KNOTTED EMBEDDINGS IN DIMENSIONS 3 AND 4

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Abstracts

Jae Choon Cha

Milnor's question on transfinite invariants

Abstract: In his 1957 paper entitled "Isotopy of Links", John Milnor defined invariants using lower central series of the fundamental group, and asked a question to find a method to extract invariants from the transfinite lower central series. We present an affirmative answer for 3-manifolds, by developing a general theory of transfinite Milnor invariants and realizing nontrivial values for the transfinite case. This talk is based on joint work with Kent Orr.

David Cimasoni

Covering spaces and spanning trees

Abstract: The aim of this talk is to show how basic notions traditionally used in the study of "knotted embeddings in dimensions 3 and 4", such as covering spaces and representation theory, can have non-trivial applications in combinatorics and statistical mechanics. For example, we will show that for any finite covering G' of a finite edge-weighted graph G, the spanning tree partition function on G divides the spanning tree partition function on G' (in the polynomial ring with variables given by the weights). Setting all the weights equal to 1, this implies a theorem known since 30 years: the number of spanning trees on G divides the number of spanning trees on G'. Other examples of such results will be presented.
Joint work (in progress) with Adrien Kassel.

Peter Feller

Three-dimensional characterizations of the Z-slice genus

Abstract: We consider knotted embeddings in dimension 4: for a fixed knot K in the 3–sphere, we look at surfaces in the 4–ball with boundary K. Here, we restrict to oriented surfaces for which the complement has cyclic fundamental group, Z–surfaces for short.

Famously, Freedman's disk theorem implies that a knot K bounds a locally flat Z–surface of genus 0 (aka a disk) if and only if K has Alexander polynomial one. For all positive integers g, we provide three 3–dimensional characterizations of when a fixed knot arises as the boundary of a locally flat Z–surfaces of genus g. These characterizations use genera of Seifert surfaces, crossing changes, and Seifert pairing (recast as the Blanchfield pairing), respectively.

The talk will focus on 3–dimensional inputs such as a characterization of the algebraic unknotting number due to Borodzik-Friedl and a simple trick turning crossing changes between two knots into Seifert surfaces with said knots as their boundary.
Based on work in progress with Lukas Lewark.

Roger Fenn

Generalized knot theories and their invariants

Abstract: Recently, in mathematical terms, it has become clear that there are many different objects which deserve the title "knots" other than the classical ones. In this talk I will give a unifying definition and briefly discuss how these new knots have given rise to new algebraic objects and unforeseen mathematical connections.

Vincent Florens

Slopes of colored links

Abstract: tba

Stefan Friedl

Epimorphisms of knot groups and the genus

Abstract: Let K and J be knots. Simon conjectured that if there exists an epimorphism from the knot group of K onto the knot group of J, then the genus of K is greater or equal than the genus of J. We will discuss a generalization of the conjecture to 3-manifolds and we will present several positive results.
This is based on joint work with Wolfgang Lück.

David Gay

Trisections diagrams and surgery operations on embedded surfaces

Abstract: Various surgery operations on dimension four begin with a 4–manifold X and an embedded surface S, then remove a neighborhood of S and replace it with something else to produce an interesting new 4–manifold. In a few standard surgery constructions, especially the Gluck twist operation, I will show how, given a trisection diagram of X with decorations that describe the embedded surface S, to produce a trisection diagram for the new 4–manifold.
This is joint work with Jeff Meier.

Naoko Kamada

Coherent double coverings of virtual links

Abstract: A virtual link diagram is called normal if the associated abstract link diagram is checkerboard colorable, and a virtual link is normal if it has a normal diagram as a representative. Normal virtual links have some properties similar to classical links. In this talk, we introduce a method of converting a virtual link diagram to a normal virtual link diagram. We show that the normal virtual link diagrams obtained by this method from two equivalent virtual link diagrams are equivalent. We discuss some applications of our method.

Seiichi Kamada

On embedded/immersed surfaces in 4–space, their braid presentations and multiplications

Abstract: An embedded/immersed surface-link is an embedded or generically immersed closed surface in Euclidian 4–space. We discuss isotopic deformations of immersed surface-links and braid presentations of oriented surface-links. A theorem of Markov type for braid presentations of immersed surface-links is presented. A multiplication of a surface-link is discussed.

Louis H. Kauffman
Cobordism and concordance of virtual links

Abstract: This talk will define cobordism of virtual link diagrams by adding the operations of oriented saddle points (reconnections), and births and deaths of trivial circles. A sequence of such operations creates a virtual oriented surface between the initial and final diagrams. This surface has a genus g that we call the genus of the cobordism. When the genus is 0, we say that the cobordism is a concordance. The 4–ball genus, $g_4(K)$ of a virtual link K is the least genus virtual surface that bounds K. That is, it is the least genus among surfaces generated by saddle points, births and deaths that results in the empty link. Along with numerous examples of phenomena of virtual link concordance, we will discuss the determination by Dye, Kaestner and Kauffman of the 4–ball genus of positive virtual links. This work uses integral Khovanov homology for virtual links and a generalization of the Rasmussen invariant to this category.

