Titles and abstracts for "Spin geometry and analysis on manifolds", CIRM, 6-10/10/2014

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Abstract: Green-hyperbolic operators form a large class of differential operators on Lorenzian manifolds many of which are of central importance in physics. Examples include wave operators, Dirac-type operators, symmetric hyperbolic systems and the Proca operator. We will describe some general solution theory. For subclasses like wave operators we will describe the relevant function spaces and will discuss well-posedness of the Cauchy and the Goursat problem.

3 Gérard Besson (Université Joseph Fourier, Grenoble)

Title: Quelques variétés ouvertes de dimension 3

Abstract: On présentera quelques exemples intéressants de variétés ouvertes de dimension 3 dont la géométrie riemannienne est vraiment inconnue. Ce sera l'occasion de poser plusieurs questions relatives à la construction de métriques riemanniennes sur ces objets. Certaines de ces variétés sont des ouverts de la sphère S^3.

4 Jean-Pierre Bourguignon (IHES)

Title:

Abstract:

5 César Del Corral (Universidad de los Andes)

Title:

Abstract:

6 Sylvestre Gallot (Université Joseph Fourier, Grenoble)

Title: Barycentre methods and comparison between manifolds

Abstract:

7 Rod Gover (University of Auckland)

Title: Boundary calculus on conformally compact manifolds, and a boundary Yamabe problem

Abstract: On conformally compact manifolds of arbitrary signature I will describe a natural boundary calculus for computing the asymptotics of a class of natural boundary problems. This is applied to the non-linear problem of finding, conformally, a conformally compact constant scalar curvature metric on the interior of a manifold with boundary. This problem was studied from a different point of view by Andersson, Chruściel, Friedrich (ACF) in 1992. They identified a conformal submanifold invariant that obstructs smooth boundary asymptotics for the problem on 3-manifolds (and gave some information on the obstructions in other dimensions). This invariant is the same as that arising from the variation of the Willmore energy. We find higher order submanifold invariants that generalise that curvature quantity found by ACF. This construction also leads to a route for manufacturing large classes of other conformal submanifold invariants.

This is joint work with Andrew Waldron

8 Ursula Hamenstädt (Universität Bonn)

Title: Almost totally geodesic closed minimal surfaces in rank one locally symmetric manifolds

Abstract: Given a closed rank one locally symmetric manifold M, we show how convexity can be used to construct a family of immersed minimal surfaces in M which are almost totally geodesic and which are injective on fundamental groups. We also discuss some applications and open questions.

9 Blaine Lawson (Stony Brook University)

Title: Potential Theory for Nonlinear PDE's

Abstract: There is an interesting potential theory associated to each degenerate elliptic, fully nonlinear equation of the form $f(D^2u) = 0$. For the stan-

dard complex Monge-Ampère equation, it is just the classical pluripotential theory. I will explain how these theories are defined in general. Fundamental to the analysis is a new invariant of such equations, called the *Riesz characteristic*, which governs asymptotic structures. The notions of tangents to subsolutions and densities will be introduced. Results concerning existence and uniqueness of tangents, the structure of sets of high density points, and the regularity of subsolutions for certain Riesz characteristics, will be discussed. The Dirichlet problem with prescribed asymptotics will be treated. I will also touch upon the question of removable singularities. Examples include real, complex and quaternionic Hessian equations, the p-convexity equation, and equations from calibrated geometry. In particular, this establishes a potential theory on every calibrated manifold.

10 Jean-Louis Milhorat (Université de Nantes)

Title: Dirac operator and holonomy on compact symmetric spaces

Abstract: Let G/K be a spin compact symmetric space, endowed with a Kähler or Quaternion-Kähler structure. With the help of an explicit formula for the first eigenvalue of the Dirac operator, we examine how this lowest eigenvalue is linked to a "lowest action" of the holonomy.

11 Sebastián Montiel (Universidad de Granada)

Title: Cheeger is a hologram of Yamabe: Solution to a problem posed by John Lee in the context of Poincaré-Einstein spaces

Abstract: Let M be an $(n+1)$-dimensional asymptotically locally hyperbolic (ALH) manifold with a conformal compactification whose conformal infinity is $\partial_\infty M$. We will observe that $\mathcal{C}h(M) \leq n$, where $\mathcal{C}h(M)$ is the Cheeger constant of M, and prove that, if the Ricci curvature of M is bounded from below by $-n$ and its scalar curvature approach $-n(n+1)$ fast enough at infinity, $\mathcal{C}h(M) = n$ if and only $\mathcal{Y}(\partial_\infty M) \geq 0$, where $\mathcal{Y}$ denotes the Yamabe type of a conformal manifold. In this way, we give an answer to a question raised by J. Lee.

12 Sergiu Moroianu (IMAR, Bucharest)

Title: Positivity of the renormalized volume of almost-Fuchsian hyperbolic 3-manifolds

Abstract: Almost-Fuchsian manifolds are those quasi-Fuchsian hyperbolic 3-manifolds admitting a minimal surface with principal curvatures bounded in absolute value by 1. The renormalized volume of quasi-Fuchsian manifolds provides a Kähler potential for the Weil-Petersson metric on the Teichmüller space. Together with Corina Ciobotaru, we prove that the renormalized volume of almost-Fuchsian hyperbolic 3-manifolds is non-negative, with equality only for Fuchsian 3-manifolds.

