# Traversing regions of supersolvable hyperplane arrangements and their lattice quotients

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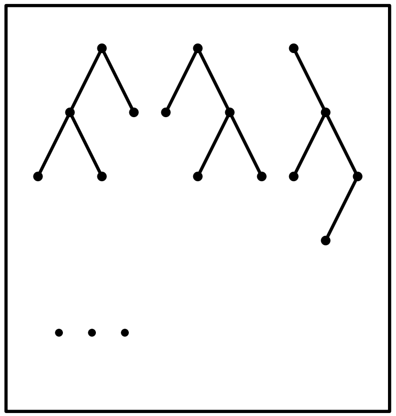
Universität Kassel

joint work with Sofia Brenner (U Kassel), Jean Cardinal (U Libre Bruxelles), Thomas McConville (Kennesaw SU) and Arturo Merino (U O'Higgins) [SODA 2026]

CIRM 2025

# Introduction

- **Combinatorial (exhaustive) generation:**
  list a family of combinatorial objects, each object exactly once



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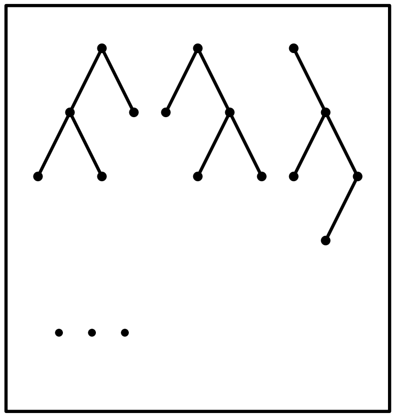

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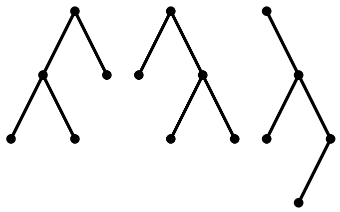

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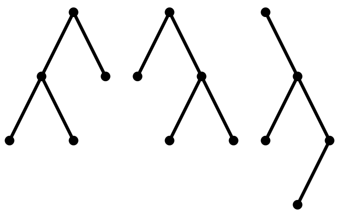

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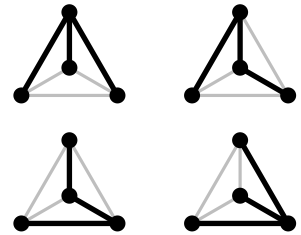

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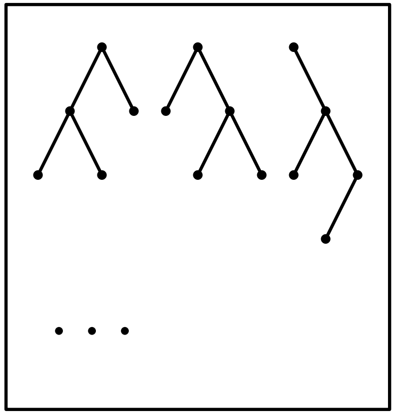

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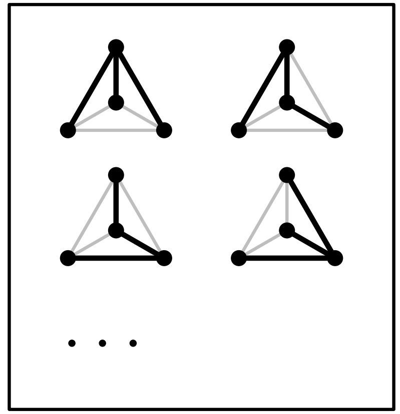
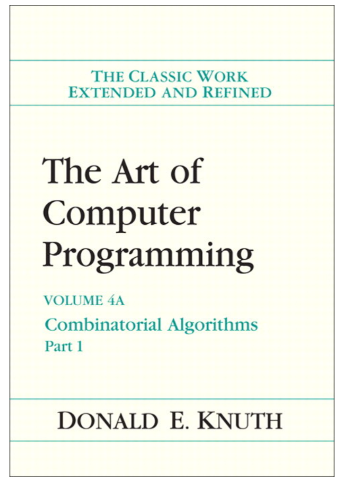


spanning trees

- Covered in depth in Donald Knuth's book 'TAOCP Vol. 4A'



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  - spanning trees by **edge exchanges** **[Smith 97]**

