Analyticity of the Stokes operator with Navier or Navier-type boundary conditions on $\boldsymbol{L}^p$-spaces

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The aim of this work is to study the analyticity of the Stokes operator with Navier or Navier-type boundary conditions on $\boldsymbol{L}^p$-spaces in order to get strong, weak and very weak solutions to the following initial boundary Stokes problem:

$$\begin{cases} \frac{\partial \boldsymbol{u}}{\partial t} - \Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{0}, \quad \operatorname{div} \boldsymbol{u} = 0 & \text{in} \quad \Omega \times (0,T), \\ \boldsymbol{u}(0) = \boldsymbol{u}_0 & \text{in} \quad \Omega, \end{cases} \tag{1}$$

with the following Navier-type boundary condition:

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0, \qquad \mathbf{curl}\, \boldsymbol{u} \times \boldsymbol{n} = \boldsymbol{0} \quad \text{on } \Gamma \times (0,T), \tag{2}$$

or with the Navier boundary condition:

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0, \qquad [\mathbf{D}(\boldsymbol{u})\boldsymbol{n}]_{\boldsymbol{\tau}} = \boldsymbol{0} \quad \text{on } \Gamma \times (0,T). \tag{3}$$

In this work we prove that the Stokes operator with Navier-type boundary conditions generates a bounded analytic semi-group on the space

$$\boldsymbol{L}^p_{\sigma,T}(\Omega) = \big\{ \boldsymbol{v} \in \boldsymbol{L}^p(\Omega);\ \operatorname{div} \boldsymbol{v} = 0 \text{ in } \Omega \quad \text{and} \quad \boldsymbol{v} \cdot \boldsymbol{n} = 0 \text{ on } \Gamma \big\}.$$

The idea is to study the resolvent of the Stokes operator:

$$\lambda \boldsymbol{u} - \Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f}, \quad \operatorname{div} \boldsymbol{u} = 0 \qquad \text{in } \Omega, \tag{4}$$

with the boundary conditions (2) or (3), and where $\lambda \in \mathbb{C}^*$ satisfies $\operatorname{Re} \lambda \geq 0$. We prove the existence of weak, strong and very weak solutions to Problem (4),(2) or Problem (4),(3) satisfying the following resolvent estimate

$$\|\boldsymbol{u}\|_{\boldsymbol{L}^p(\Omega)} \leq \frac{C(\Omega, p)}{|\lambda|} \|\boldsymbol{f}\|_{\boldsymbol{L}^p(\Omega)}.$$

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Liouville theorems for a shallow water equation and related models

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The Euler-α equation plays an important role in modelling and in numerical regularizations of turbulent flows. The goal of this talk is to provide a few Liouville theorems in the one-dimensional case, *i.e.* for the Camassa–Holm equation, as well as for related models of nonlinear dispersive wave propagation (the rod equation, etc.). For such equations, we prove that the only global strong, spatially periodic solution vanishing in at least one point $(t_0, x_0) \in \mathbb{R}^+ \times \mathbb{S}^1$ is the identically zero solution. Such conclusion holds provided a physical parameter of the model (related to the Finger deformation tensor) is outside some neighborhood of the origin. We also establish the analogue of this result in the case of non-periodic solutions defined on the whole real line with vanishing boundary conditions at infinity. Our analysis relies on the application of new local-in-space blowup criteria and involves the computation of several best constants in convolution estimates and weighted Poincaré inequalities.

This is a partially joint work with Fernando Cortez.

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On the Hall-MHD equations

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In this talk we present recent results on the Hall-MHD system. We consider the incompressible MHD-Hall equations in $\mathbb{R}^3$.

$$
\begin{aligned}
&\partial_t u + u \cdot \nabla u + \nabla p = (\nabla \times B) \times B + \nu \Delta u, \\
&\nabla \cdot u = 0, \quad \nabla \cdot B = 0, \\
&\partial_t B - \nabla \times (u \times B) + \nabla \times ((\nabla \times B) \times B) = \mu \Delta B, \\
&u(x,0) = u_0(x) \quad ; \quad B(x,0) = B_0(x).
\end{aligned}
$$

Here $u = (u_1, u_2, u, u_3) = u(x,t)$ is the velocity of the charged fluid, $B = (B_1.B_2, B_3)$ the magnetic field induced by the motion of the charged fluid, $p = p(x,t)$ the pressure of the fluid. The positive constants ν and μ are the viscosity and the resistivity coefficients. Compared with the usual viscous incompressible MHD system, the above system contains the extra term $\nabla \times ((\nabla \times B) \times B)$, which is the so called Hall term. This term is important when the magnetic shear is large, where the magnetic reconnection happens. On the other hand, in the case of laminar flows where the shear is weak, one ignores the Hall term, and the system reduces to the usual MHD. Compared to the case of the usual MHD the history of the fully rigorous mathematical study of the Cauchy problem for the Hall-MHD system is very short. The global existence of weak solutions in the periodic domain is done in [1] by a Galerkin approximation. The global existence in the whole domain in $\mathbb{R}^3$ as well as the local well-posedness of smooth solution is proved in [2], where the global existence of smooth solution for small initial data is also established. A refined form of the blow-up criteria and small data global existence is obtained in [3]. Temporal decay estimateof the global small solutions is deduced in [4]. In the case of zero resistivity we present finite time blow-up result for the solutions obtained in [5]. We note that this is quite rare case, as far as the authors know, where the blow-up result for the *incompressible* flows is proved.

