Actualité de la philosophie de Cavaillès pour une réflexion sur les mathématiques contemporaines ?

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Histoire des mathématiques, philosophie des mathématiques et mathématiques: quelles interactions?

Luminy, 26 novembre 2025

Relevance of Cavaillès' philosophy when thinking on contemporary mathematics?

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History of Mathematics, Philosophy of Mathematics, and Mathematics: Which Interactions?

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I. Introduction

Prejudices and naivety of a mathematician

- Cavaillès is a philosopher-mathematician.
- *History* of mathematics integrated to *philosophy* of mathematics.
  Interplay between the developments of mathematics *per se*, and of foundational studies and mathematical logic.
- Style !
  Comparison with E. Artin.

What is the relevance of Cavaillès philosophy of mathematics, for mathematicians today, outside the circle of problems that were the starting points of Cavaillès' investigations, namely the beginning of set theory and related foundational issues ?

A few words of warning

- Exploratory investigation !

  Want to indicate some directions of inquiry, and formulate a few specific questions...

- JBB takes full responsibility for insisting on presenting “well-known facts,” and possibly making parts of this talk boring for some parts of the audience.

- Introduction
- An introduction to Jean Cavaillès...
  - Cavaillès' life and personality
  - On Cavaillès' philosophy of mathematics
- Addressing the "relevance" question
  - Modern model theory.
  - Bias towards/against number theory ?
  - Cavaillès' philosophy and "novel mathematics" of the second half of the XX-th century.

II. An Introduction to Jean Cavaillès

Jean Cavaillès: a chronology I

- Born in May 1903, at Saint-Maixent. Calvinist and republican family. Father and grand-father, and several earlier ancestors, officers in the French army.
- 1923: École Normale Supérieure (Philosophie)
- 1927: Agrégation de Philosophie
- 1928 – 1936: Agrégé-répétiteur (*caïman*) à l'École Normale Supérieure
- 1928 – 1931: travels in Germany (Rockefeller Foundation scholarship), where he discovers:
  - set theory, which would be the subject of his secondary thesis;
  - the flowering of axiomatic, which would be a major theme of his main thesis;
  - the correspondence between Cantor and Dedekind; which he would edit in collaboration with Emmy Nœther;
  - the youth and social movements and the rise of fascism.
- 1928 – 1937: work on his doctoral theses under the "supervision" of Léon Brunschwicg, defended in January 1938: *Méthode axiomatique et formalisme* et *Remarques sur la formation de la théorie abstraite des ensembles.*

Jean Cavaillès: a chronology II

- 1936 – 1938 : teaches philosophy at the Lycée d'Amiens.
- September 1939 – August 1940: lieutenant in the French army. Prisoner of war, but escapes during transfer to Germany.
- September 1940 – August 1943: plays a key role in the French *Résistance*, while *professeur de logique* at the Sorbonne.
- 1942 : creates the *Cohors* espionage and sabotage network, while continuing to teach at the Sorbonne.

Lucy Aubrac appreciation of Jean Cavaillès

Intervention au colloque d'Amiens
(Centre National de Documentation Pédagogique, 1984)

Il était capable en même temps de concevoir un cours, de rédiger un article, de trouver un moyen de se procurer des renseignements, d'organiser un sabotage et de mettre au point une méthode de codage.

Ses capacités organisationnelles et tactiques, son sens des responsabilités et d'entreprise, son idée de la grandeur de la France lui valent parmi ses amis le surnom de Sully.

N.B.: Maximilien de Béthune, duc de Sully, was a minister of the king of France Henri IV, of protestant faith, widely appraised for his role in France's "reconstruction" after the wars of religion of the XVI-th century.

Jean Cavaillès: a chronology III

- 1936 – 1938 : teaches philosophy at the Lycée d'Amiens.
- September 1939 – August 1940: lieutenant in the French army. Prisoner of war, but escapes during transfer to Germany.
- September 1940 – August 1943: plays a key role in the French *Résistance*, while *professeur de logique* at the Sorbonne.
- 1942 : creates the *Cohors* espionage and sabotage network, while continuing to teach at the Sorbonne.
- September 1942: prisoner at Saint-Paul d'Eyjaux; begins to write *Sur la logique et la théorie de la science*; delivers a lecture to other prisoners on *Descartes et le Discours de la Méthode*.
- December 1942: escapes from Saint-Paul d'Eyjaux.
- August 1943: arrested by the German counter-espionage.
- February/April(?) 1944: executed by a firing squad in Arras.

