Recent progress in mathematical and numerical analysis of inverse problems

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Conference Abstracts

GENERAL PLENARY SESSIONS

Emerging imaging approaches in medicine

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Imaging techniques in medicine are for visualizing contrast information on the electrical, acoustic, optical, mechanical properties of tissues. They can be formulated as nonlinear inverse problems. The goal is to achieve good resolution, good stability, and high specificity. A standard approach for solving imaging problems is nonlinear optimization. In this talk, I will discuss emerging alternative approaches which include differential imaging, physics-based classification and identification approaches, and hybrid imaging. I will show resolution, stability, and specificity enhancements for a few modalities. These include spectroscopic admittivity imaging, conductivity imaging by Lorentz force, ultrasound modulated electromagnetic tomography, and optical cohence elastography.

High-contrast high-resolution coupled physics imaging modalities

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Several recent coupled-physics medical imaging modalities aim to combine a high-contrast, low-resolution, modality with a high-resolution, low-contrast, modality and ideally offer high-contrast, high-resolution, reconstructions. Such modalities often involve the reconstruction of constitutive parameters in partial differential equations (PDE) from knowledge of internal functionals of the parameters and PDE solutions. This talk presents several recent results of uniqueness, stability and explicit reconstructions for several hybrid inverse problems. In particular, we provide explicit characterizations of what can (and cannot) be reconstructed and offer optimal (elliptic) stability estimates for a large class of coupled-physics imaging modalities including Magnetic Resonance Elastography, Transient Elastography, Photo-Acoustic Tomography and Ultrasound Modulation Tomography. Numerical simulations confirm the high-resolution, high-contrast, potential of these novel modalities.

Approximate globally convergent and adaptive finite element methods in imaging of targets from experimental data

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In this talk we will briefly present two-stage numerical procedure for the solution of hyperbolic coefficient inverse problem. On the first stage an approximate globally convergent algorithm rigorously guarantee obtaining at least one point in a small neighborhood of the exact solution without any advanced knowledge of that neighborhood. On the second stage an adaptive finite element method refines the solution obtained on the first stage.

We will demonstrate numerical verification of the two-stage procedure implemented in the software package WavES (waves24.com) on the reconstruction of refractive indices of inclusions from backscattered experimental data provided by the Optoelectronics and Optical Communications Center of University of North Carolina at Charlotte, Charlotte, USA.

Stability for an inverse boundary value problem for the Helmholtz equation

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In the talk I will present some recent results concerning stability for an inverse boundary value problem for the Helmholtz equation. More precisely one wants to determine the speed of propagation from knowledge of the Dirichlet to Neumann map. First I will overview the main results concerning uniqueness and continuous dependence.
In the second part of the talk I will concentrate on the issue of continuous dependence, crucial for effective reconstruction, describing some recent results contained in [1], [2], [3] where Lipschitz continuous dependence estimates have been derived in the case of coefficients that are finite linear combinations of bounded functions ψ_j, $j = 1, \cdots, N$, defined on a Lipschitz partition $\mathcal{D}_N = \cup_{j=1}^N D_j$ of Ω with supp $\psi_j \subset D_j$, $j = 1, \cdots, N$, for example when $\psi_j = \chi_{D_j}$, $j = 1, \cdots, N$. These assumptions are quite natural having in mind an approximation scheme for the reconstruction of an arbitrary coefficient. A crucial role is played by the Lipschitz constant appearing in the estimates, in particular its dependence on the partitioning number N. First I will consider the case where the functions ψ_j, $j = 1, \cdots, N$ are known, giving an explicit bound of the Lipschitz constant in terms of the frequency and on the partitioning number N and which turns out to be optimal with respect to N. I will consider the case where $\psi_j = \chi_{D_j}$, $j = 1, \cdots, N$ are unknown and $\mathcal{D}_N$ is a regular tetrahedron's partition of Ω proving that it is possible to recover the partition in a Lipschitz way form the Dirichlet to Neumann map.

References:

[1] E. Beretta, M. V de Hoop and L. Qiu "Lipschitz stability of an inverse boundary value problem for a Schrödinger type equation", *SIAM J. Math. Anal.* Vol. 45, No. 2, pp. 679-699 (2013)

[2] E. Beretta, M. V. de Hoop, L. Qiu and O. Scherzer "An inverse boundary value problem for the Helmholtz equation with multi-frequency data" *preprint* (2014)

[3] E. Beretta, E. Francini ,M. V. de Hoop and S. Vessella "On the determination of interfaces from boundary measurements " *in preprint* (2014)

Aymptotics of the spectrum of the Neumann-Poincaré operator, for 2 close-to-touching inclusions in 2D

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The Neumann-Poincaré operator is an integral operator that appears in the integral representation of solutions to transmission problems in piecewise homogeneous media. We study its spectral properties for a system of 2 smooth inclusions in 2D. As the inter-inclusion distance δ vanishes and as the coefficient contrast degenerates, the behavior of the spectrum is related to the possible blow-up of the solutions to the corresponding elliptic transmission problem. We report numerical calculations that illustrate our results on the asymptotics of the Neumann-Poincaré eigenvalues.