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On the isotropic nature of the possible blow up for 3D Navier-Stokes

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The purpose of the talk will be the proof of the following result for the homogeneous incompressible Navier-Stokes system in dimension three: given an initial data v_0 with vorticity $\Omega_0 = \nabla \times v_0$ in $L^{\frac{3}{2}}$, (which implies that v_0 belongs to the Sobolev space $H^{\frac{1}{2}}$), we prove that the solution v given by the classical Fujita-Kato theorem blows up in a finite time $T^\star$ only if, for any p in $]4,6[$ and any unit vector e in $\mathbb{R}^3$, there holds

$$\int_0^{T^\star} \|v(t) \cdot e\|^p_{\frac{1}{2}+\frac{2}{p}}\, dt = \infty.$$

We remark that all these quantities are scaling invariant under the scaling transformation of Navier-Stokes system.

Maximum modulus estimate of Stokes flow

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In this talk, we extend the maximum modulus estimate for the space variables of the nonstationary Stokes equations in the bounded C^2 cylinders to time estimate. We show that if the boundary data is L^∞ and the normal part of the boundary data has log-Dini continuity with respect to time, then the velocity is bounded. We emphasize that there is no continuity assumption on space variables in the new maximum modulus estimate. This completes the maximum modulus estimate.

We also discuss of counter examples which show the theorem is almost optimal.

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Low Mach number limit: old and new

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A number of recent works have been devoted to the rigorous justification of the low Mach number limit for the compressible Navier-Stokes equations. In this talk, we consider the so-called ill-prepared data case, and prove the convergence to the incompressible Navier-Stokes equations. Compared to prior papers, we here succeed in justifying the asymptotics in a L^p-related framework, even though acoustic waves have to be considered. This result is a work in progress with Lingbing He (Tsinghua University, Beijing).

Global L_2–solvability of an interface problem governing two-phase capillary fluid motion

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We deal with the motion of two immiscible fluids in a container. The liquids are separated by a close unknown interface on which surface tension is taken into account. This flow is governed by an interface problem for the non-homogeneous Navier–Stokes system. We prove that the problem is uniquely solvable in an infinite time interval provided that the initial velocity of the liquids and mass forces are small enough, the initial configuration of the inner fluid is close to a ball. Moreover, we show that the velocity decays exponentially at infinity with respect to time and that the interface between the fluids tends to a sphere of the certain radius. The proof is based on an exponential estimate of a generalized energy [1] and on a local existence theorem for the problem in anisotropic Sobolev–Slobodetskiĭ spaces [2]. We reduce the regularity of an initial interface demanded earlier for local L_2–solvability.

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Spatial decay of weak solutions of the 3D time-dependent Navier-Stokes system.

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We consider weak solutions of the 3D time-dependent Navier-Stokes system in an exterior domain. Our main result states that if the initial data decays sufficiently fast, the solution exhibits a spatial decay of a certain rate. Within a certain range of rates, the solution reproduces the decay rate of the initial data.

The proof of this result is based on suitable representation formulas for solutions of the time-dependent Stokes system.

A unified framework for parabolic equations with mixed boundary conditions and diffusion on interfaces

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We consider scalar parabolic equations in a general non-smooth setting with emphasis on mixed interface and boundary conditions. In particular, we allow for dynamics and diffusion on a Lipschitz interface and on the boundary, where diffusion coefficients are only assumed to be bounded, measurable and positive *semi*-definite. In the bulk, we additionally take into account diffusion coefficients which may degenerate towards a Lipschitz surface. For this problem class, we introduce a unified functional analytic framework based on sesquilinear forms and show maximal L^p-regularity for the abstract Cauchy problem in L^q. Important ingredients of the proof are the definition and analysis of (possibly degnerate) diffusion along Lipschitz hypersurfaces and extension, embedding and trace theorems for Muckenhoupt-weighted Sobolev spaces.

Stability issues in the theory of complete fluid systems

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We discuss some stability problems related to the Navier-Stokes-Fourier system describing the motion of a compressible, viscous, and heat conducting fluids. We introduce the concept of relative entropy/energy and present some applications that concern:

- Existence and conditional regularity of weak solutions;
- singular limits;
- existence and regularity for the inviscid system.

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Some results on global solutions to the Navier-Stokes equations

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In this talk we shall present some results concerning global smooth solutions to the three-dimensional Navier-Stokes equations set in the whole space $\mathbb{R}^3$:

$$\partial_t u + u \cdot \nabla u - \Delta u = -\nabla p \,, \quad \operatorname{div} u = 0 \,.$$

We shall more particularly be interested in the geometry of the set $\mathcal{G}$ of initial data giving rise to a global smooth solution.

The question we shall address is the following: given an initial data u_0 in $\mathcal{G}$ and a sequence of divergence free vector fields converging towards u_0 in the sense of distributions, is the sequence itself in $\mathcal{G}$? The related question of strong stability was studied in [1] and [2] some years ago; the weak stability result is a recent work, joint with H. Bahouri and J.-Y. Chemin (see [3]-[4]). As we shall explain, it is necessary to restrict the study to sequences converging weakly *up to rescaling* (under the natural rescaling of the equation). Then weak stability can be proved, using profile decompositions in the spirit of P. Gérard's work [5], in an anisotropic context.

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Womersley Flow of Generalized Newtonian Liquid

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We show that in an infinite straight pipe of arbitrary cross section with Lipschitz boundary, a generalized Newtonian liquid admits one and only one fully developed time-periodic flow, either when the flow rate or the axial pressure gradient is prescribed. In addition, the flow depends continuously upon the data. We deal with fluids with either shear-thickening or shear-thinning behaviour. In this last case we can prove our results also for singular shear viscosity.

Polymeric fluids

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The essence of modelling polymeric fluids is encapsulated in the coupling of equations describing the evolution of macroscopic quantities (like velocity, pressure and eventually also density and temperature) with an additional equation describing the microscopic structure. The influence of the processes of polymerization and fragmentation will be accounted through the dependence of viscosity on the level of polymerization and/or appearance of the extra stress tensor. We concentrate on a dilute solution of polymer chains suspended in a non-Newtonian solvent, and we assumed that individual polymer chains do not interact with one another, but can be convected by the macroscopic velocity field, and are also subject to polymerization and fragmentation processes.

