

La conjecture de Zilber-Pink 16–20 mai 2011
Programme

	Lundi	Mardi	Mercredi	Jeudi	Vendredi
9h–9h55	Yafaev	Scanlon	Habegger	Rémond	Ullmo
10h–10h55	Scanlon	Ullmo	Ullmo	Scanlon	Rémond
11h25–12h25	Rémond	Habegger	Scanlon	Yafaev	Habegger
14h–14h55					Scanlon
15h–15h55					Yafaev
17h–18h	Ullmo	Yafaev		Table	
18h25–19h25	Habegger	Rémond		ronde	

Philippe Habegger : Bounded height conjectures.

Lecture 1: Historic overview of “unlikely intersections” in semi-abelian varieties
 Lecture 2: Height upper bounds on algebraic tori I: overview of the proof
 Lecture 3: Height upper bounds on algebraic tori II: questions of effectivity
 Lecture 4: Consequences of the higher dimensional case for the Zilber-Pink Conjecture

Gaël Rémond : Problèmes de Lehmer, de Bogomolov et applications.

Ce mini-cours présente diverses minoration de hauteurs généralisant le problème de Lehmer ainsi que leurs applications à la conjecture de Zilber-Pink (sur les variétés abéliennes et les tores).

Cours 1 : Notion de hauteur. Problème de Lehmer. Théorème de Dobrowolski.

Cours 2 : Extension du problème de Lehmer en dimension supérieure et aux variétés abéliennes. Problème de Lehmer relatif. Problème de Bogomolov effectif. Formulation générale.

Cours 3 : Application du problème de Lehmer relatif à la conjecture de Zilber-Pink.

Cours 4 : Application du problème de Bogomolov effectif à la conjecture de Zilber-Pink.

Thomas Scanlon : Theorems on unlikely intersections by counting points in definable sets.

Pila has proven, unconditionally, the André-Oort conjecture for products of modular curves by implementing a strategy proposed by Zannier whereby lower bounds on the size of the Galois orbit of a special point are leveraged against upper bounds obtained from a counting theorem of Pila and Wilkie on rational points in definable sets. This method has already been exploited by several authors (Daw, Habegger, Masser, Peterzil, Pila, Starchenko, Yafaev, and Zannier) to prove other instances of the Pink-Zilber conjecture on unlikely intersections.

With this lecture series, I plan to present a high level overview of the strategy and then to present many of the details of these proofs. Specifically, I will begin with an overview of the method sketching Pila’s proof of the André-Oort conjecture but delaying a discussion of the key results to the later lectures. With the remaining lectures, I will give a detailed accounts of the following topics: o-minimality, subanalytic and exponential-algebraic geometry, cell decomposition, the Pila-Wilkie counting theorem, functional Ax-Lindemann-Weierstrass theorems, Masser-Zannier theorem on simultaneous torsion, Pila on Zilber-Pink. The lectures will close with a prospectus for extensions of this method.

Emmanuel Ullmo and Andrei Yafaev : The André-Oort conjecture under GRH.

Lecture 1 (Yafaev): Special points, special subvarieties: an introduction to André-Oort and Zilber-Pink conjectures for Shimura varieties.

Lecture 2 (Ullmo): Strategy of the proof. The case of products of modular curves and the Manin-Mumford conjecture.

Lecture 3 (Ullmo): Preliminaries on Shimura varieties.

Lecture 4 (Yafaev): Lower bounds for Galois orbits of special subvarieties.

Lecture 5 (Ullmo): Ergodic theory on homogeneous spaces $\Gamma \backslash G(\mathbf{R})$.

Lecture 6 (Yafaev): Hecke orbits and characterisation of special subvarieties.

Lecture 7 (Ullmo): Equidistribution of special subvarieties.

Lecture 8 (Yafaev): An overview of the proof of the André-Oort conjecture under GRH.