Henri Cartan sur Jean Cavaillès, en 1981

Préface à la seconde édition de *Méthode axiomatique et formalisme*

[...] les réflexions du jeune Cavaillès de 1937 n'ont nullement perdu de leur intérêt aujourd'hui : elles nous aident à faire le point sur l'histoire de l'évolution des idées à une époque qui fut fertile en controverses. Cavaillès était peut-être le seul qui fût alors capable de dresser un tableau d'ensemble de cette évolution puisqu'il alliait à sa culture philosophique une solide formation mathématique ; il avait aussi pris la peine d'étudier les travaux des logiciens et d'en assimiler la substance.

[...] La "Conclusion" du philosophe-mathématicien Cavaillès se déroule en une vingtaine de pages que chacun, qu'il soit mathématicien ou philosophe prendra le plus grand intérêt à lire, quel que soit par ailleurs le jugement personnel vers lequel il incline.

Concerning Cavaillès' philosophy

Some key points of Cavaillès' three books:

- *Remarques sur la formation de la théorie abstraite des ensembles*: the originality of set theory does not reside so much in its objects as in its methods (notably Dedekind chain, transfinite iteration, diagonal procedure).
- *Méthode axiomatique et formalisme* (= *MAF*): Can the axiomatic method provide a foundation for mathematics ?
  If yes, foundation is not given a priori; it does not come before and from the outside of mathematics.
- *Sur la logique et la théorie de la science*: Cavaillès questions the capacity of a philosophy of consciousness to account for mathematical work.
  For Cavaillès, as for Bolzano and also (independently) for Hegel, probably influenced by Tarski's formal semantic, objectivity does not lie in the expression of thought, but in the meaning of thought. And the meaning/content is the concept.

Sur la logique et la théorie de la science and the philosophy of concepts

- Proposing a philosophy of concepts was a way to question the notion of object as a uniquely and definitely determined thing.
- Cavaillès aimed to introduce a type of structural philosophy on the model of structural mathematics. At the same time, he adopted Hegel's replacing the Aristotelian philosophy of "being qua being" with a philosophy of becoming.
- Conceptual becoming is a source of the unforeseen, but not a sign of contingency.
  On the contrary, it develops according to an *internal necessity*, which does not originate in consciousness, but is the product of a "conceptual dialectic".
- Cavaillès merged his understanding of Hegel's *Wissenschaft der Logik*, which he read in Saint-Paul d'Eyjaux, with his knowledge of Spinoza's system and with his reading of Dedekind's habilitation lecture, in which he could find again the expression of 'internal necessity'.

  Hegelian philosophy permeated large domains of German culture and the Hegelian lexicon penetrated even mathematics — for instance Dedekind's habilitation lecture (1854) and *Was sind und was sollen die Zahlen?*.

  Dedekind attended the lectures by Hermann Lotze on *Geschichte der deutschen Philosophie seit Kant* in Göttingen in 1852; his notes have been recently transcribed by Rachel Rudolph and Dirk Schlimm.

Jean-Toussaint Desanti dans *Souvenir de Jean Cavaillès*

[...] Il ne nous tenait pas de discours "à propos des mathématiques", ni "à propos de la logique". Il nous faisait entrer dedans avec lui. [...] Il n'exposait pas une "philosophie des mathématiques" qui eût livré sur l'objet une vue extérieure. Il s'efforçait de montrer l'objet lui-même, selon les exigences nécessaires de son mouvement de constitution. C'était difficile. Énigmatique parfois. Bienheureuse énigme qui nous sollicitait et nous arrachait à nos habituelles rhétoriques d'apprentis philosophes.

Méthode axiomatique et formalisme, 2ème éd., Introduction

The conclusion of *Sur la logique et la théorie de la science*

The last sentences *Sur la logique et la théorie de la science* are:

> *Ce n'est pas une philosophie de la conscience mais une philosophie du concept qui peut donner une doctrine de la science.*

followed by the statement:

> *La nécessité génératrice n'est pas celle d'une activité, mais d'une dialectique.*

III. Addressing the "relevance" question

III.1 Modern model theory

A striking illustration of Cavaillès' perceptiveness as a mathematician and of the relevance of some key ideas of his philosophy of mathematics.

III.2 Bias towards/against number theory ?

