Real root classification via Hermite quadratic forms: recent advances and applications

Weijia Wang
joint work with Mohab Safey El Din
Sorbonne Université, CNRS, LIP6

Counting and classifying real solutions of polynomial systems is a fundamental problem in real algebraic geometry, with increasing impact in optimization and control. In the parametric setting, the goal is to partition parameter space according to the number of real solutions that satisfy prescribed semi-algebraic constraints. A powerful approach is based on Hermite quadratic forms: to a suitable polynomial g, one associates a symmetric bilinear form whose signature encodes the number of real solutions counted with the sign of g. By combining signatures for a judiciously chosen family of such polynomials, one can recover a full classification of the real solutions.

This talk surveys recent algorithmic and implementation advances for this framework. I will discuss improvements that reduce the number of Hermite quadratic forms that must be computed, leading to substantial complexity gains under mild genericity assumptions. I will also discuss efficient algorithms for constructing the relevant Hermite quadratic forms and for computing their signatures. Finally, I will explain how this real root classification machinery can be used as a computational primitive in the analysis of linear matrix inequalities (LMIs) arising in control, for instance by certifying feasibility regions and stratifying parameter-dependent matrix inequalities.