

Spatio-temporal risk modeling
Cirm, Luminy. April 26-30, 2010

Organizers: Cécile Mercadier and Philippe Soulier

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SCHEDULE

MONDAY 26TH		TUESDAY 27TH		WEDNESDAY 28TH		THURSDAY 29TH		FRIDAY 30TH	
9h30 - 9h45	Welcome	9h00 - 10h00	Course 1.2	9h00 - 10h00	Course 2.3	9h00 - 10h00	Course 1.3	9h00 - 10h00	Course 1.4
9h45-10h45	Course 1.1	10h15-11h15	Course 2.2	10h15-11h00	Balan	10h15-11h15	Course 2.4	10h15-11h15	Drees
11h15-12h15	Course 2.1	11h30-12h30	Loisel	11h15-12h00	Nolan	11h30-12h30	Samorodnitsky	11h30-12h30	Beirlant
Lunch		Lunch		12h00-12h30	Stoev	Lunch		Lunch	
15h30-16h15	Leipus	16h00-16h45	Maume-Deschamps	Lunch		16h00-16h45	Kulik	13h45-14h30	Puccetti
16h15-16h45	Siaulyis	16h45-17h15	Bäuerle	Free Afternoon		16h45-17h15	Di Bernardino	14h30-15h00	Robert
16h45-17h30	Collamore	17h30-18h00	Constantinescu			17h30-18h00	Wintenberger		
18h00-18h30	Mikosch	18h00-18h30	Cossette			18h00-19h00	Davis		
18h30-19h15	Tang	18h30-19h15	Marceau						

Courses

- 1/ Henrik Hult: Four lectures on importance sampling
- 2/ David Stanford: Matrix Methods for Fluid Queues and Erlangization

Henrik Hult: Four lectures on importance sampling

- *Importance sampling in rare event simulation* The first lecture gives an introduction to importance sampling with applications to rare event simulation. Examples include hitting probabilities of a random walk, including both the light-tailed and heavy-tailed case.
- *Importance sampling for computation of risk measures* In the second lecture we consider non-linear functions of the empirical measure. Examples include computation of financial risk measures such as Value-at-Risk and Expected Shortfall.
- *Importance sampling and stochastic control* There is a close connection between importance sampling and stochastic control theory. The choice of sampling distribution can be seen as the control and the objective is to minimize the relative error.
- *Sequential importance sampling in large state spaces* We provide examples of importance sampling in large state spaces. Examples include the number of non-intersecting paths from $(0,0)$ to (n,n) in an n -by- n lattice and related problems.

David Stanford: Matrix Methods for Fluid Queues and Erlangization

- Quasi Birth and Death Processes and their solution, (entailing the key matrices R , G , and U), and the role of uniformization in obtaining the solution.
- Fluid flow models: existence and computation of the stationary distribution, and introduction to the determination of transient probabilities using QBD methods.
- Continuation of the determination of transient probabilities; reversed fluid flows, and Erlangization.

MONDAY 26TH APRIL

15h30-16h15 Remigijus Leipus *Asymptotics of random sums of heavy-tailed negatively dependent random variables with applications*

Let X_1, X_2, \dots be negatively dependent and identically distributed random variables having dominatedly varying tails, and let N be a counting random variable independent of X_i 's. In this paper, we obtain the asymptotics for the tail probability of the random sum $S_N = \sum_{k=1}^N X_k$, where the tail of X_1 is comparable with, heavier or lighter than that of N .

16h15-16h45 Jonas Siaulys *Local precise large deviation results for sums of random variables with O -regularly varying densities*

We establish local precise large deviation results for sums $S_n, n = 1, 2, \dots$ of independent and identically distributed random variables X_1, X_2, \dots with O -regularly varying density f and distribution function F . The asymptotic behavior of the probability $\mathbb{P}(x < S_n - \mathbb{E}S_n \leq x + T)$ is comparable, for fixed T , with quantities $nTf(x)$ or $nT(F(x+T) - F(x))$.