A small puzzle, at the border of mathematics, history, and philosophy.

III.3 Cavaillès' philosophy and "novel mathematics" of the second half of the XX-th century.

A "stress test," focusing on some specific features of contemporary mathematics.

III.1 Model theory: a very short sketch of "recent" developments

"Classical" model theory: direct relations with algebra in real/valued fields.

"Modern" model theory:

- *In vitro* period $\gtrsim 1960$

  categoricity = classification of all models of a given (possibly large) cardinal of a given theory.

  - M. Morley, *Categoricity in power*, TAMS, 1965
  - S. Shelah $\sim 1970 - 1980$
  - E. Hrushovski, B. Zilber $\gtrsim 1985$, Zariski geometries

  $\longrightarrow$ stable and simple theories

- *In vivo* period: E. Hrushovski $\gtrsim 1960$

  deep impact on "classical" problems:

  III.1 Diophantine geometry over function fields of positive characteristic

  - Algebraic differential equations
  - Algebraic dynamics
  - Combinatorics

III.2 Bias towards/against number theory ?

Naive observation: most of the mathematicians whose works, concerning *foundational questions*, have been closely investigated by Cavaillès also made major contributions to *number theory*.

main examples: Dedekind, Kronecker, Hilbert, Herbrand

(weak) counterexample: Cantor

counter-counterexample: Weierstrass

Mathematics and botany: Arnold's mushrooms I

When you are collecting mushrooms, you only see the mushroom itself. But if you are a mycologist, you know that the real mushroom is in the earth. There's an enormous thing down there, and you just see the fruit, the body that you eat. In mathematics, the upper part of the mushroom corresponds to theorems that you see. But you don't see the things which are below, namely problems, conjectures, mistakes, ideas, and so on.

You might have several apparently unrelated mushrooms and are unable to see what their connection is unless you know what is behind.

V. I. Arnold, *From Hilbert's Superposition Problem to Dynamical Systems*, 1997.

Mathematics and botany: Arnold's mushrooms II

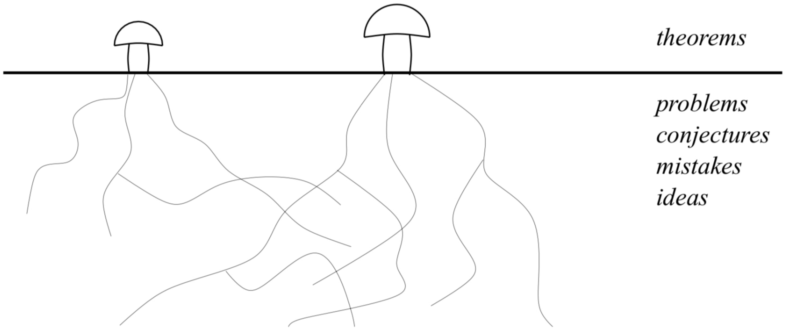

Fig. 1. The mathematical mushroom

From V. I. Arnold, *From Hilbert's Superposition Problem to Dynamical Systems*, 1997.

Mathematics and botany: the number theorists' point of view I

Je crois donc que l'Analyse la plus abstraite est en grande partie une science d'observation, j'assimile absolument le complexe des notions connues et à connaître dans ce domaine de l'analyse, à celles des sciences naturelles, les notions de l'analyse ayant leur individualité propre, leur figure si je puis dire, et leurs corrélations multipliées, au même degré que les animaux et les plantes.

Ch. Hermite, letter to L. Königsberger, March 2, 1876.

Sie sollten sich ein Gärtchen anlegen, ein mathematisches Gärtchen, in dem Sie spazieren gehen können.

E. Hecke, letter to his PhD student W. Maak, $\sim$ 1935.

Mathematics and botany: the number theorists' point of view II

Some readers will find that I have given too free rein to a lamentable tendency to argue from the general to the particular, and have obfuscated them by interjecting unfamiliar concepts of representation theory into what could be a purely geometric discussion. My intention is not that, but rather to equip myself, and perhaps them as well, for a serious study of the Shimura varieties in higher dimensions. We are in a forest whose trees will not fall with a few timid hatchet blows. We have to take up the double-bitted axe and the cross-cut saw, and hope that our muscles are equal to them.

R. P. Langlands, On the zeta-functions of some simple Shimura varieties, 1979.

III.3 “Novel mathematics” I: precise dictionaries and conjectures

Well-known role of analogies in mathematics !!!

