Topologie Algébrique et Quantique

(en l'honneur de Pierre VOGEL)

CIRM, 19 – 23 avril 2010

Final Program[1] **:**

Lundi/ Monday 19 April

9h15	- 10h15	M. Boileau : *Algebraic properties of 3-manifolds fundamental groups*
11h	- 12h	R. Oliver : *Equivalences of fusion between finite Lie groups*
16h30	- 17h30	F. Costantino : *On "quantum" polyhedra*
17h45	- 18h45	A. Touzé : *Cohomological finite generation for reductive groups*

Mardi/ Tuesday 20 April

9h15	- 10h15	D. Chataur : *Bivariant chains and duality*
11h	- 12h	J. Marché : *Localization of colored Jones polynomials*
16h30	- 17h30	C. Cazanave : *The* $\mathbf{A}^1$*-homotopy type of Atiyah-Hitchin schemes*
17h45	- 18h45	B. Audoux : *Heegaard-Floer homology and finite type properties*

[1] Because of major flight restrictions in Europe caused by ash from Iceland's Eyjafjallajokull volcano, the following speakers have not been able to come to CIRM and their talks have therefore been cancelled : *J. E. Andersen, D. Bar-Natan, P. Gilmer, T. Le, K. Orr, A. Ranicki, H. Reich, V. Turchin.*

Mercredi/ Wednesday 21 April

9h00	-	9h50	G. Massuyeau : *Homology cylinders and the tree-level of the LMO homomorphism*
10h20	-	11h10	J. Davis : *A two component link with Alexander polynomial one is concordant to the Hopf link*
11h30	-	12h20	E. Wagner : *Variations around Khovanov link homologies*
			(free afternoon)

Jeudi/ Thursday 22 April

9h00	-	9h50	E. Farjoun : *Homotopy Normal maps, principal fibration and monoidal functors.*
10h20	-	11h10	R. Kashaev : *Triangulations, Hamiltonian paths, and knot invariants*
11h30	-	12h20	K. Habiro : *On certain limits of the reduced colored Jones polynomials of knots*
16h30	-	17h30	P. Vogel : *The universal Lie algebra : properties and conjectures*
17h45	-	18h45	J.-C. Hausmann : *Conjugation spaces and manifolds*
		19h30	Conference Dinner

Vendredi/ Friday 23 April

9h15	-	10h15	R. Budney : *A universal algebra for (spaces of) knots*
11h	-	12h	M. Kreck : 3-*manifolds, codes, and arithmetic*

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Résumés/ Abstracts :

Benjamin Audoux : *Heegaard-Floer homology and finite type properties*

Heegaard-Floer homology is a closed 3-manifold invariant which has been introduced by P. Ozsvath and Z. Szabo. It can be refined into a link invariant which categorifies the Alexander polynomial. These invariant are very powerful and they carry many geometrical information. My talk will begin by a short review of the definition and some properties of Heegaard-Floer homology. Then I will discuss my present work which aims at determining in which way do the finite type properties of Alexander polynomial survive their categorification.

Michel Boileau : *Algebraic properties of 3-manifolds fundamental groups*

The solution of the geometrisation conjecture by G. Perelman shows that 3-manifolds fundamental groups share some important algebraic properties that will be presented. In connection with the problem of characterizing these groups, properties of groups satisfying a Poincaré duality in dimension 3 will be discussed.

Ryan Budney : *A universal algebra for (spaces of) knots*

In 1949 Schubert gave the connect-sum decomposition for knots. One way to say this is that oriented smooth embeddings of S^1 in S^3 taken up to ambient isotopy form a free commutative monoid under the connect-sum operation. Alternatively, let $K_{3,1}$ denote the space of smooth embeddings of $\mathbb{R}$ in $\mathbb{R}^3$ which agree with a fixed linear embedding outside of a ball. Then $\pi_0 K_{3,1}$ is a free commutative monoid with respect to connect-sum. There is a homotopy-associative "space-level" connect-sum mapping $K_{3,1} \times K_{3,1} \to K_{3,1}$. This mapping can be enhanced to an action of the operad of 2-cubes on $K_{3,1}$ and in 2006 I showed that $K_{3,1}$ is a free object over the 2-cubes operad, with free generating subspace the space of prime knots $P \subset K_{3,1}$, i.e. an operadic space-level analogue of Schubert's theorem. In 1953 Schubert generalized the connect-sum operation, creating what are known as 'satellite knots', but unlike the connect-sum operation Schubert noticed satellite knots do not decompose in a unique way. In 1979 Larry Siebenmann noticed that Schubert's satellite constructions fit with the JSJ-decomposition of 3-manifolds, giving the appropriate uniqess statement for Schubert's satellite operation thought of as a decomposition of knots. In this talk I will describe a new operad, 'the splicing operad' which encodes splicing for knots at the 'spaces of knots' level. The main theorem is that $K_{3,1}$ is free with respect to the splicing operad's action, and the free generating subspace is the subspace of $K_{3,1}$ consisting of torus and hyperbolic knots. The splicing operad itself also has a pleasant structure as an operad, although it's not quite free.