References:

[1] E. Bonnetier and F. Triki, *Pointwise bounds on the gradient and the spectrum of the Neumann–Poincaré operator: The case of 2 discs*, AMS, Contemporary Math.. (2012) 577.81–92.

[2] E. Bonnetier, and F. Triki. *On the spectrum of the Poincaré variational problem for two close-to-touching inclusions in 2D*. Archive for Rat. Mech. Anal. (2013) 209. 541–567.

[3] E. Bonnetier, F. Triki, and C.H. Tsou, *Eigenvalues of the Neumann-Poincaré operator of 2 inclusions with contact of order m: a numerical study*, in preparation.

Inverse scattering in a periodic waveguide from far field data

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In this talk we apply the sampling methods (such as the Colton-Kirsch's Linear Sampling Method or the Kirsch's Factorization Method) to identify some unknown inhomogeneities in a known 2D periodic waveguide from scattering data. Quite recently, the notion of far field in a periodic waveguide was properly defined by Fliss-Joly, which enables us to solve the inverse scattering problem by using far field data. We prove that the far field formulation of sampling methods amounts to consider the propagating Floquet modes as incident waves and we show the feasibility of such method with the help of some numerical experiments. This work is a collaboration with Sonia Fliss.

Inverse problems for parabolic operators of Grushin type

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The Carleman estimates strategy introduced by Imanuvilov and Yamamoto in 1998 to prove Lipschitz stability results for uniformly parabolic equations, seems hard to apply to parabolic equations of Grushin-type. Indeed, Carleman estimates are still missing for operators that degenerate in the interior of the space domain such as the Grushin operator. Nevertheless, we will discuss how to recover Lipschitz stability for the determination of the source term and degenerate coefficients of such equations from locally distributed measurements of the solution. Our approach combines a method due to Lebeau and Robbiano, which relies on Fourier decomposition, with classical Carleman inequalities for the heat equations with nonsmooth coefficients that are solved by the Fourier modes (joint work with K. Beauchard and M. Yamamoto).

The anisotropic Calderon problem

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The anisotropic Calderon problem is whether it is possible to determine a Riemannian metric (modulo the natural invariance by isometries) on a compact Riemannian manifold with boundary from knowledge of the Cauchy data of harmonic functions. This problem is solved in dimension two, as well as in the conformal class of the Euclidean metric and for analytic metrics, but remains challenging for smooth metrics in dimension higher than three. In this talk I will present recent results in the case of a conformal class of metrics presenting a special structure of (warped) product with an Euclidean factor. I will explain how the identifiability of the conformal factor can be deduced from the injectivity of a certain geodesic ray transform in the transversal part of the manifold. In the case of independence with respect to the Euclidean variable, a direct link with the corresponding hyperbolic problem (which can be solved using the Boundary control method) can be made. This extends previous results of C. Kenig, M. Salo, G. Uhlmann and myself where stronger geometric assumptions were made. This is a joint work with S. Kurylev (University College London), M. Lassas (University of Helsinki) and M. Salo (University of Jyvaskyla).

The stability issue for the inverse conductivity problem

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We discuss the stability issue for Calderon inverse conductivity problem. We suppose that the unknown conductivity has the structure $\sigma(x) = \sum_{j=1}^{n} \sigma_j(x)\chi_{D_j(x)}$

for almost every $x \in \Omega$ where the domains D_j are known Lipschitz domains and $j(x)$ are unknown (matrix) valued functions of a certain type. More precisely, we extend here the stability result obtained by Alessandrini and Vessella in 2005 in Advances in Applied Mathematics 35: 207-241, where the authors considered the case of $j(x)$ unknown real numbers.

Selecting anomalous components in unknown complex background

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We shall present a new qualitative method capable of selecting defects in complex and unknown background from differential measures of farfield operators: i.e. measures of the farfield operator for the cases with and without defects. Indeed, the main difficulty is that the background physical properties are unknown. Our approach exploits the principle of the Generalized Linear Sampling method previously introduced by the authors as a rigorous alternative to so-called Linear Sampling Method and the link with solutions to the interior transmission problems. We shall present the theoretical foundations of the method and some validating numerical experiments. The motivation behind this work is the identification of cracks in concrete like materials. Simulations of such configurations will also be discussed.