16h45-17h30 Jeffrey Collamore *On Cramér-Lundberg theory with stochastic investments and its dual financial process*

This talk will be concerned with risk estimates relating to a class of random recurrence equations. Our original motivation came from the ruin problem with investments, where an insurance company invests its excess capital and earns stochastic interest on these investments. A similar problem arises when studying the stationary tail behavior of the GARCH(1,1) financial process. Both processes exhibit temporal dependence, which cannot be analyzed by classical techniques. The tail behavior for these processes is usually obtained by observing that they satisfy a random recurrence equation, namely, $Y \stackrel{d}{=} \Phi(Y)$, where Φ is a random function and Y is a random variable on \mathbf{R} .

A well-known result of Goldie (1991) then states that (1) $\mathbf{P}\{Y > u\} \sim \mathbf{C}u^{-R}$ as $u \rightarrow \infty$. In this estimate, the constant R is generally known explicitly, but not the

constant C . In this talk, we will introduce an alternative approach to Goldie's. In particular, we will begin by describing a duality connecting Cramér-Lundberg models with stochastic investments to an extended GARCH(1,1) financial process. Using this duality, we then establish the sharp upper bound $\mathbf{P}\{\text{ruin}\} \leq \bar{C}(u)u^{-R}$, for all u , and provide a new proof of (1). Here, however, we improve upon (1) by providing a characterization of the constant C , and by relating the constants appearing in (1) to the constants of decay appearing in a classical Cramér-Lundberg problem. Finally, we develop statistical methods for inference concerning R . We conclude with an application of these ideas to importance sampling. (Joint work with Anand Vidyashankar.)

18h00-18h30 Thomas Mikosch *Modeling of claims reserves in a cluster Poisson process*

We consider two different frameworks for modeling claims reserves in a non-life insurance portfolio. Due to the Poisson structure of the underlying claim counting processes one can calculate explicit expressions for the claims reserves which are understood as mean squares prediction of claim numbers and payments, as well as for the prediction error. (Joint work with Anders H. Jessen, Muneya Matsui and Gennady Samorodnitsky.)

18h30-19h15 Qihe Tang *Which one of the insurance risk and financial risk plays a dominating role on ruin?*

Consider an insurer who is exposed to a stochastic economic environment. Such an environment contains two kinds of risk. The first kind, called insurance risk, is the traditional liability risk related to the insurance portfolio and the second kind, called financial risk, is the asset risk related to the investment portfolio. Under certain regular variation conditions on the tail probabilities of the two kinds of risk, we derive some exact asymptotic formulas for the ruin probability. The formulas confirm that the ruin probability is mainly determined by the one of the two kinds of risk which is more heavy-tailed than the other. (This talk is based on a joint work with Jinzhu Li.)

TUESDAY 27TH APRIL

11h30-12h30 Stéphane Loisel *On some correlation aspects in longevity risk*

We introduce longevity and mortality risks and present different models to take into account spatio-temporal dependence in stochastic mortality models. Various examples illustrate the specificity of mortality patterns in different countries and also for different sets of policyholders. Some consequences on the design of longevity derivatives are studied.

16h00-16h45 Véronique Maume-Deschamps *Multivariate risk indicators: estimation and application to optimal reserve allocation.*

We consider some risk indicators of vectorial risk processes. These indicators are expected sums of some penalties that each line of business would have to pay due to its temporary potential insolvency. The dependency between lines of business is taken into account. By using stochastic algorithms, we may estimate the minimum of these risks indicators, under a fixed total capital constraint. This minimization may apply to optimal reserve allocation. (Joint work with Peggy Cenac and Clémentine Prieur.)

16h45-17h15 Nicole Bäuerle *Multivariate Risk Processes with Interacting Intensities*

The classical models in risk theory consider a single type of claims. In the insurance business, however, several business lines with separate claim arrival processes appear naturally, and the individual claim processes may not be independent. We introduce a new class of models for such situations, where the underlying counting process is a multivariate continuous time Markov chain of pure birth type and the dependency of the components arises from the fact that the birth rate for a specific claim type may depend on the number of claims in the other component processes. Under certain conditions we obtain a fluid limit, i.e. a functional law of large numbers for these processes. We also investigate the consequences of such results for questions of interest in insurance applications. Several specific subclasses of the general model are discussed in detail and the Cramér asymptotics of the ruin probabilities are derived in particular cases. (This is a joined work with Rudolf Grübel.)