13 Sylvie Paycha (Universität Potsdam)

Title: Traces on toroidal pseudodifferential operators and conformal invariance on the non-commutative torus

Abstract: Trace-class pseudodifferential operators on a closed manifold of dimension n are those of order smaller than $-n$. Zeta-regularised traces extend the ordinary trace to linear forms defined on a larger class of pseudodifferential operators, those whose symbols have a polyhomogeneous asymptotic behaviour at infinity. On operators of non-integer order, zeta-regularised traces coincide with Kontsevich and Vishik's canonical trace.
These constructions hold in particular for tori; we extend them to noncommutative tori using a *global symbol calculus* for pseudodifferential operators on tori inspired from the work of Ruzhanski and Turunen.
We derive "defect formulae" for zeta-regularized traces which are useful to measure anomalies, in particular conformal anomalies. The conformal invariance of the ζ-function at zero of the Laplacian on the noncommutative torus is then a straightforward consequence.
Inspired by the work of Connes and Tretkhoff, Fathizadeh and Khalkhali, Connes and Moscovici on a noncommutative version of the Gauss-Bonnet theorem, and more recently on the (noncommutative version of the) scalar curvature, we further investigate the conformal anomaly of the zeta-determinant of the Laplacian on the noncommutative torus, which on an ordinary closed surface yields back the Gauss-Bonnet theorem.

This is based on joint work with Cyril Lévy and Carolina Neira.

14 Mihaela Pilca (Universität Regensburg)

Title: Homogeneous Almost Quaternion-Hermitian Manifolds

Abstract: An almost quaternion-Hermitian structure on a $4n$-dimensional Riemannian manifold is a reduction of the structure group of M to the subgroup $\mathrm{Sp}(n)\mathrm{Sp}(1) \subset \mathrm{SO}(4n)$. We show that a compact simply connected homogeneous almost-Hermitian manifold of non-vanishing Euler characteristic is either a Wolf space, or $\mathbb{S}^2 \times \mathbb{S}^2$, or the complex quadric $\mathrm{SO}(7)/\mathrm{U}(3)$. This talk is based on joint work with Andrei Moroianu and Uwe Semmelmann.

15 Harold Rosenberg (IMPA)

Title: Minimal surfaces S in hyperbolic 3-manifolds N of finite volume, and in $M \times S^1$, M a hyperbolic surface of finite area.

Abstract: We prove such surfaces S, that are properly immersed and of finite topology, have finite total curvature equal to 2π times the Euler characteristic of S. We show the ends of S are asymptotic to standard model ends. We will give examples and applications. We prove that properly embedded stable minimal surfaces in N, have finite topology and their ends are asymptotic to cusp ends: a quotient of a horo-disk by a parabolic isometry. This is joint work with Pascal Collin and Laurent Hauswirth. In collaboration with Laurent Mazet, we prove existence theorems for embedded minimal submanifolds in N: there is always a compact embedded minimal surface in N.

16 Simon Salamon (King's College London)

Title: Index theory and special geometries

Abstract: I shall discuss contraints on Betti numbers that arise on symmetric spaces and in the presence of special holonomy, and related rigidity results for coupled Dirac operators.

17 Alessandro Savo (Sapienza Università di Roma)

Title: Constant heat flow and the isoparametric property

Abstract: Let Ω be a compact domain with smooth boundary in a Riemannian manifold. We study the solution of the heat equation on Ω having constant unit initial conditions and Dirichlet boundary conditions. The aim of this talk is to discuss the geometry of domains for which, at any fixed value of time, the normal derivative of the solution (heat flow) is a constant function on the boundary. We express this fact by saying that such domains have the *constant flow property*. In constant curvature spaces known examples of such domains are given by geodesic balls and, more generally, by domains whose boundary is connected and isoparametric. The question is: are they all like that?
We will examine this and other questions relating heat diffusion with the r-mean curvatures of the boundary and the isoparametric property. We will also discuss the constant flow property in relation to other well-known overdetermined problems involving the Laplace operator, like the Serrin problem or the Schiffer problem.

18 Uwe Semmelmann (Universität Stuttgart)

Title: Generalized Killing spinors

Abstract:

19 Robert Stanton (Ohio State University)

Title: Spinor constructions for exceptional Lie algebras

Abstract: Let V be a $2n$-dimensional vector space over a field k of characteristic not 2 or 3 and let g be a non-degenerate symmetric bilinear form on V of maximal Witt index. We will describe a construction of exceptional Lie algebras and some of their fundamental finite dimensional representations using the spinors of V. This is joint work with Marcus Slupinski.

20 Xiao Zhang (Chinese Academy of Sciences, Beijing)

Title: Spin geometry and the energy-momentum inequality for asymptotically AdS spacetimes

Abstract: The positive energy theorem plays a fundamental role in general relativity. It was first proved by Schoen-Yau in 1979 using the method of geometric analysis in the case of zero cosmological constant, where initial data sets are asymptotically flat. Later Witten used spin geometry to give another proof. When the cosmological constant is negative and spacetimes are asymptotically AdS, initial data sets are asymptotically hyperbolic. In 1989, Min-Oo extended Witten's method to asymptotically hyperbolic spin manifolds and proved rigidity of hyperbolic spaces. Min-Oo's method was lately used by several authors to provide the complete and rigorous proof of the positive energy theorem for asymptotically AdS spacetimes. In this talk, we will give a short review of the topic. In particular, we will discuss the recent proof by Wang, Xie and the author on the relevant energy-momentum inequality in the most general case as well as the finding of the invariant mass for asymptotically AdS spacetimes.