A significant instance, already mentioned this week: analogy between number fields and function fields (Hermite, Kronecker, Dedekind-Weber, Hensel, Hasse, Artin, Weil,...)

Since the middle of the XX-the century, transformation of analogies: modern versions of those as extremely precise and predictive dictionaries.

Two examples from Deligne’s work:

- *le yoga des poids*:
  Hodge theory $\longleftrightarrow$ Étale cohomology of algebraic varieties over finite fields and Frobenius action;
- $\mathbb{C}$-analytic connections and irregular singularities $\longleftrightarrow$ étale sheaves and wild ramification.

Closely related to diverse “modern” conjectures, such as the ones constituting the Langlands program.

???

"Novel mathematics" II: A feeling of discomfort...

Arguably one of most novel area of mathematics, when compared to pre-war mathematics are category theory, homological algebra, homotopical algebra,...

An extremely intriguing fact from the perspective of mathematical practice: at various important steps of the developments of these domains, some of its main contributors expressed some very strong reservations about their scope and significance:

- Well-known story, told by MacLane, of himself rebuking Kan, who was trying to get him interested in the adjointness properties of $\otimes$ and Hom.
- Extremely bad documentation of the theory of derived categories (!!!!).
- Quillen's comments to Ken Brown on his own work on *Homotopical Algebra* (LNM 43), when Brown did use it to solve the PhD problem that Quillen had asked him.
- ...

“Novel mathematics” II: A feeling of discomfort...

What does this mean ?

1. Nothing ?
2. Serious weakness in foundations ?
3. Reluctance to develop theories that would be both “classical mathematics” (say, to solve problems, *à la* Hilbert), and “contributions to foundations” ?

Would remarkably confirm some key points of Cavaillès’ (ultimate) philosophy, focusing on concepts, on the *enchaînement des contenus*, and its views concerning mathematics, integrating foundational issues, as a *devenir*.

Suggestions for further reading

A mathematician (:=JBB)'s point of view
more geometrico, à la Thomason...

If you are an:

- Honest scholar, with a special interest in the role of history of mathematics both for mathematics and its philosophy.
  Get a copy of *Jean Cavaillès, Œuvres complètes de Philosophie des Sciences*, Paris, Hermann, 1994, and first read:
  Méthode axiomatique et formalisme (1938).
  Then read:
  Hourya Benis-Sinaceur, Jean Cavaillès — Philosophie Mathématique, 2nd edition, Paris, Vrin, 2019.

- Reckless cheat, willing to *briller en société* with a modicum of actual effort, just read Hourya's book. Then (you shall) repent, become honest, and read Cavaillès' and Hourya's other books and papers.

- Philosophical thrill seeker, read directly:
  Sur la logique et la théorie de la science (1943).

The previous slides are a slightly edited version of the ones used as a support of the joint oral presentation by HB-S and J-BB in Luminy on November 26-th, 2025.

The first half of this presentation – An Introduction to Jean Cavaillès – *was delivered by HB-S, and a more detailed transcript of this part appears in the next pages. The introduction and the second half of the presentation* – Addressing the "relevance" question – *were delivered by J-BB.*

These slides were prepared jointly by HB-S and J-BB, with the exception of the last slide – Suggestions for further readings – *that had been prepared at the last minute by J-BB, who takes full responsibility for its content and for plagiarizing (in blue) a famous paper by Thomason in its wording.*

HB-S' notes for *An Introduction to Jean Cavaillès...*

Jean Cavaillès was born in 1903. His father taught at the Military Academy. His family, Calvinist and deeply religious, was committed to the values of rigor, integrity, republican patriotism, and honour. They also valued justice: they were Dreyfusards.

Music held a significant place in Cavaillès' youth and life; it was a profound source of inspiration for his thought.

In 1923, Cavaillès was admitted to the ENS after a year of solitary preparation for the entrance exam. He studied Greek and mathematics while also attending philosophy lectures by E. Bréhier and L. Brunschvicg. He was already a prestigious person, admired "from afar" (R. Aron) by his fellow students and surrounded by mystery.