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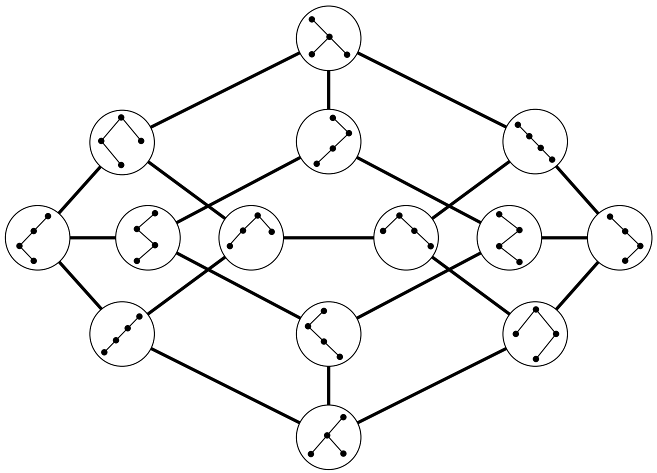

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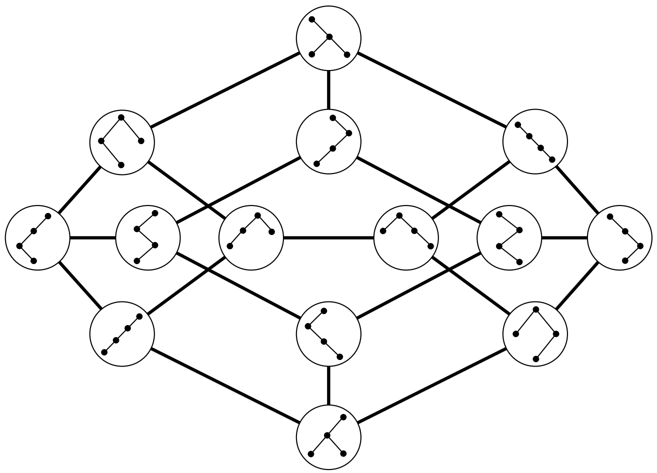

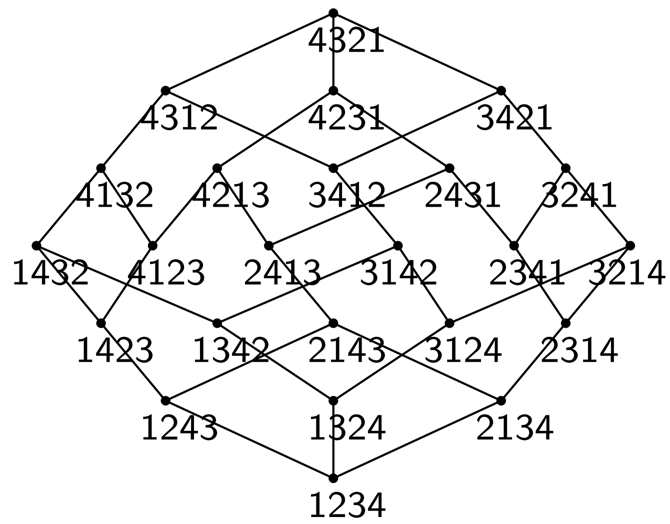

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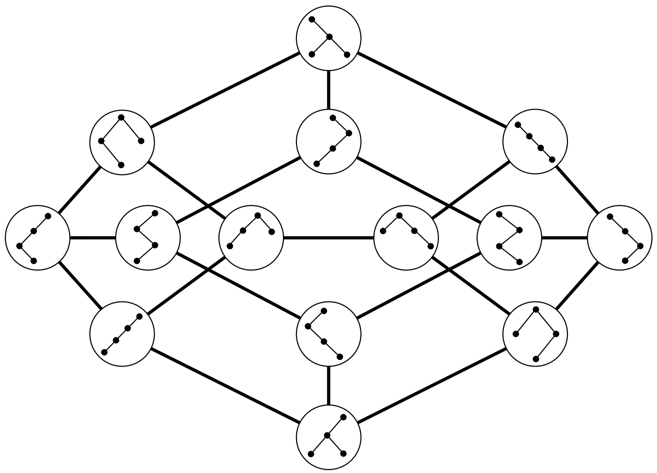

