SESSION DE PRINTEMPS du Séminaire Commun d'Analyse Géométrique
(CIRM, 5-6 Mars 2010)
ATELIER sur le thème:
'Equations hyperboliques en métrique de Kerr'
coordonné par Dietrich HÄFNER (Institut Fourier, Grenoble)

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Programme & modalités
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Un des problèmes ouverts majeurs en Relativité Générale est la question de la stabilité non linéaire de la métrique de Kerr qui décrit des trous noirs en rotation. L'obtention d'estimations dispersives pour l'équation des ondes dans cet espace-temps est un premier pas dans cette direction. Récemment de telles estimations ont été obtenues par plusieurs auteurs; le but de l'atelier sera d'étudier ces travaux en détail. Nous reviendrons également sur la preuve de la stabilité non linéaire de l'espace-temps de Minkowski pour discuter quelle stratégie devrait être adaptée pour la métrique de Kerr.

VENDREDI 5 MARS 2010
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Exposé 1 (14:30 - 15:45)
Romain GICQUAUD (Univ. Tours)
http://www.lpta.univ-montp2.fr/users/gicquaud/

Titre: " Stabilité non linéaire de l'espace de Minkowski, d'après H. Lindblad et I. Rodnianski "

Résumé:
Dans cet exposé, j'introduirai la condition de nullité faible satisfaite par les équations d'Einstein du vide en coordonnées harmoniques. Je montrerai ensuite comment cette condition permet tout d'abord d'établir l'existence globale de solutions pour des données initiales proches de la métrique de Minkowski. Puis je montrerai ensuite comment la condition de nullité faible permet d'obtenir une nouvelle preuve de la stabilité non linéaire de l'espace-temps de Minkowski simplifiant la preuve originale de D. Christodoulou et S. Klainerman.

Références:
Hans Lindblad, Igor Rodnianski, Global existence for the Einstein vacuum equations in wave coordinates.
arXiv:math/0312479

Hans Lindblad, Igor Rodnianski, The global stability of the Minkowski space-time in harmonic gauge, *Annals of Math.* (to appear), arXiv:math/0411109

Exposé 2 (16:00 - 17:15)
Erwann AUBRY (Univ. Nice Sophia Antipolis)
http://math.unice.fr/~eaubry/

Title: " Boundedness of the solutions of the Klein Gordon equation on the Schwarzschild metric "

Abstract:
Following Dafermos-Rodnianski, we will prove a classical L^∞ bound on the solutions of the Klein Gordon equation on Scharzschild exterior backgrounds due to Kay and Wald. The original proof is based on vector field commutators and multipliers, elliptic estimates and Sobolev inequality. We will also present another proof due to Dafermos and Rodnianski which improves the result.

Reference:
Kay, Bernard S.; Wald, Robert M. Linear stability of Schwarzschild under perturbations which are nonvanishing on the bifurcation 2-sphere. *Classical Quantum Gravity* **4**:4 (1987) 893-898.

Exposé 3 (17:30 - 18:45)
Laurent MICHEL (Univ. Nice Sophia Antipolis)
http://math.unice.fr/~lmichel/

Titre: " Résonances et estimations de la résolvante pour des trous noirs à symétrie sphérique "

Résumé:
Le but de cet exposé est de montrer comment certains résultats sur l'équation de Schrödinger semiclassique entrent dans la description des solutions de l'équation des ondes pour la métrique de De Sitter-Schwarzschild (en termes de résonances).

Références:
Sá Barreto, Antônio; Zworski, Maciej. Distribution of resonances for spherical black holes. *Math. Res. Lett.* **4**:1 (1997)103-121.

Bony, Jean-François; Michel, Laurent. Microlocalization of resonant states and estimates of the residue of the scattering amplitude. *Comm. Math. Phys.* **246**:2 (2004) 375-402.