In 1927 he obtained the agrégation in philosophy and began a series of stays in Germany financed by Rockefeller scholarships. There he discovered:

- set theory which would be the subject of his secondary thesis,
- the flowering of axiomatic which would be a major theme of his main thesis,
- the correspondence between Cantor and Dedekind which he would edit in collaboration with Emmy Nœther,
- the youth and social movements and the rise of fascism. He listened to Hitler's speech in Munich in 1931 and read *Mein Kampf.*

A resistance fighter from the very beginning, he co-founded, with E. d'Astier de la Vigerie, the newspaper *Libération*, whose first issue's editorial proposed outlining "the tasks incumbent upon the French who have not given up." In 1942, he created the *Cohors* espionage and sabotage network while continuing to teach logic and philosophy at the Sorbonne.

Lucie Aubrac, great resistance fighter and close friend of Cavaillès, wrote:

> He was capable of simultaneously designing a course, writing an article, finding a way to obtain information, organizing sabotage, and developing a coding method.

She also said:

> His organizational and tactical skills, his sense of responsibility and enterprise, his idea of the greatness of France earned him the nickname Sully.[1]

Arrested by the French police while attempting to embark for London, he was interned at the French camp of Saint-Paul d'Eyjeaux in September 1942. There he began to write down his treatise on logic, which would be published posthumously in 1947 under the title *Sur la logique et la théorie de la science*, and he gave the prisoners a lecture on *Descartes et le Discours de la Méthode*. Arrested by German counter-espionage on August 28, 1943, Cavaillès continued to work in solitary confinement at Fresnes jail, where his family brought him books lent by Gaston Bachelard. Interrogated by the Germans, he acknowledged the acts "that concerned him alone," and explained that he had only conformed to the values of great German thinkers, such as Kant and Beethoven. In February 1944, he was executed by firing squad in Arras and buried anonymously. He was posthumously awarded the title of Companion of the Liberation and Knight of the Legion of Honour.

His work is small in volume, but, in my view, dense and profound. It encompasses themes that run through the history of mathematics and logic in the 19th and early 20th centuries.

I am quoting, for example: the birth of projective geometry, non-Euclidean and non-Archimedean geometries, group theory, the axiomatic method, Lebesgue measure theory, questions of definition and predicativity (by studying the works of Émile Borel), questions of computability (by evoking the results of Herbrand, Church, Kleene and Gödel), axioms of choice and good ordering, hypothesis of the continuum (by describing in detail the arguments of Cantor, Zermelo, Gödel), problems of consistency, completeness and categoricity, of transfinite induction (by summarizing the proof of consistency of Gentzen's elementary arithmetic with whom he discussed at length in Göttingen in the same year as this proof), etc.

We should add the discussions of more strictly epistemological questions posed by Russell's theory of types, Hilbert's metamathematical theory of signs, Brouwer's theory of the dyad, Tarski's semantics, finitism, constructivism, the link between intuition and abstraction, contents and forms, subject and objects, philosophy of consciousness and philosophy of the concept, etc.

Cavaillès possessed a remarkable understanding of what was new and significant in "modern" mathematics of his time. All those who knew him,

[1] the most famous of Henri IV's ministers.

all those who read him, were and are struck by the extraordinary acuity of his analyses. In his preface to the second edition of *Méthode axiomatique et formalisme* (1981), Henri Cartan acknowledged that

> the reflections of the young Cavaillès from 1937 have by no means lost their relevance today: they help us to take stock of the history of the evolution of ideas in an era that was rich in controversies. Cavaillès was perhaps the only one who was then capable of drawing up an overall picture of this evolution since he combined his philosophical culture with a solid mathematical training; he had also taken the trouble to study the works of logicians and to assimilate their substance.

With a sharp mind and visionary intelligence, Cavaillès was able to identify the driving role of structures quickly, to discern the novelty of the semantic point of view introduced by Tarski, and finally to propose a philosophy of the concept well before philosophers announced the death of the subject.

I will summarize Cavaillès' key points of each of his three books.

• *Remarques sur la formation de la théorie abstraite des ensembles*: according to Cavaillès, the originality of set theory does not reside so much in its objects as in its methods, which were new and characteristic: Dedekind chain, transfinite iteration, diagonal procedure. Cavaillès asks whether set theory was a "necessary occurrence" or a "contingent historical creation".

• *MAF* : The question is whether, as Hilbert argued in 1904 in his address to the Third International Congress of Mathematicians in Heidelberg, axiomatic can provide a foundation for mathematics.