Spectral theory and inverse problems for Laplacians on prturbed lattices

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We study spectral properties for the Laplacian on a lattice perturbed on its finite part. We shall prove Rellich type theorem for the Helmholtz equation, absence of embedded eigenvalues, and the limiting absorption principle for the resolvent satisfying the radiation condition. Constructing the spectral representaion, we characterize the solution space for the Helmholtz equation in a suitable Besov space on the torus, and the S-matrix is defined through the singularity expansion of solutions. In some cases, we can reconstruct the perturbation on a finite part of the lattice from the S-matrix of one fixed energy. Our theory covers the square, triangular, hexagonal, Kagome, diamond, subdivision lattices as well as ladder and graphite. This is a joint work with K. Ando and H. Morioka.

Review of some inverse problems in periodic cylindrical wave guide

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In this talk we present some results about determination of periodic parameter in an infinite cylindrical domain $\Omega = \mathbb{R} \times \omega$ of $\mathbb{R}^3$, also called cylindrical wave guide, with ω a bounded domain of $\mathbb{R}^2$. We consider an inverse spectral problem and two inverse boundary value problems related to this topic.

Inverse problems for non-linear wave equations and the Einstein equations

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We consider inverse problem for a non-linear wave equation with a time-depending metric tensor on manifolds. In addition, we study the question, do the observation of the solutions of coupled Einstein equations and matter field equations in an open subset U of the space-time M corresponding to sources supported in$U$ determine the properties of the metric in a larger domain $W \subset M$ containing U.

To study these problems we define the concept of light observation sets and show that these sets determine the conformal class of the metric.

The results have been done in collaboration with Yaroslav Kurylev and Gunther Uhlmann.

The Finite Element Inverse Conductivity Problem

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The forward problem of electrical impedance tomography is often approximated using the finite element method on a tetrahedral mesh. and the resulting finite dimensional formulation is equivalent to the Ohm-Kirchhoff system for a resistor network with the same topology. The inverse problem for the resistor network from the Dirichlet -to-Neumann map is often over determined. Given the vertex positions of the mesh in $\mathbb{R}^3$ the edge conductances do not uniquely determine isotropic (let alone anisotropic) conductivities. Furthermore the Dirichlet-to-Neumann map is invariant under piecewise linear homeomorphisms that move the interior vertex positions. An important step in understanding this problem is a complete description of the constraints on the dihadral angles and the solution of the equivalent of the triangulation plane survey problem for tetrahedral meshes. We will show some numerical results on constraints on the conductivity tensors on each tetrahedron which result in a locally unique solution.

On Inverse Problems Associated with Vlasov Equilibria

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The Vlasov system is a nonlinear integrodifferential equation that is a most important equation of plasma physics. It is composed of a transport equation for a phase spaced density coupled to electromagnetic fields. The equilibria (steady states) of the Vlasov system are used to model both laboratory and naturally occurring plasmas. However, given field quantities there is a high degree of degeneracy in the equilibrium phase space density. A variety of inverse problems, for which this degeneracy is removed, will be discussed. The results obtained are applicable to a variety of Vlasov-like transport systems.

D-bar Methods in Electrical Impedance Tomography

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D-bar methods for the solution of nonlinear inverse problems arise from the use of complex geometrical optics solutions to PDE's associated with the inverse problem at hand. Such solutions have provided a means of proving global uniqueness for the inverse conductivity problem and a framework for direct reconstruction algorithms. Electrical impedance tomography is an important application of the inverse conductivity problem, with applications in medical imaging, nondestructive evaluation, and geophysics. In this talk, direct reconstruction algorithms based on D-bar methods will be discussed from the point of view of theory, numerical implementation, and their practical use for clinical electrical impedance imaging.

Stability and convergence results for an inverse problem for the wave equation

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Using uniform global Carleman estimates for discrete elliptic and semi-discrete hyperbolic equations, we study Lipschitz and logarithmic stability for the inverse problem of recovering a potential in a semi-discrete wave equation, discretized by finite differences in a 2-d uniform mesh, from boundary or internal measurements. The discrete stability results, when compared with their continuous counterparts, include new terms depending on the discretization parameter h. From these stability results, we design a numerical method to compute convergent approximations of the continuous potential.

References:

Baudouin, L.; S. Ervedoza, S.; Osses, A.; Stability of an inverse problem for the discrete wave equation and convergence results. http://arxiv.org/abs/1310.5092

Localization Analysis for Large-Scale Inverse Problems in Numerical Weather Prediction

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We study large-scale dynamic inverse problems as they appear in Numerical Weather Prediction (NWP) using an Ensemble Kalman Filter (EnKF/LETKF). Ensemble methods provide a flexible alternative to large-scale variational systems. Many different versions of ensemble filters have been suggested and tested over the last years.