L^∞-type Estimates for the Stokes Equation

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In this talk we show that the solution of the Stokes equation is governed by a bounded analytic semigroup on $L^\infty_\sigma(\Omega)$ of angle $\pi/2$ for a large class of domains Ω. We further discuss various decay properties of this solution. This is joint work with K. Abe, Y. Giga and P. Maremonti and M. Rapp.

Stability of time-dependent Navier-Stokes flow and algebraic energy decay

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Let $V = V(x,t)$ be a given time-dependent Navier-Stokes flow of an incompressible viscous fluid in the whole space $\mathbb{R}^3$. As important examples of this basic flow V, we have the following in mind: forward self-similar solution, time-periodic solution and global mild (eventually strong) solution of the Cauchy problem. It is thus reasonable to assume that $V \in L^\infty(0,\infty; L^{3,\infty}) \cap C_w([0,\infty); L^{3,\infty})$, where $L^{3,\infty}$ denotes the weak-L^3 space. The energy stability of small V in this class with respect to any initial disturbance in L^2_σ has been recently investigated by Karch, Pilarczyk and Schonbek. It would be interesting to find how fast the disturbance decays in L^2 as $t \to \infty$ when the initial disturbance possesses better summability at space infinity. In this presentation it is proved that any weak solution $u(x,t)$ with the strong energy inequality to the perturbed Navier-Stokes system, which the disturbance should obey, enjoys $\|u(t)\|_2 = O\big(t^{-\frac{3}{2}(1/q-1/2)}\big)$ as $t \to \infty$ provided the initial disturbance is taken from $L^q \cap L^2_\sigma$ for some $q \in [1,2)$. The proof relies on a development of the Fourier splitting method combined with L^q-L^r estimate of the evolution operator generated by the linearized operator around the basic flow V.

This talk is based on a joint work with Maria E. Schonbek (University of California, Santa Cruz).

A Liouville theorem for the planar Navier-Stokes equations with the no-slip boundary condition and its application to a geometric regularity criterion

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In this talk, we establish a Liouville type result for a backward global solution to the Navier-Stokes equations in the half plane with the no- slip boundary condition. No assumptions on spatial decay for the vorticity nor the velocity field are imposed. We study the vorticity equations instead of the original Navier-Stokes equations. As an application, we extend the geometric regularity criterion for the Navier -Stokes equations in the three dimensional half space under the no-slip boundary condition. This is a joint work with Yoshikazu Giga (University of Tokyo) and Yasunori Maekawa (Tohoku University).

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L^p theory for generalized Newtonian flows

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I will present results about regularity of weak solutions to the system describing a steady flow of a generalized Newtonian fluid. The model example of considered equations is the system $-\operatorname{div} A(Du) + \nabla p = -\operatorname{div} A(G)$, $\operatorname{div} u = 0$ with $A(D) = |D|^{p-2}D$ for some $p > 1$ in a domain $\Omega \subset \mathbb{R}^n$. I focus on optimal local estimates in Lebesgue spaces. It will be shown that if $G \in L^q(2Q)$, for some cube Q and $q \in [p, pn/(n-2)]$ then $Du \in L^q(Q)$. These results were published in [1].

Also a recent result from [2] on the local estimates up to the boundary of Ω under perfect slip boundary conditions will be discused.

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Global L^2–asymptotic stability of solutions to Navier-Stokes system

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It is well-known that the following initial value problem for the Navier–Stokes system for an incompressible fluid in the whole three dimensional space

$$\begin{aligned} u_t - \Delta u + \nabla\cdot(u\otimes u) + \nabla p &= F, \quad (x,t) \in \mathbf{R}^3 \times (0,\infty) \\ \operatorname{div} u &= 0, \\ u(x,0) &= u_0(x). \end{aligned}$$

has a global-in-time solution in several cases *e.g.* for a sufficiently small initial condition u_0 and for a small external force F in suitable scaling invariant spaces or if we limit ourselves to a two-dimensional flow. In the talk, I will explain how to show that these solutions are asymptotically stable under arbitrary large perturbations (in the $L^2(\mathbf{R}^3)$-norm) of their initial conditions. This is a joint work with Dominika Pilarczyk, Maria Elena Schonbek, and Xiaoxin Zheng.

Convergence of $\mathcal{D}$-solutions in constant vectors at infinity of the stationary Navier-Stokes equations past an obstacle

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This is the joint work with Horst Heck (Bern Univ., Switzerland) and Hyunseok Kim (Sogan Univ., Korea).

In an exterior domain $\Omega \subset \mathbb{R}^3$, we consider the stationary Navier-Stokes equations

$$\text{(N-S)} \qquad \begin{cases} -\Delta u + u \cdot \nabla u + \nabla p = \mathrm{div}\ F \quad \text{in } \Omega, \\ \mathrm{div}\ u = 0 \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega, \\ u(x) \to u^\infty \quad \text{as } |x| \to \infty \end{cases}$$

where F and u^∞ are the given 3×3-tensors of functions on Ω and the prescribed constant vector in $\mathbb{R}^3$ at infinity, respectively. For a family $\{F_n, u_n^\infty\}_{n=1}^\infty$ of given data, we denote by $\{u_n\}_{n=1}^\infty$ the family of $\mathcal{D}$-solutions corresponding to (N-S). In this talk, we shall show that there is a constant $\delta > 0$ such that if $\|F_n\|_{L^{3/2,\infty}} + |u_n^\infty| \le \delta$ holds for all $n = 1, 2, \cdots$ and if $F_n \rightharpoonup F$ weakly-$*$ in $L^{3/2,\infty}(\Omega) \cap L^2(\Omega)$, $u_n^\infty \to u^\infty$ in $\mathbb{R}^3$ as $n \to \infty$, then we have that $\nabla u_n \rightharpoonup \nabla u$ weakly-$*$ in $L^{3/2,\infty}(\Omega) \cap L^2(\Omega)$, where u is a unique $\mathcal{D}$-solution of (N-S). Furthermore, if $F_n \to F$ strongly in $L^2(\Omega)$, then it holds that $\nabla u_n \to \nabla u$ strongly in $L^q(\Omega)$ for all $3/2 < q \leqq 2$. This result may be regarded as generalization of Shibata-Yamazaki [2] since we may include such a critical case as $u^\infty = 0$. It should be noted that the solution u of (N-S) with $u^\infty = 0$ behaves completely different from those of the case $u^\infty \neq 0$.