Bony, Jean-François; Häfner, Dietrich. Decay and non-decay of the local energy for the wave equation on the de Sitter-Schwarzschild metric. *Comm. Math. Phys.* **282**:3 (2008) 697-719.

SAMEDI 6 MARS 2010
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Exposé 4 (9:30 - 10:30)
Dietrich HÄFNER (Institut Fourier, Grenoble)
http://www.math.u-bordeaux1.fr/~hafner/

Title: " Introduction to the Kerr metric "

Abstract:
This talk is an introduction to the Kerr metric. This metric describes eternal rotating black holes. We discuss its form in Boyer-Lindquist coordinates, the construction of the horizons, the ergosphere and null geodesics.

Reference:
Barrett O'Neill : The geometry of Kerr black holes. A.K. Peters Wellesley, Mass. (1995)

Exposé 5 (10:45 - 12:00)
Dietrich HÄFNER (Institut Fourier, Grenoble)
http://www.math.u-bordeaux1.fr/~hafner/

Title: " The redshift effect "

Abstract:
This talk is based on work by Dafermos and Rodnianski. We start by explaining the classical red-shift from a physical point of view. We explain how this effect can be used to show decay estimates for the wave equation on the Schwarzschild metric. This is linked to a "vector field version" of the classical red-shift effect. Eventually we explain how the redshift effect can be used to treat the superradiant modes of the wave equation in the Kerr geometry for small angular momentum of the space-time.

References:
Dafermos, Mihalis; Rodnianski, Igor. The red-shift effect and radiation decay on black hole spacetimes. Comm. Pure Appl. Math. **DXII**:7 (2009) 859-919.
Dafermos, Mihalis; Rodnianski, Igor. Lectures on black holes and linear waves, arXiv:0811.0354

Exposé 6 (13:15 - 14:15)
Colin GUILLARMOU (Ecole Normale Supérieure)
http://www.dma.ens.fr/~guillarmou/

Title: " Local energy estimate on Kerr black hole backgrounds (after Tataru-Tohaneanu)"

Abstract:
This talk is based on work by Tataru and Tohaneanu.
We study dispersive properties for the wave equation in the Kerr space-time with small angular momentum. The main results are uniform energy bounds and local energy decay for such backgrounds.

Reference:
Daniel Tataru, Mihai Tohaneanu, Local energy estimate on Kerr black hole backgrounds, arXiv: 0810.5766

Exposé 7 (14:30 - 15:30)
Pieter BLUE (Univ. Edinburgh)
http://www.maths.ed.ac.uk/~pblue

Title: " Decay for the wave equation outside a Kerr black hole " (joint work with L. Andersson)

Abstract:
The wave equation outside a Kerr spacetime has been actively studied in the last decade. It provides a model for the open problem of stability of the Kerr spacetime and is itself a challenging mathematical problem. The linear wave equation is challenging because it lacks a positive, conserved energy, because there is a complicated trapping set, and because it lacks a full set of symmetries with which to generate Sobolev norms. In this talk, I will summarise our recent work on extending the vector-field method to prove decay estimates for the wave equation. I will also compare our results and methods with the Fourier transform methods used by others.

Reference:
Andersson, Lars; Blue, Pieter: Hidden symmetries and decay for the wave equation on the Kerr spacetime, arXiv:0908.2265.

Exposé 8 (15:45 - 16:45)
Lars ANDERSSON (MPI, Golm)
http://www.math.miami.edu/~larsa/

Title: " Hidden symmetries and linear fields on Kerr "

Abstract:
In this talk I will discuss the field equations for linear fields on the Kerr spacetime, including solutions to the scalar wave equation, the Maxwell equation and linearized gravity.

The Kerr spacetime possesses a hidden symmetry in the form of a Killing tensor. I will give an overview of its properties and its role in the analysis of the field equations on Kerr. Among the important consequences of the presence of the Killing tensor are that it allows the linear field equations to be separated. Further, it can be used to construct symmetry operators and potentials, which allows one to define generalized generalized currents with useful positivity properties.