Paolo Lisca
Symmetric unions and topological spin models

Abstract: An open question close to the slice-ribbon conjecture asks whether every ribbon knot can be represented as a symmetric union. Next to this basic existence question sits the question of uniqueness of such representations. Eisermann and Lamm introduced a notion of symmetric equivalence among symmetric union diagrams, showing that inequivalent diagrams can be detected using a refined version of the Jones polynomial. I will talk about joint work in progress with Carlo Collari. We prove that every topological spin model gives rise to invariants of symmetric equivalence and that such invariants can be used to distinguish inequivalent symmetric union diagrams, providing a partial answer to a question left open by Eisermann and Lamm.

Andrew Lobb
Quantum $\mathfrak{sl}_n$ knot cohomology and the slice genus

Abstract: We will give an overview of the information about the smooth slice genus so far yielded by the quantum $\mathfrak{sl}_n$ knot cohomologies.

Bruno Martelli
Shadow complexity of smooth closed four-manifolds

Abstract: Turaev's shadows can be used to define a complexity on any smooth closed four-manifold. This "shadow complexity" is a natural number that measures how complicated the two-skeleton of a four-manifold is. This complexity has the advantage of being tightly related to well-studied problems in three-dimensional topology: decomposing a three-manifold along spheres and tori, and classifying the exceptional Dehn fillings of a (multi-cusped) hyperbolic three-manifold. We present a description of all the smooth closed orientable four-manifolds with complexity zero and one.
This is joint work with Koda and Naoe.

Jeffrey Meier

Bridge trisections of knotted surfaces in four-manifolds

Abstract: In this talk, we will develop the theory of generalized bridge trisections for smoothly embedded closed surfaces in smooth, closed four-manifolds. The main results is that any such surface can be isotoped to lie in bridge trisected position with respect to a given trisection of the ambient four-manifold. In the setting of knotted surfaces in the four-sphere, this gives a diagrammatic calculus that offers a promising new approach to four-dimensional knot theory. However, the theory extends to other ambient four-manifolds, and we will pay particular attention to the setting of complex curves in simple complex surfaces, where the theory produces surprisingly satisfying pictures and leads to interesting results about trisections of complex surfaces.
This talk is based on various joint works with Dave Gay, Peter Lambert-Cole, and Alex Zupan.

Jean-Baptiste Meilhan

Knotted surfaces up to link homotopy

Abstract: Milnor introduced in the 50's a family of link invariants which are invariant under "link homotopy", *i.e.* under continuous deformations where distinct components remain disjoint. A full link homotopy classification of links was achieved 40 years later by Habegger and Lin, using a refinement of Milnor invariants for "string links". The situation seems drastically different in higher dimensions: Bartels and Teichner indeed showed that any embedding of a disjoint union of 2–spheres in 4–space is link homotopic to the trivial one. In this talk, we consider higher order analogues of string links, and give a classification up to link homotopy using a 4–dimensional version of Milnor invariants. The proof combines the result of Bartels and Teichner and diagrammatic approaches to knotted surfaces in 4–space.
This is a joint work with B. Audoux and E. Wagner.

Mark Powell

Surface systems for links

Abstract: A surface system for a link in S^3 is a collection of embedded Seifert surfaces for the components, that are allowed to intersect one another. When do two n–component links with the same pairwise linking numbers admit homeomorphic surface systems? It turns out this holds if and only if the link exteriors are bordant over the free abelian group $\mathbb{Z}^n$. In this talk we characterise these geometric conditions in terms of algebraic link invariants in two ways: first the triple linking numbers and then the fundamental groups of the links. This involves a detailed study of the indeterminacy of Milnor's triple linking numbers.
Based on joint work with Chris Davis, Matthias Nagel and Patrick Orson.

Peter Teichner

The group of disjoint 2–spheres in 4–space

Abstract: Will discuss recent joint work with Rob Schneiderman, computing the group of link homotopy classes of link maps of two 2-spheres in 0-4-space. By definition, a link map is required to keep the two components disjoint and a link homotopy is a homotopy through link maps. As a consequence of our computation, we show that any such link map with one topologically embedded component is link homotopic to the unlink!

Raphael Zentner

Irreducible $SL(2,\mathbb{C})$–representations of integer homology 3-spheres

Abstract: We prove that the splicing of any two non-trivial knots in the 3–sphere admits an irreducible $SU(2)$–representation of its fundamental group. This uses instanton gauge theory, and in particular a non-vanishing result of Kronheimer–Mrowka and some new results that we establish for holonomy perturbations of the ASD equation. Using a result of Boileau, Rubinstein and Wang (which builds on the geometrization theorem of 3–manifolds), it follows that the fundamental group of any integer homology 3–sphere different from the 3–sphere admits irreducible representations of its fundamental group in $SL(2,\mathbb{C})$.