17h30-18h00 Corina Constantinescu *Symbolic Computation for Boundary Problems in Risk Theory*

In this talk we will present a symbolic computation approach to boundary problems, based on operators, that is applicable in risk theory. The main idea is to reduce the integral equations satisfied by functions of the risk processes to boundary problems. Further, one can factorize these problems into first order boundary problems, which will often allow to derive explicit expressions for the functions considered. For instance, one can find explicit expressions for the Gerber-Shiu functions in terms of the penalty function, in quite general settings.

18h00-18h30 Helene Cossette *Discrete-time risk models based on time series for count random variables*

In this talk, we consider risk models based on time series models for count random variables, which can be applied in the context of accidents. Examples of time series

models for count data are integer value moving average models and integer value autoregressive models. We examine the properties of the total amount of (discounted or not) claims over a fixed number of periods. We analyze the dangerousness of the risk models through the measurement of the adjustment coefficient. Ruin measures are also examined.

18h30-19h15 Etienne Marceau *Agrégation des risques dépendants et allocation du capital*

Dans cet exposé, nous considérerons un portefeuille constitué de risques dépendants (contrats d'assurance, lignes d'affaires, etc.). La relation de dépendance entre les risques est fondée sur des copules, des modèles multivariés composés ou des modèles avec mélange commun. Nous examinons diverses méthodes d'agrégation des risques dépendants adaptées aux modèles de dépendance considérés. Ces méthodes nous permettent d'aborder l'évaluation des mesures de risque Value-at-Risk (VaR) et Tail-Value-at-Risk (TVaR) et l'allocation du capital aux composantes du portefeuille de risque. Dans le cas des modèles de dépendance fondés sur les mélanges communs, nous examinons aussi la part de capital associé au risque systématique associé aux facteurs aléatoires communs induisant la dépendance entre les risques. Des exemples numériques sont présentés dans le but d'illustrer les notions présentés.

WEDNESDAY 28TH APRIL

10h15-11h00 Raluca Balan *A cluster limit theorem for infinitely divisible point processes*

In this talk, we examine the connection between the limit representation of an infinitely divisible point process, and its cluster representation. Our result identifies some explicit conditions for the convergence of the sequence of point processes associated to a triangular array of random variables, in terms of the probabilistic behavior of the variables in the array. As applications, we discuss the exceedance processes and the extremal index. (Joint work with Sana Louhichi.)

11h15-12h00 John Nolan *Classes of multivariate max stable distributions and their relationships*

We examine classes of multivariate max stable distributions and look at their relationships to each other. One group of models is found by directly starting with a known angular measure. Here the class of discrete angular measures is particularly tractable in any dimension. We then explore a class of tractable (at least in two dimension) models with piecewise polynomial density for the angular measure.

Another group of models is the family of common models is the family of generalized asymmetric logistic models. We detail the connection between this class of models and the closely related class of generalized stable mixture class. (Joint work with Anne-Laure Fougères and Cécile Mercadier)

12h00-12h30 Stilian Stoev *Tail behavior of Holder norms and limit theorems for maxima in Holder spaces*

We discuss some work in progress on functional limit theorems of maxima in Holder spaces. It turns out that the classical tightness conditions of Lamperti readily apply, provided that one can control the tail-behavior of Holder norms of certain random processes. A powerful isomorphism theorem of Ciesielski allows one to obtain useful bounds on the tails of these Holder norms. Some implications on the path regularity of max-stable processes will be discussed.

THURSDAY 29TH APRIL

11h30-12h30 Gennady Samorodnitsky *Long Strange Segments, Ruin Probabilities and the Effect of Memory on Moving Average Processes*

We obtain the rate of growth of multivariate long strange segments and the rate of decay of infinite horizon multivariate ruin probabilities for a class of infinite moving average processes with exponentially light tails. The rates are computed explicitly. We show that the rates are very similar to those of an i.i.d. process as long as moving average coefficients decay fast enough. If they do not, then the rates are significantly different. This demonstrates the change in the length of memory in a moving average process associated with certain changes in the rate of decay of the coefficients. (Jointly with Souvik Ghosh.)