If to ground means to provide an absolute beginning or an ultimate justification, the answer is no. But if to ground means to isolate principles/axioms and highlight the links of logical dependence or independence between principles and theorems, the answer may be yes. Hence the importance given by Cavaillès to the theory of proof inaugurated by Hilbert and to Gentzen's consistency proof.

Anyway, foundation is not a priori, as traditional philosophy pretended; it does not come before and from the outside of mathematics.

• In *Sur la logique et la théorie de la science*, Cavaillès questions the capacity of a philosophy of consciousness to account for mathematical work. He did not adopt Hilbert's idea of objectivity conferred by "language and writing." For him, as for Bolzano and also (independently) for Hegel, and probably influenced by Tarski's formal semantic, objectivity does not lie in the expression of thought, but in the meaning of thought. And the meaning/content is the concept.

Proposing a philosophy of the concept was a way to question the notion of object as a uniquely and definitely determined thing. This is clear in the last page of the booklet. — But remind that Cavaillès'philosophy of the concept was an uncompleted open-ended program.

Cavaillès aimed to introduce a kind of structural philosophy corresponding to structural mathematics. He thus set the tone to several subsequent attempts, including those of G.-G. Granger, J. Vuillemin, G. Deleuze. At the same time, Cavaillès adopted Hegel's replacing the Aristotelian philosophy of "being qua being" with a philosophy of becoming. He thus combined the structural with becoming, in an original way.

In a Hegelian view, concepts are linked and transformed in a specific process, the "conceptual becoming" as Cavaillès calls it. Cavaillès proposed replacing the philosophy of consciousness with a philosophy of the concept, moving from the subjectivity of the mathematician to the objectivity of concepts (groups, fields, orders, etc.) and methods (axiomatization, formalization, etc.).

Conceptual becoming is a source of the unforeseen but not a sign of contingency. On the contrary, it develops according to an internal necessity, which does not originate in consciousness, but is the product of a "conceptual dialectic". The last sentence of *Sur la logique... is as follows*: "it is not a philosophy of consciousness but a philosophy of the concept that can provide a doctrine of science," followed by the specification: "Generative necessity is not that of an activity, but of a dialectic."

The Hegelian reminiscence is evident. But there is still a difference on which I will not comment today: Cavaillès uses an indefinite article (a dialectic), while Hegel proposed the dialectic as movement of both things themselves and knowledge.

In his *Souvenir de Jean Cavaillès* (MAF, 1981, Introduction), Jean-Toussaint Desanti testifies:

> He did not give us a discourse 'about mathematics', nor 'about logic'. He took us into it with him.... He did not expound a 'philosophy of mathematics' that would have given an external view of the object. He strove to show the object itself, according to the necessary requirements of its movement of constitution.

Desanti is thus describing a typically Hegelian turn of thought.

Cavaillès merged his understanding of *Hegel's Wissenschaft der Logik*, which he read in Saint-Paul d'Eyjaux, with his knowledge of Spinoza's system and with his reading of Dedekind's habilitation lecture in which he could find again the expression of "internal necessity".

It should be emphasized that Hegelian philosophy permeated large domains of German culture and the Hegelian lexicon with its specific turns of thought penetrated even mathematics: revealing indications of this can be found in Dedekind's writings, notably in his habilitation lecture (1854) and in *Was sind und was sollen die Zahlen?* Dedekind may well not have read Hegel extensively, but he knew something from his philosophy through the lecture by Hermann Lotze, which he listened to in Göttingen in 1852 and of which he took notes preserved in his Nachlaß and recently transcribed by Rachel Rudolph and Dirk Schlimm.

Nevertheless Cavaillès did not give an explicit answer to the question he posed: is set theory a necessary development, as one might think from his simultaneous invention by Dedekind and Cantor, or is it a free creation?

Before the war, Cavaillès was preparing a book whose title would have been *Mathematical Experience.* "Mathematical experience", term and concept, was coined by Léon Brunschvicg, who thus broadened the meaning of "experience" beyond empirical life and physical or chemical experimentation; the aim was defining a new empiricism. Cavaillès planned to explore this avenue further by defining experience not as the conscious initiative of a subject but as an activity governed by rules. Surprisingly enough, *Sur la Logique...* is not a book about mathematical experience, but an essay on a conceptual dialectic. I am leaving aside today the reasons why Cavaillès abandoned questioning the mathematical experience.

Question:

Is the dynamical process of knowledge a move towards what is, or presumed to be epistemologically basic and ontologically primary ?