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On bounded solutions to the compressible isentropic Euler system

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We analyze the Riemann problem for the compressible isentropic Euler system in the whole space $\mathbb{R}^2$. Using the tools developed by De Lellis and Székelyhidi for the incompressible Euler system we show that for every Riemann initial data yielding the self-similar solution in the form of two admissible shocks there exist in fact infinitely many admissible bounded weak solutions. Moreover for some of these initial data such solutions dissipate more total energy than the self-similar solution which might be looked at as a natural candidate for the "physical" solution.

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Strong solutions of the Navier-Stokes equations with Navier's boundary conditions

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In this contribution we deal with a system of the Navier-Stokes equations with Navier's boundary condition or with Navier-type boundary conditions. We study perturbations of initial conditions of strong solutions of our system. We prove that if these perturbations are sufficiently small in L^3- norm then corresponding solutions are strong too.

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On long-time asymptotics of the Navier-Stokes flows in a two-dimensional exterior domain

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We consider the incompressible Navier-Stokes equations in a two-dimensional exterior domain Ω, with no-slip boundary conditions. Our initial data are of the form $u_0 = \alpha\Theta_0 + v_0$, where Θ_0 is the Oseen vortex with unit circulation at infinity and v_0 is a solenoidal perturbation belonging to $(L^2(\Omega))^2$. We show that the solution behaves asymptotically in time like the self-similar Oseen vortex with circulation α, when $|\alpha|$ is sufficiently small. This is a global stability result, in the sense that the perturbation v_0 can be arbitrarily large.

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Stochastic Three-Dimensional Rotating Navier-Stokes Equations: Averaging, Convergence and Regularity

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In this talk we consider the Stochastic 3D Rotating Navier-Stokes Equations

$$\partial_t U - \nu \Delta U + (U \cdot \nabla) U + \frac{1}{\epsilon} e_3 \times U = -\nabla \pi + \sqrt{Q} \frac{\partial \mathcal{W}}{\partial t}, \tag{5}$$

$$\operatorname{div} U = 0, \tag{6}$$

$$U|_{t=0} = U^0(x_1, x_2, x_3), \tag{7}$$

where $U(t,x) = (U_1, U_2, U_3)$, $x = (x_1, x_2, x_3)$ is the velocity field (a random 3D vector field), $\pi(t,x)$ is the pressure (a random scalar field), $\nu > 0$ is the kinematic viscosity, $\mathcal{W}(t)$ is a cylindrical Wiener process which injects energy in x_3 dependent modes, defined on a filtered probability space $(\Omega, F_t, \mathcal{P})$, in the Hilbert space $H = L^2_s$ of square integrable solenoidal vector fields; Q is a non-negative, symmetric, trace class operator in H. In (5), e_3 is the vertical axis, $e_3 \times U = (-U_2, U_1, 0) = JU$; J is the non-local Poincaré-Coriolis-Riesz rotation operator on solenoidal vector fields. Eqs. (5)-(7) are fundamental in meteorology and geophysical fluid dynamics where the rotation plays an essential role.

For the stochastic three-dimensional rotating Navier-Stokes equations (1)-(3), we prove averaging theorems for stochastic dynamics in the case of strong rotation. Regularity results are established by bootstrapping from global regularity of the limit stochastic equations and convergence theorems. The energy injected in the system by the noise including x_3-dependent modes is large, the initial condition has large energy, and the regularization time horizon is long. Regularization is the consequence of precise mechanisms of relevant three-dimensional nonlinear interactions. We establish multiscale averaging and convergence theorems for the stochastic dynamics.

To understand reality, we deal with stochastic three-dimensional fluid dynamics having intense activity and energy (solutions depend on three space variables and dynamics is far from 2D for all times), and prove that energy and vorticity remain bounded due to fast rotation. Since rotation (global or local) is one of the most common features of real fluids, the principle that fast rotation has a smoothing effect is important for applications.

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A regularity result for suitable weak solutions to the three-dimensional Navier-Stokes initial value problem

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We consider the three-dimensional Navier-Stokes initial value problem. Let $V \subseteq \mathbb{R}^3$ be an open set. We introduce the space of functions

$$J_V(\mathbb{R}^3) := \{u(y) : \|u\|^2_{2,w(x)} := \int_\Omega |u(y)|^2 |x-y|^{-1} dy < \infty, \text{ a.e. in } x \in V\}.$$

Moreover, by $J(\mathbb{R}^3)$ we mean $\{u(y) : u \in L^2(\mathbb{R}^3)$ and $\nabla \cdot u = 0$ in w. s.$\}$.

By a suitable weak solution to the Navier-Stokes initial value problem we mean a pair (u, π_u) which is a weak solution in the sense of the definition given in [1].

We establish the following

Theorem *Let $V \subseteq \mathbb{R}^3$ be an open set. There is $\varepsilon_0 > 0$, independent of V, such that for all $u_0 \in J_V(\mathbb{R}^3) \cap J^2(\mathbb{R}^3)$ with $\sup\limits_{x \in V} \|u_0\|_{2,w(x)} < \varepsilon_0$, there exists a suitable weak solution (u, π_u) enjoying the property, for all $B(x_0, R) \subseteq V$ and $\delta > 0$,*

$$\|u(t)\|_{L^\infty(B(x_0,R-3\delta))} \leq c\,(\sup_{B(x_0,R)} \|u_0\|_{2,w(x)}, \|u_0\|_2) \min\{t^{-\frac{1}{2}}, t^{-\frac{3}{4}}\},$$

a.e. in $t > 0$. This result should be compared with the ones proved in [1,2].