16h00-16h45 Rafal Kulik *Tail empirical process for some long memory sequences*

We describes limiting behaviour of tail empirical process associated with some long memory models. We show that such process has dichotomous behaviour, according to an interplay between a Hurst parameter and a tail index. In particular, the limit may be non-Gaussian and/or degenerate, indicating an influence of long memory. On the other hand, tail empirical process with random levels never suffers from long memory. This is very desirable from a practical point of view, since such the process may be used to construct Hill estimator of the tail index. To prove our results we need to establish several new results for regularly varying distribution functions, which may be of independent interest. (This is a joint work with Philippe Soulier.)

16h45-17h15 Elena Di Bernardino *Estimating Bivariate Tails*

In this work we consider the general problem of estimating the tail of a bivariate distribution. An extension of the threshold method for extreme values is developed, using a two-dimensional version of the Pickands-Balkema-de Hann Theorem. We construct a two-dimensional tail estimator and we provide its asymptotic properties. The dependence structure between the marginals is described by a copula. Simulations are implemented.

17h30-18h00 Olivier Wintenberger *Limit for sums of dependent time series with infinite variance*

For some classical time series (univariate GARCH(1,1), ARCH(1)), we characterize the asymptotic distribution of partial sums. It is an alpha-stable law which depends on the marginal distribution and on the law of the stationary time series. Multivariate regular variation properties are also used to derive the result. (Joint work with K. Bartkiewicz, A. Jakubowski and T. Mikosch.)

18h00-19h00 Richard Davis *Measuring Extremal Dependence for Time Series and Spatial Processes via the Extremogram*

The extremogram was developed as a tool for assessing various types of extremal dependence in a multivariate time series. The use of the extremogram in applications arising in both financial and environmental contexts will be illustrated. Currently, bootstrapping methods are being adapted to the extremogram in order to construct more meaningful and useful inference procedures. These techniques, as well as permutation procedures, will be demonstrated in several examples. (This is joint work with Thomas Mikosch and Ivor Cribben.)

FRIDAY 30TH APRIL

10h15-11h15 Holger Drees *Bootstrapping Blocks Estimators of the Extremal Index: How Empirical Cluster Processes Make Your Life Easy.*

Recently Drees and Rootzén (2010) have introduced a very general class of empirical processes (indexed by functions) which describe certain aspects of the extreme value behavior of time series. Moreover they have proved the asymptotic normality of these processes under suitable mixing conditions.

We apply this theory to examine the asymptotic properties of smoothed blocks estimator of the extremal index. In addition, we discuss how the distribution of their estimation error can be approximated using a bootstrap approach. To this end, the limiting behavior of multiplier block bootstrap versions of the empirical processes conditional on the original data turns out to be vitally important.

Drees, H., and Rootzén, H. (2010). *Limit Theorems for Empirical Processes of Cluster Functionals*, to appear in the [*Annals of Statistics*](#).

11h30-12h30 Jan Beirlant *Bias reduction in extreme value methods: a personal perspective*

The last decade several bias-correction methods have appeared in the estimation of the extreme value index (positive or real-valued), extreme quantiles or large return periods. One important motivation is to construct estimation methods where the results are less dependent on the choice of the threshold or the number of extremes used in the estimation. We review different methods and discuss different points of interest in this matter: methods based on quantile models versus probability models; what about increase of variance; sensitivity with respect to the underlying second-order regular variation models; other tail models such as Weibull-type models; bias reduction in goodness-of-fit testing; bivariate extensions. The material will be illustrated using simulation results and practical examples.

13h45-14h30 Giovanni Puccetti *The AEP algorithm for the fast computation of the distribution of the sum of dependent random variables*

We propose a new algorithm to compute numerically the distribution function of the sum of d dependent, non-negative random variables with a given joint distribution. (Joint work with P. Arbenz and P. Embrechts.)

14h30-15h00 Christian Yann Robert *Pseudo-empirical likelihood inference for clusters of rare events*

Events with low probability but disastrous impact are of particular interest to a large variety of applied sciences. The analysis of such rare events entails the understanding of the way in which they cluster in time. Especially it is very important to measure the strength of dependence between these events. We present a new statistical methodology based on an empirical likelihood approach to estimate the distribution of the size of the clusters. Our results are illustrated through simulations and by applications to real data.