Christophe Cazanave : *The $\mathbf{A}^1$-homotopy type of Atiyah-Hitchin schemes*

Given an algebraic variety Y, the Atiyah-Hitchin schemes associated to Y are a family of new algebraic varieties indexed by a positive integer n. The motivating example, associated to the case $Y = \mathbf{A}^1 - \{0\}$, is given by the schemes of pointed degree n rational functions, say $\mathcal{F}_n$. G. Segal and F. Cohen et al. studied the homotopy type of the space of complex rational functions and proved that, as n goes to infinity, the sequence $\mathcal{F}_n(\mathbf{C})$ "tends to" $\Omega^2\mathrm{S}^3$. I conjecture that this result extends to the algebraic context of $\mathbf{A}^1$-homotopy theory : the sequence of Atiyah-Hitchin schemes associated to Y "tends to" $\Omega^{\mathbf{P}^1}(\mathbf{P}^1 \wedge Y)$ when n goes to infinity.

David Chataur : *Bivariant chains and duality*

In this talk we review some recent results on Poincaré duality at the chain level. More precisely, if M is a smooth closed oriented manifold singular geometric chains together with the intersection product is a partial commutative algebra (this follows from J. McClure's works). Introducing bivariant chains we prove that the cap product with the fundamental class induces a multiplicative quasi-isomorphism between geometric chains with the intersection product and singular cochains with the cup product. Motivations come from the study of free loop and path spaces.

François Costantino : *On "quantum" polyhedra*

I will first recall the definition of Kauffman brackets of embeddings of colored graphs in S^3, provide some examples and sketch the connection of these constructions with ideas coming from physics. I will then introduce the problem of studying the asymptotical behavior of these invariants and, restricting to the case of trivalent planar graphs, relate it to the geometry of hyperbolic polyhedra. If time permits, I will then report on some recent advances (joint with F. Gueritaud and R. Van der Veen) showing how Schläfli-like differential equations can be recovered from the combinatorial properties of the invariants.

J. Davis : *A two component link with Alexander polynomial one is concordant to the Hopf link*

M. Freedman showed that a knot with Alexander polynomial one is topologically slice, i.e. bounds a locally flat 2-disk in the 4-disk. I show an analogous result for 2-component links.

Emmanuel Farjoun : *Homotopy Normal maps, principal fibration and monoidal functors*

This is a report on work with K. Hess and M. Prezma and Bill Dwyer. Given a map of groups or loop spaces $n : G \to H$ we consider the problem of putting "compatible" group structures on the homotopy quotient $H//G = hocolim_G(H)$. A necessary and sufficient condition is developed that allows control over the application of product preserving functors to normal maps. Similar question can be considered for dga's, hopf algebras and other group-like structures. Application to principal fibration and classical questions will be considered.

Kazuo Habiro : *On certain limits of the reduced colored Jones polynomials of knots*

Dasbach and Lin studied the "head" and "tail" of the colored Jones polynomials. They proved that, up to sign, the last and the first three coefficients of the colored Jones polynomials of an alternating knot converge, and conjectured that the other coefficients also converge and yield two power series in q and in q^{-1}. In this talk, I consider similar limits for the reduced colored Jones polynomials, which are certain linear combinations of the colored Jones polynomials.

Jean-Claude Hausmann : *Conjugation spaces and manifolds*

There are well known spaces X with an involution τ such that the mod 2-cohomology ring of X most ressembles that of its fixed point set X^τ, via a ring isomorphism $H^{2*}(X) \approx H^*(X^\tau)$ dividing the degrees in half. Examples include complex Grassmannians, toric manifolds, polygon spaces, etc. These examples enjoy further properties making them *conjugation spaces* (or *conjugation manifolds*), a concept introduced in a joint paper by the author, T. Holm and V. Puppe (AGT, 2005).