The theorem is contained in the paper

- F. Crispo and P. Maremonti, *A remark on the partial regularity of a suitable weak solution*, forthcoming.

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The Navier-Stokes system with time-dependent Robin-type boundary conditions

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We show that the incompressible 3D Navier-Stokes system in a $\mathscr{C}^{1,1}$ bounded domain or a bounded convex domain Ω with a non penetration condition $\nu \cdot u = 0$ at the boundary $\partial\Omega$ together with a time-dependent Robin boundary condition of the type $\nu \times \operatorname{curl} u = \beta u$ on $\partial\Omega$ admits a solution with enough regularity provided the initial condition is small enough in an appropriate functional space.

More precisely: let $\tau > 0$ and $\beta : [0,\tau] \times \partial\Omega \to \mathscr{M}_3(\mathbb{R})$ bounded measurable on $[0,\tau] \times \Omega$ preserving normal vector fields at the boundary, such that $\beta(t,x)$ is symmetric for almost all (t,x) and satisfying, for a constant $M > 0$, $0 \le \beta(t,x)\xi \cdot \xi \le M|\xi|^2$ for almost all (t,x) and all $\xi \in \mathbb{R}^3$. We assume in addition that $t \mapsto \beta(t,x)$ is piecewise Hölder continuous of order $\alpha > \frac{1}{2}$. Then there exists $\varepsilon > 0$ such that for all $u_0 \in L^2(\Omega,\mathbb{R}^3)$ with $\operatorname{div} u_0 = 0$ in Ω, $\operatorname{curl} u_0 \in L^2(\Omega,\mathbb{R}^3)$, $\nu \cdot u_0 = 0$ on $\partial\Omega$ and $\|u_0\|_2 + \|\operatorname{curl} u_0\|_2 < \varepsilon$, the problem

$$\left\{\begin{array}{rclll} \partial_t u - \Delta u + \nabla\pi - u \times \operatorname{curl} u & = & 0 & \text{in} & (0,\tau)\times\Omega, \\ \operatorname{div} u & = & 0 & \text{in} & (0,\tau)\times\Omega, \\ \nu \cdot u & = & 0 & \text{on} & (0,\tau)\times\partial\Omega, \\ \nu \times \operatorname{curl} u & = & \beta\, u & \text{on} & (0,\tau)\times\partial\Omega, \\ u(0,\cdot) & = & u_0 & \text{in} & \Omega. \end{array}\right.$$

admits a solution $u \in H^1(0,\tau,L^2(\Omega,\mathbb{R}^3))$ with $-\Delta u + \nabla\pi \in L^2((0,\tau)\times\Omega)$.

On the existence of weak solutions to Kolmogorov's two-equation model of turbulence

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In 1942, Kolmogorov postulated the following two-equation model for the turbulent motion of an incompressible fluid

$$\operatorname{div} \boldsymbol{u} = 0, \quad \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \operatorname{div}\Big(\frac{k}{\omega} D(\boldsymbol{u})\Big) - \nabla p + \boldsymbol{f}\,, \tag{1}$$

$$\frac{\partial \omega}{\partial t} + \boldsymbol{u} \cdot \nabla \omega = \operatorname{div}\Big(\frac{k}{\omega} \nabla \omega\Big) - \omega^2\,, \tag{2}$$

$$\frac{\partial k}{\partial t} + \boldsymbol{u} \cdot \nabla k = \operatorname{div}\Big(\frac{k}{\omega} \nabla k\Big) + \frac{k}{\omega}\,|D(\boldsymbol{u})|^2 - k\omega\,. \tag{3}$$

Here, the unknown functions are $\boldsymbol{u} = (u_1, u_2, u_3)$ mean velocity, p mean pressure, $k \geq 0$ mean turbulent kinetic energy, $\omega > 0$ rate of dissipation; $D(\boldsymbol{u}) = \frac{1}{2}(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\mathrm{T}})$ mean strain-rate tensor. The function $l := \frac{k^{1/2}}{\omega}$ ("external length scale") characterizes the part of the turbulent flow that carries most of the turbulent energy.

For sufficiently small $T > 0$, we present an existence result for weak solutions $(\boldsymbol{u}, k, \omega)$ to (1)–(3) in $\Omega \times \,]\,0, T[$ ($\Omega := (\,]\,0, L[\,)^3$) under spatial periodic boundary conditions at $\partial\Omega \times \,]\,0, T[$ and given initial data $(\boldsymbol{u}_0, k_0, \omega_0)$ in $\Omega \times \{0\}$.

Low Mach number limit and diffusion limit in a model of radiative flow

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We consider an asymptotic regime for a simplified model of compressible Navier-Stokes-Fourier system coupled to the radiation, when hydrodynamical flow is driven to incompressibility through the low Mach number limit. We prove a global-in-time existence for the primitive problem in the framework of weak solutions and for the incompressible target system and we study the convergence of the primitive system toward its incompressible limit. Moreover,we investigate the cases when the radiative intensity is driven either to equilibrium or to non-equilibrium diffusion limit, depending the scaling performed, and we study the convergence of the system toward the aforementioned limits.

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Some topics in the mathematical thermodynamics of compressible fluids

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We will talk about several issues related to the notions of weak solutions, dissipative solutions and stability properties to the compressible Navier-Stokes system and its approximations.