However, although Ensemble Data Assimilation (EDA) has high potential, its theoretical justification is still under development, both on the application side testing the development and operational setup of EnKF systems for different data types as well as the mathematical analysis looking into approximation properties of ensembles with ensemble-size small compared to the total number of degrees of freedom in the model. It is very common to use space localization in order to reduce the effect of spurious long range correlations, leading to the Localized Ensemble Transform Kalman Filter (LETKF).

It is the goal of our analysis to understand the basic properties of localization. We derive deterministic error estimates for the EnKF and study its dependence on localization. In particular, we investigate the convergence properties of the localized EnKF when the localization radius tends to zero.

Then, we demonstrate the practical meaning of the analysis by derivation of a formula for an optimal localization radius depending on the observation density and the observation error. The validity of the formula is also demonstrated by numerical experiments for some simple assimilation systems. Applications of the results to convective-scale Numerical Weather Prediction are shown.

Spectral analysis on interior transmission eigenvalues

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We give some result on the interior transmission eigenvalues. First we prove that there exists a infinity of such eigenvalues, the space spanned by the generalized associated eigenfunctions is dense in the natural space associated with the problem. We prove an estimate on the Weyl counting function and in some cases we can give an asymptotic formula.

Uniqueness of coefficients by strong maximum principle

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In this talk, I will present a method introduced in 2010 by Cristofol and Roques to obtain uniqueness results in inverse problems of determining non-constant coefficients of nonlinear parabolic equations. This method is mainly based on the Hopf's lemma and on the parabolic maximum principle. It can be applied to several types of 1D reaction-diffusion equations and systems, such as the Fisher-KPP equation and Lotka-Volterra competition systems.

Recent developments in tensor tomography

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The standard X-ray transform, where one integrates functions over straight lines, is a well-studied object and forms the basis of medical imaging techniques such as CT and PET. This transform has useful generalizations involving other families of curves, weight factors, and integration of tensor fields. These more general transforms come up in seismic imaging (travel time tomography), in medical imaging (SPECT and ultrasound), and in the mathematical analysis of other inverse problems such as the Calderon and Gel'fand problems and in variants of the question "Can one hear the shape of a drum?'".

In this talk we discuss recent progress and open questions related to ray transforms, focusing on the case where one integrates tensor fields over a family of geodesics. The talk is based on joint works with Gabriel Paternain (Cambridge) and Gunther Uhlmann (Washington/Helsinki).

Optimising the optimisers - what is the right image and data model?

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When assigned with the task of reconstructing an image from given data the first challenge one faces is the derivation of a truthful image and data model. Such a model can be determined by the a-priori knowledge about the image, the data and their relation to each other. The source of this knowledge is either our understanding of the type of images we want to reconstruct and of the physics behind the acquisition of the data or we can thrive to learn parametric models from the data itself. The common question arises: how can we optimise our model choice? Starting from the first modelling strategy this talk will lead us from the total variation as the most successful image regularisation model today to non-smooth second- and third-order regularisers, with data models for Gaussian and Poisson distributed data as well as impulse noise. Applications for image denoising, inpainting and surface reconstruction are given. After a critical discussion of these different image and data models we will turn towards the second modelling strategy and propose to combine it with the first one using a bilevel optimisation method. In particular, we will consider optimal parameter derivation for total variation denoising with multiple noise distributions and optimising total generalised variation regularisation for its application in photography.

Local lens and boundary rigidity

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We study the boundary rigidity problem with partial data consisting of determining locally the Riemannian metric of a Riemannian manifold with boundary from the distance function measured at pairs of points near a fixed point on the boundary. We show that one can recover uniquely and in a stable way a conformal factor near a strictly convex point where we have the information. In particular, this implies that we can determine locally the isotropic sound speed of a medium by measuring the travel times of waves joining points close to a convex point on the boundary.

The local results lead to a global lens rigidity uniqueness and stability result assuming that the manifold is foliated by strictly convex hypersurfaces.

This is a joint work with Gunther Uhlmann and Andras Vasy.

Multifrequency inverse medium problem

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In the talk I will present results of uniqueness and stability related to the reconstruction of the refractive index of a medium using multifrequency scattering data.

Inverse problems for fluid dynamics

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I discuss several types of inverse problems for fluid dynamics such as Navier-Stokes equations. I prove uniqueness and conditional stability for the formulations by Dirichlet-to-Neumann map and Carleman estimates. This is a joint work with Prof. O. Imanuvilov (Colorado State Univ.)