After a survey of conjugation spaces and their main porperties, we shall discuss the problem of existence and classification of conjugation manifolds having a given fixed point set, with recent results of M. Olbermann, J. Scherer–W. Pitsch and a joint work with I. Hambleton.

Rinat Kashaev : *Triangulations, Hamiltonian paths, and knot invariants*

I will describe a construction of state sum knot invariants, which uses the combinatorial setting of triangulated manifolds, where knots are realised by Hamiltonian paths.

M. Kreck : 3-*manifolds, codes, and arithmetic*
(joint work with Volker Puppe)

Let M be a closed odd-dimensional manifold with involution τ with finitely many fixed points. Puppe has associated to M a self-dual binary code $C(M, \tau)$ and asked the question which codes occur in this way.

I will explain self-dual codes, their relation to arithmetic and answer the question above and related questions. Motivated by the answer we define a (new ?) construction of self-dual codes. If there is time, I will also study codes over the Gaussian integers mod 2 which come from orientation-preserving involutions on 3-manifolds.

Julien Marché : *Localization of colored Jones polynomials*

Topological Quantum Field Theory associates to a knot complement an element of a vector space associated to the boundary. These vectors come in sequences indexed by the level of the theory. Using geometric quantization, the vector space of the boundary is generated by theta functions and the coefficents are evaluations of the colored Jones polynomial of the knot. We investigate the asymptotic behaviour of these vectors when the level goes to infinity.

Gwenael Massuyeau : *Homology cylinders and the tree-level of the LMO homomorphism*

Let S be a compact oriented surface with connected boundary. Homology cylinders over S form a monoid which contains the Torelli group of S. The LMO homomorphism is a diagrammatic representation of that monoid, which arises from the theory of finite-type invariants. In that talk, we shall give a topological interpretation of the full tree-reduction of the LMO homomorphism. In particular, we will connect that invariant of homology cylinders to nilpotent homotopy types of closed oriented 3-manifolds.

Bob Oliver : *Equivalences of fusion between finite Lie groups*

Fix a prime p. Two finite groups G and H will be called p-locally equivalent if there is an isomorphism from a Sylow p-subgroup S of G to a Sylow p-subgroup T of H which preserves all conjugacy relations between elements and subgroups of S and T. Anyone who works much with finite Lie groups notices that there are many cases of p-local equivalences between them. For example, if q and q' are two prime powers such that $q^2 - 1$ and $(q')^2 - 1$ have the same 2-adic valuation, then $SL_2(q)$ and $SL_2(q')$ are 2-locally equivalent.

In work with Carles Broto and Jesper Møller, we proved a very general theorem about such p-local equivalences between finite Lie groups. Our methods were topological : we did it by showing that the p-completed classifying spaces have the same homotopy type, and then applying tools based in part on the Sullivan conjecture and the work of Lannes. This theorem is not surprising to the group theorists, but the ones we have asked do not know of any purely algebraic proof of the result.

Antoine Touzé : *Cohomological finite generation for reductive groups*

This talk deals with the representation theory of algebraic group schemes G (or equivalently of finitely generated commutative Hopf algebras). We will present the following result (T-van der Kallen) : if G is reductive and acts on a finitely generated algebra A, the cohomology algebra $H^*(G, A)$ is finitely generated.

P. Vogel : *The universal Lie algebra : properties and conjectures.*

The properties of simple Lie algebras or simple Lie superalgebras can be globalized in a categorical way in order to produce an object called the universal Lie algebra. This object is a linear tensor category over a commutative ring Λ. It is useful for understanding many properties of Lie algebras and representations of Lie algebras. It is also useful in order to produce finite type invariants for knots, links and 3-dimensional manifolds. The coefficient ring Λ is highly unknown but a complete description of it is conjectured. This conjectural description implies a conjecture of Deligne about exceptional Lie algebras.

E. Wagner : *Variations around Khovanov link homologies*

In 1999, Khovanov constructed a link homology theory which is now known as the categorification of the Jones polynomial.

In the first part of this talk, I will describe this construction, paying particular attention to the algebraic structures of the Frobenius algebra and the $(1+1)$ TQFT underlying the construction.