Heat-conducting, compressible mixtures with multicomponent diffusion

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We study a model for heat conducting compressible chemically reacting gaseous mixture, based on the coupling between the compressible Navier–Stokes–Fourier system and the full Maxwell-Stefan equations. The viscosity coefficients are density-dependent functions vanishing on vacuum and the internal pressure depends on species concentrations. More precisely, the model reads

$$\left.\begin{aligned} \partial_t \varrho + \operatorname{div}(\varrho \mathbf{u}) &= 0 \\ \partial_t(\varrho \mathbf{u}) + \operatorname{div}(\varrho \mathbf{u} \otimes \mathbf{u}) - \operatorname{div} \mathbf{S} + \nabla \pi &= \varrho \mathbf{f} \\ \partial_t E + \operatorname{div}(E\mathbf{u}) + \operatorname{div}(\pi \mathbf{u}) + \operatorname{div} \mathbf{Q} - \operatorname{div}(\mathbf{S}\mathbf{u}) &= 0 \\ \partial_t \varrho_k + \operatorname{div}(\varrho_k \mathbf{u}) + \operatorname{div}(\mathbf{F}_k) &= \varrho \vartheta \omega_k, \quad k \in \{1, ..., n\} \end{aligned}\right\} \quad \text{in } (0,T) \times \Omega. \tag{4}$$

We consider the question of existence of a solution to this system and based on several levels of approximations we construct a weak solution without any restriction on the size of the data.

Stokes- and Navier-Stokes equations on domains with edges and corners

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Fluid flow in singular domains has quite some significance in applications. For instance, models of cyclones, such as hurricanes or tornados, are considered in a cylinder which is, by the presence of edges, a so-called weakly singular domain. Another important application are wetting and de-wetting phenomena. Here in general three-phase dynamic contact lines (gas/fluid/solid, fluid/fluid/solid, etc.) appear, which at the end lead to systems of equations on wedge type domains. In spite of their significance there still exist fundamental open problems concerning their rigorous mathematical treatment. The aim of the lecture, hence, is to present some recent progress in this direction. We intend to present maximal regularity type results for the Stokes equations subject to Navier slip on cylindrical domains, on wedge type domains, and on (graph) Lipschitz domains. These results are based on parabolic cylindrical theory, the commuting and non-commuting operator sum method, and off-diagonal estimates. As a consequence of the linear outcome we can recover well-known results for the Navier-Stokes equations on smooth domains.

Maximal L^p-Regularity of the Spatially Periodic Stokes Operator

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We investigate Navier-Stokes flows in the whole space $\mathbb{R}^n$, $n \geq 3$, subject to a spatial periodicity in one direction. To this end we prove maximal L^p-regularity in $L^q(G)$-spaces of the abstract Stokes operator $\mathcal{A}_{G,q}$ in the locally compact abelian group $G := \mathbb{R}^{n-1} \times T$, where T is the one-dimensional torus and the abstract Stokes operator is defined in an obvious way on $L^q_\sigma(G)$. This is achieved by using abstract harmonic analysis in order to establish a concept of the class of Muckenhoupt weights on the group G. With the Muckenhoupt weights at hand we prove an extrapolation theorem for groups, corresponding to the classical extrapolation theorem due to García-Cuerva and Rubio de Francia (1985). This enables us to show $\mathcal{R}$-boundedness of the operator family $\lambda(\lambda - \mathcal{A}_{G,q})^{-1}$ in the weighted space $L^q_{\omega,\sigma}(G)$ of solenoidal vector functions.
As an application we obtain local-in-time existence of spatially periodic solutions to a quasilinear parabolic system describing the dynamics of nematic liquid crystal flows. The underlying model is a simplified Ericksen-Leslie model coupling the Navier-Stokes equation for the velocity with a diffusion equation for the macroscopic orientation d of a molecule.

The Stokes and Navier-Stokes Equations with Navier and Navier-type Boundary Conditions

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In a three dimensional bounded eventually multiply-connected domain, we present some results concerning the stationary Stokes and Navier Stokes equations with different boundary conditions of the form:

$$\boldsymbol{u}\cdot\boldsymbol{n} = g, \quad \mathbf{curl}\,\boldsymbol{u}\times\boldsymbol{n} = \boldsymbol{h}\times\boldsymbol{n} \tag{5}$$

$$\boldsymbol{u}\times\boldsymbol{n} = \boldsymbol{g}\times\boldsymbol{n} \quad \text{and} \quad \pi = \pi_0 \tag{6}$$

and the Navier boundary condition:

$$\boldsymbol{u}\cdot\boldsymbol{n} = 0 \qquad \text{and} \qquad 2[\boldsymbol{D}(\boldsymbol{u})\boldsymbol{n}]_{\boldsymbol{\tau}} = 0 \tag{7}$$

We prove the existence and uniqueness of weak, strong and very weak solutions correspending to the boundary conditions (5) and (6) in L^p theory. Next, we give the relationship between the Navier boundary condition (7) and the Navier-type boundary condition (5). To prove the solvability, we study the well-posedeness of some elliptic systems. For this end, it is necessary to establish inf-sup conditions which play an essential role in our proofs.

End-point maximal L^1-regularity of the Cauchy problem for a parabolic equation

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In this talk, we consider maximal L^1-regularity of the Cauchy problem for a parabolic equation in the homogeneous Besov space. In the preceding paper [3], we obtained maximal L^p-regularity of the Cauchy problem for a heat equation in the homogeneous Besov space with $1 < p \leq \infty$, however we did not cover maximal regularity for $p = 1$. Although the estimate is known by Danchin [1], we show maximal L^1-regularity by a different approach than known result and give some optimality of function spaces. We also discuss the relation between the result due to Giga-Saal [2] that shows maximal time L^1-regularity in the space of the Fourier transformed finite Radon measures. This is a joint work with Takayoshi Ogawa (Tohoku Univ.).

