove compact generation of derived categories of Grothendieck categories, based upon certain coverings by localizations. This theorem follows from an application of Rouquier's cocovering theorem in the triangulated context, and it implies Neeman's result on compact generation of quasi-compact separated schemes. We give an application of our theorem to non-commutative deformations of such schemes.

16. **Ieke Moerdijk:** *Dendroidal sets and simplicial operads II*

    The category of dendroidal sets is an extension of that of simplicial sets, suitable for defining nerves of operads rather than just of categories. In part I, we will introduce this category of dendroidal sets, and prove that it carries the structure of a monoidal Quillen model category. The Quillen structure extends the Joyal structure on simplicial sets, while the monoidal structure is reminiscent of the Boardman-Vogt tensor product between operads. The fibrant objects in this model category provide a model for the notion of infinity-operad. In part II, we will show that there is Quillen equivalence between this model category and the category of simplicial (or topological) coloured operads. The proof of this equivalence uses a dendroidal version of the Rezk theory of complete Segal spaces, which (if time) we will also explain.

17. **Anatoly Preygel:** *To be announced.*

18. **Chris Schommer-Pries:** *The unicity of the homotopy theory of higher categories.*

    We will describe joint work with Clark Barwick in which we provide a complete axiomatization of the homotopy theory of (infty,n)-categories. The space of theories satisfying our axioms is a $B(Z/2)^n$ and includes the models of Barwick, Bergner, Joyal, Kan, Lurie, Rezk, and Simpson, among others. This generalizes a theorem of Toën when n = 1, and it verifies two conjectures of Simpson.

19. **Timo Schürg:** *Orientations for quasi-smooth morphisms.*

    Quasi-smooth morphisms between derived schemes lead to natural orientations in various homology theories. The universal homology theory having such orientations is derived algebraic bordism. This is a homology theory defined similar to algebraic bordism constructed by Levine and Morel, but using quasi-smooth derived schemes as generators. Comparing the canonical orientations obtained in the homology theories K_0 and Chow groups leads to a virtual version of the Grothendieck-Riemann-Roch theorem.

20. **Aaron Smith:** A_∞*-connections in the higher Riemann-Hilbert correspondence.*

    I will recapitulate the results of previous work with Jonathan Block in which we prove a higher homotopical version of the classical Riemann-Hilbert correspondence. Then, I will introduce some recent calculations which extend this framework to a correspondence between flat A_∞-connections and infinity-local systems valued in A_∞ algebras.

21. **Paolo Stellari:** *Derived categories of cubic hypersurfaces and ACM bundles.*

    In this talk we use Kuznetzov's description of the derived categories of smooth cubic hypersurfaces to answer some questions concerning the geometry of such manifolds. We pay particular attention to moduli problems and we give new constructions of stable ACM bundles on cubic threefolds and fourfolds. This is a joint work with M. Lahoz and E. Macri.

22. **Constantin Teleman:** *Gauge theory, Mirror symmetry and Langlands duality.*

23. **Boris Tsygan:** *Microlocal determinant formulas.*

    In addition to index and Riemann-Roch theorems that provide a topological formula for the Euler characteristic of a complex of geometric origin, there are analogous results that compute the determinant line of the cohomology of this complex. They include: a) Beilinson's microlocal formula for the determinant of cohomology of a constructible sheaf; b) Patel's formula for the determinant of cohomology of a D-module; c) a conjectural microlocal formula for the determinant of cohomology of an elliptic pair. The compatibility between a) and b), proved by Beilinson, implies product formulas for determinants of period matrices (the simplest case being the expression of the Beta function as a product of Gamma functions). The conjectural c) would be a multiplicative analog of the index theorem for elliptic pairs (Schapira-Schneiders; Bressler-Nest-Tsygan) and could be related to regularized determinants of elliptic differential operators on real analytic manifolds. We will review the subject and formulate the conjecture.

24. **James Wallbridge:** *Towards a theory of derived geometric quantization.*

25. **Chenchang Zhu:** *Proper actions of topological 2-groupoids.*

Properness of topological groupoid makes it easy to transfer corresponding objects from differentiable geometry to non-commutative geometry. Since C^*-algebras form a 2-category, 2-groupoids acts naturally on them. Thus it is considerable to bring higher groupoids as symmetry in the world of non-commutative geometry. This right away clarifies several existing concepts such as twisted actions in the sense of Green or BusbySmith therein. Moreover, via desingularization, 2-groupoid also helps in the case of Connes' example on non-Hausdorff 1-groupoids coming from foliations. Our long term goal is to use 2-groupoids to overcome some difficulties in Baum-Connes conjecture.

In this talk, we describe actions of topological ∞-groupoids in the sense of Kan complex on themselves via Kan fibration similar to that of Glenn's treatment. We apply this to topological 2-groupoid actions on groupoids. The simplicial method allows us to describe naturally what free and proper, or proper actions are. The notion of properness includes desired examples motivated by non-commutative geometry.

This talk is based on joint works with Alcides Buss, Du Li and Ralf Meyer.