In the last part, I will enlarge the former algebraic setting to include the Odd Khovanov homology introduced by Ozsvath, Rasmussen, and Szabo, as well as a nested Khovanov homology proposed by Stroppel and Webster. This is joint work with Anna Beliakova.

Abstracts of talks which were cancelled because of the volcano :

Joergen Ellegaard Andersen : *TQFT, Hitchin's connection and Toeplitz operators*

Dror Bar-Natan : *18 Conjectures*

I will state 18=3x3x2 "fundamental" conjectures on finite type invariants of various classes of virtual knots. This done, I will state a few further conjectures about these conjectures and ask a few questions about how these 18 conjectures may or may not interact. (Handout available at `http://www.math.toronto.edu/~drorbn/Talks/Luminy-1004`.)

Patrick M. Gilmer : *Topological applications of TQFT*

We will review the properties of TQFT and the universal construction due to Blanchet Habegger, Masbaum and Vogel. We will discuss some lower bounds on the Heegaard genus of 3-manifolds (due to Garoufalidis) and tunnel number of knots (due to Kohno) coming from TQFT. We will also discuss the Integral TQFT which is defined over a ring of cyclotomic integers that we have been studying with Masbaum. We will mention some applications of integral TQFT including obstructions to embedding 3-manifolds and bounds on Heegaard genus.

Thang Le : *The AJ conjecture for two-bridge knots*

We explain the AJ conjecture which relates the Jones polynomial and the A polynomial of a knot, and show that the conjecture holds for two-bridge knots with irreducible A polynomial.

Kent Orr : *Knots concordance, L^2 methods, and amenable groups*

Pierre's work on localization underpins current techniques on embedding theory and concordance. In joint work with Jae Choon Cha, we combine this perspective on localization and embedding theory with the von Neumann ρ-invariant to study knot concordance, a classical relation on knots first defined and studied by Fox and Milnor, and closely allied with deep considerations in singularity theory and the classification of 4-manifolds. Using a new approach which subsumes past results, we extend prior techniques to a broader class of problems, and extend key results concerning invariance of L^2 signatures and betti numbers under homology cobordism.

Andrew Ranicki : *The algebraic η-invariant*

The signature of a 4k-dimensional Riemannian manifold with boundary $(P, \partial P)$ was proved by Atiyah, Patodi and Singer (1973) to be the sum of the Hirzebruch L-genus and the η-invariant of ∂P. Many authors (Neumann, Meyer, Cappell-Lee-Miller, Bunke, Nemethi, ...) have subsequently given a more algebraic treatment of the η-invariant, defining it in particular for a closed (4k-1)-dimensional manifold N with a separating hypersurface $M \subset N$ and a complex structure J on $H^{2k-1}(M)$. The talk will describe an even more algebraic treatment of the η-invariant, using notions of the algebraic theory of surgery. This is related to the Wall non-additivity of the signature, the Maslov index, the von Neumann ρ-invariant etc. The ρ-invariant is a high-dimensional knot concordance invariant which is useful in the Cochran-Orr-Teichner calculations of the classical knot concordance group.

Holger Reich : *Lattices in rank one and K-theory*

The Farrell-Jones conjecture predicts that the algebraic K-theory of a group Γ can be built up from the K-theory of virtually cyclic subgroups of Γ. The talk will give an introduction to this Conjecture and its consequences.

We prove the new result that the conjecture holds for lattices in rank one, that is for discrete subgroups Γ of a Lie group G of rank one for which G/Γ has finite invariant volume. This extends results of Farrell/Jones and Berkove/Farrell/Juan-Pineda/Pearson. The result holds also for higher K-theory and arbitrary coefficient rings. Moreover there is an analogous result for L-theory.

Victor Turchin : *Uni-trivalent graphs and the homology of higher dimensional knot spaces*

I will describe natural graph-complexes computing the rational homology/homotopy of spaces of higher dimensional analogues of long knots. Perhaps surprisingly these graph-complexes depend only on the parities of the dimensions of the source and of the target. In particular this means that from the rational homology of $Emb(\mathbb{R}, \mathbb{R}^N)$, $N > 3$, one can easily recover that of $Emb(\mathbb{R}^m, \mathbb{R}^N)$, when m is odd and $N > 2m + 1$. It is quite interesting that the regrading in question is related to the open question whether Vassiliev invariants can distinguish knots (in $\mathbb{R}^3$) from their inverses.