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Conditional regularity for the solutions of the Navier-Stokes equations

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We study conditional regularity criteria for the nonstationary solutions of the Navier-Stokes equations in the whole three-dimensional space based on the velocity gradient. We pay a special attention to the criteria based on the regularity of the gradient of one velocity component. We use the Lebesgue and Besov spaces for the presentation of our results.

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Spatial asymptotics for time periodic solutions to the Stokes problem in a layer

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We consider the solutions to the time periodic Stokes problem in a layer $\Omega = \mathbb{R}^2 \times (0,1) \ni x = (y,z)$:

$$\begin{aligned} &u_t - \Delta u + \nabla p = f, \quad \operatorname{div} u = g \text{ in } \Omega \\ &u|_{z=1} = 0, \quad u|_{z=0} = 0, \quad u|_{t=0} = u|_{t=2\pi}, \end{aligned}$$

where the data f, g are also time periodic and smooth with bounded support for simplicity. Starting from solutions with $u \in L^2(L^2_\beta)$, $p \in L^2(L^2_\beta)$, where $L^2_\beta(\Omega)$ is a weighted L^2-space with polynomial weight at infinity, we derive the main asymptotic terms of u, p as $|y|$ tends to infinity. Observe that no compatibility condition for the data is needed because the solutions may drive flux to infinity.

Dynamics of a point vortex as limits of a shrinking solid in an irrotational fluid

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Joint work with

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In this talk I will present the joint work [1] with Olivier Glass and Alexandre Munnier. In this work we consider the motion of a rigid body immersed in a two-dimensional perfect fluid. The fluid is assumed to be irrotational and confined in a bounded domain. We prove that when the body shrinks to a pointwise massless particle with fixed circulation, its dynamics in the limit is given by the point vortex equation.

As a byproduct of our analysis we also prove that when the body shrinks with a fixed mass the limit equation is a second-order differential equation involving a Kutta-Joukowski-type lift force, which extends the result of [2] to the case where the domain occupied by the solid-fluid system is bounded.

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The h-principle and turbulence

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It is well known since the pioneering work of Scheffer and Shnirelman that weak solutions of the incompressible Euler equations exhibit a wild behaviour, which is very different from that of classical solutions. Nevertheless, weak solutions in three space dimensions have been studied in connection with a long-standing conjecture of Lars Onsager from 1949 concerning anomalous dissipation and, more generally, because of their possible relevance to the K41 theory of turbulence.

In recent joint work with Camillo De Lellis we established a connection between the theory of weak solutions of the Euler equations and the Nash-Kuiper theorem on rough isometric immersions. Through this connection we interpret the wild behaviour of weak solutions of Euler as an instance of Gromov's h-principle. In this lecture we explain this connection and outline recent progress towards Onsager's conjecture.

Long time solvability for the 3D rotating Euler equations

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In this talk, we consider the long time existence of classical solutions to the 3D incompressible rotating Euler equations for initial data in $H^s(\mathbb{R}^3)$ with $s > 5/2$. More precisely, we shall show that for given initial velocity $\phi \in H^s(\mathbb{R}^3)$ with $s > 5/2$ satisfying $\operatorname{div}\phi = 0$ and for given finite time T, there exists a positive number $\Omega_{\phi,T}$ such that the 3D rotating Euler equation admits a unique classical solution on the time interval $[0,T]$ provided that the speed of rotation $\Omega \in \mathbb{R}$ is sufficiently high, that is, $|\Omega| \geqslant \Omega_{\phi,T}$. Furthermore, we shall give an upper bound of the minimal speed of rotation $\Omega_{\phi,T}$ which ensures the long time existence of classical solutions in terms of the norm of initial data and the given time T when initial data belong to $H^{7/2}(\mathbb{R}^3)$. This also gives a lower bound for the maximal existence time of the solution in terms of the rotating speed $|\Omega|$.

Uniqueness of solutions on the whole time axis to the Navier-Stokes equations in unbounded domains

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We consider the uniqueness of bounded continuous L^3_w-solutions on the whole time axis to the Navier-Stokes equations in 3-dimensional unbounded domains. Thus far, uniqueness of such solutions to the Navier-Stokes equations in unbounded domain, roughly speaking, is known only for a small solution in $BC(R; L^3_w)$ within the class of solutions which have sufficiently small $L^\infty(L^3_w)$-norm. In this talk, we give another type of uniqueness theorem for solutions in $BC(R; L^3_w)$ using a smallness condition for one solution and a precompact range condition for the other one. The proof is based on the method of dual equations.

This is a joint work with Reinhard Farwig (TU Darmstadt) and Tomoyuki Nakatsuka (Nagoya University).

On local strong solutions of the non-homogeneous Navier-Stokes equations

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Joint work with R. Farwig (TU Darmstadt) and H. Sohr (U Paderborn)

Consider a bounded domain $\Omega \subseteq \mathbb{R}^3$ with smooth boundary $\partial\Omega$, a time interval $[0, T)$, $0 < T \leq \infty$, and in $[0, T) \times \Omega$ the non-homogeneous Navier-Stokes system

$$u_t - \Delta u + u \cdot \nabla u + \nabla p = f,\ u|_{t=0} = u_0,\ \operatorname{div} u = k,\ u|_{\partial\Omega} = g,$$

with sufficiently smooth data f, u_0, k, g. In this general case there are mainly known two classes of weak solutions, the class of global weak solutions, similar as in the well known case $k = 0$, $g = 0$, which need not be unique, and the class of local very weak solutions, see [1], [2], [3], which are uniquely determined, but need neither have differentiability properties nor satisfy the energy inequality. Our aim is to introduce a new class of local strong solutions for the general case $k \neq 0$, $g \neq 0$, satisfying similar regularity and uniqueness properties as in the known case $k = 0$, $g = 0$. For slightly restricted data this class coincides with the corresponding class of very weak solutions yielding new regularity results. Further, through the given data we obtain a control on the interval of existence of the strong solution (compare [4], [5]).

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A Serrin-type condition for the gradient on one component of the velocity field of a Leray solution

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In the theory of the Navier-Stokes equations one of the most challenging open problem is the question of the existence and uniqueness of a global smooth solution to the Navier-Stokes equations for any sufficiently regular data. In the past many results have been published concerning sufficient conditions for the regularity of a solution, starting from the well-known Serrin-Prodi condition $\boldsymbol{u} \in L^s(0,T;\boldsymbol{L}^q(\mathbb{R}^3))$ such that

$$\frac{2}{s} + \frac{3}{q} \leq 1, \quad 2 \leq s < +\infty,$$

which has been extended to corresponding conditions on the velocity gradient, the pressure and the vorticity as well. Meanwhile, alternative conditions on one component of $\boldsymbol{u}$, on several components of $\nabla \boldsymbol{u}$ or on components of the vorticity $\boldsymbol{\omega}$ have been introduced by various authors. However, it is still not known, whether Serrin's condition imposed on one component of $\boldsymbol{u}$ guarantees the regularity of a Leray solution. In our present talk we present a new result in this direction, which says that if a Leray solution $\boldsymbol{u} : \mathbb{R}^3 \times]0,T[\to \mathbb{R}^3$ satisfies the Serrin type condition

$$\nabla u^3 \in L^4(0,T;\boldsymbol{L}^2(\mathbb{R}^3))$$

then $\boldsymbol{u}$ is a regular solution.

From the compressible Navier-Stokes-Fourier system with a low Mach number to the Oberbeck-Boussinesq approximation in a whole space

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We will present the asymptotic analysis of solutions to the compressible Navier-Stokes-Fourier system, when the Mach number is small proportional to ϵ, a Froude number is proportional to $\sqrt{\epsilon}$ and $\epsilon \to 0$ and the domain containing the fluid varies with changing parameter ϵ. In particular, the fluid is driven by a gravitation generated by object(s) placed in the fluid of diameter converging to zero. As $\epsilon \to 0$, we will show that the fluid velocity converges to a solenoidal vector field satisfying the Oberbeck-Boussinesq approximation on R^3 space with a concentric gravitation force. Our approach is based on weak solutions. In order to pass to the limit in a convective term we apply the spectral analysis of the associated wave propagator (Neumann Laplacian) governing the motion of acoustic waves.

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Steady subsonic Euler flows with large vorticity past a body or through a channel

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Mathematical study for the flows, in particular, the steady flows, in the exterior of a body or in the interior of a channel has significant physical applications. In this talk, we will discuss the existence, uniqueness, and far field behavior on subsonic Euler flows in these two important classes of physical domains. The focus is on the flows with large vorticity.

The Stability of Stationary Solutions to the Two-Dimensional Navier-Stokes Exterior Problem

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This talk is concerned with the stability of stationary solutions of the two-dimensional Navier-Stokes exterior problem. The stationary solutions are assumed to be small and enjoy certain pointwise decay conditions. If the decay condition is critical, the domains and solutions are assumed to satisfy some symmetry condition as well. Under an initial perturbation in the solenoidal L^2-space, with the same symmetry if the decay order of the stationary solution is critical, the solution of the nonstationary equation tends to the stationary solution in the solenoidal L^2-class. Also given are the decay orders of the perturbation in other function spaces.

The global existence, together with asymptotic stability of the stationary solutions, is proved by the energy estimates. The precise decay orders are proved by L^q-L^r estimates of the semigroup generated by the perturbed Stokes operator. In order to obtain the L^q-L^r estimates, we make use of the square root of the original resolvent.

Hausdorff measure of the singular set in the incompressible magnetohydrodynamic equations

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We derive a local energy inequality for weak solutions of the three dimensional magnetohydrodynamic equations. Combining Biot-Savart law, interpolation inequalities and the local energy inequality, we prove a partial regularity theorem for suitable weak solutions. Furthermore, we obtain an improved estimate for the logarithmic Hausdorff dimension of the singular set of suitable weak solutions.

On a generalization of the Darcy-Forchheimer equation

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We study mathematical properties of steady flows described by the system of equations generalizing the classical porous media models of Darcy's and Forchheimer's. The considered generalizations are outlined by implicit relations between the drag force and the velocity, that are in addition parametrized by the pressure. We analyze such drag force–velocity relations which are described through a maximal monotone graph varying continuously with the pressure. Large-data existence of a solution to this system is established, whereupon we show that under certain assumptions on data, the pressure satisfies a maximum or minimum principle, even if the drag coefficient depends on the pressure exponentially.

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Collective Motion and Bifluid Models

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We will present existence result for model of motion of two-phase fluid [3] with a free boundary separating the compressible and the incompressible phase. It is obtained as a singular limit of the compressible Navier-Stokes system with a special form of pressure, which blows up for some prescribed value of ρ [2]. This model has been also recently used to model the collective motion [1]. We discuss its properties in $1D$ case.

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Time-dependent singularities in the Navier-Stokes system

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The Cauchy problem for the incompressible Navier–Stokes system in the whole three dimensional space reads

$$\begin{aligned} \partial_t u + (u \cdot \nabla)u - \Delta u + \nabla p &= 0, \quad (x,t) \in \mathbf{R}^3 \times (0,\infty), \\ \operatorname{div} u &= 0. \end{aligned}$$

We have proved that, for a given Hölder continuous curve $\{(\gamma(t),t) \in \mathbf{R}^3 \times (0,\infty) \,:\, t > 0\}$, there exists a solution to this system which is smooth outside this curve and singular on it. This is a pointwise solution of the system outside the curve, however, as a distributional solution on $\mathbf{R}^3 \times (0,\infty)$, it solves an analogous Navier-Stokes system with a singular force concentrated on the curve.
This is a joint work with Grzegorz Karch.

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