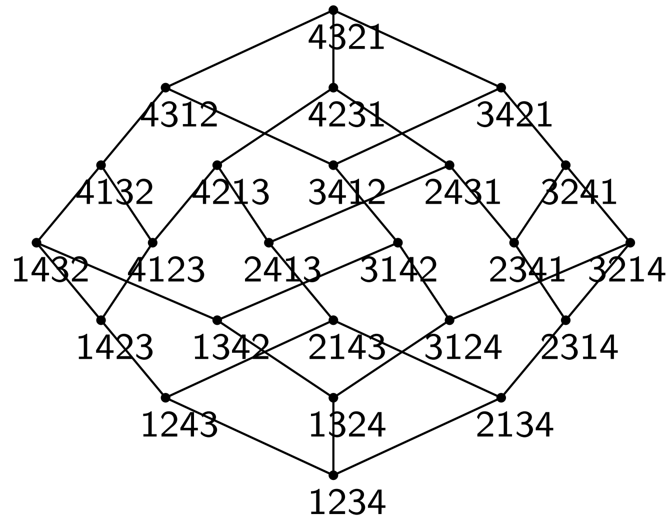


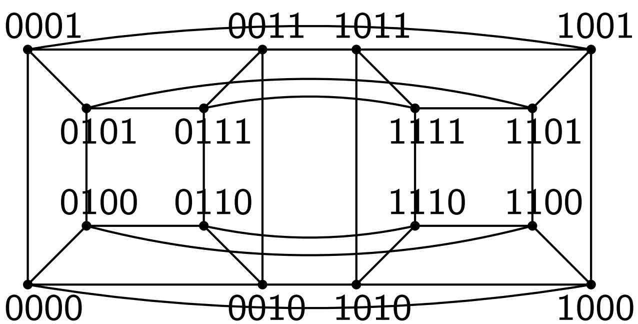

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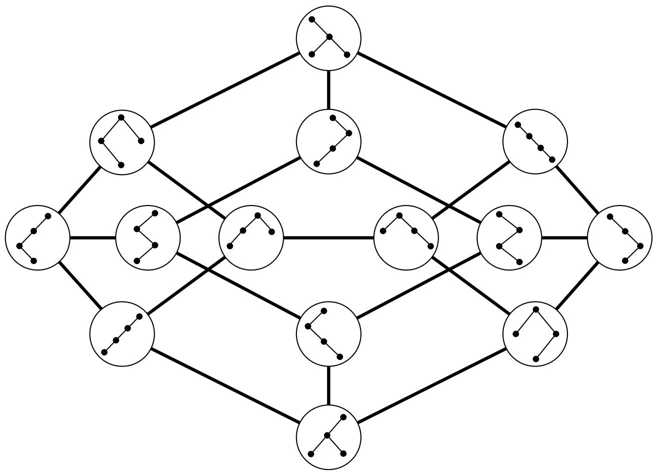

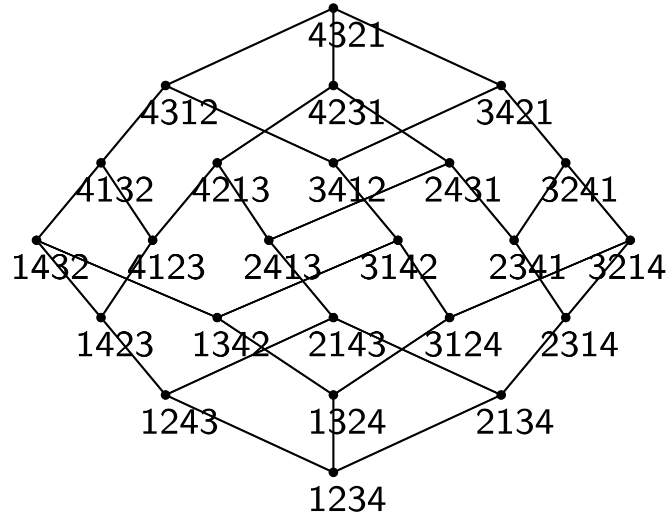


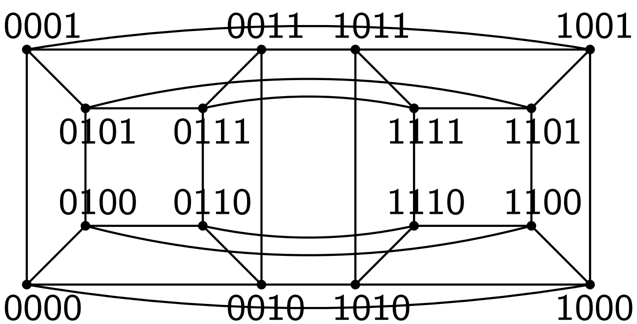


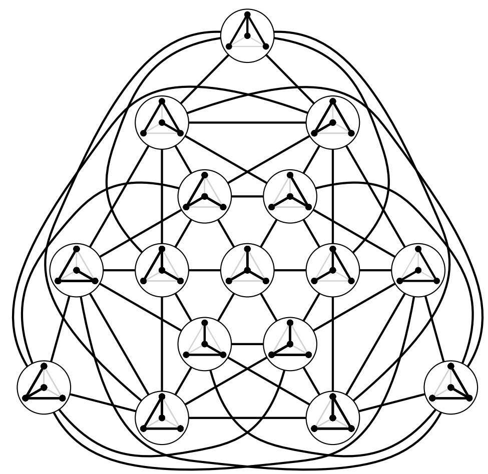

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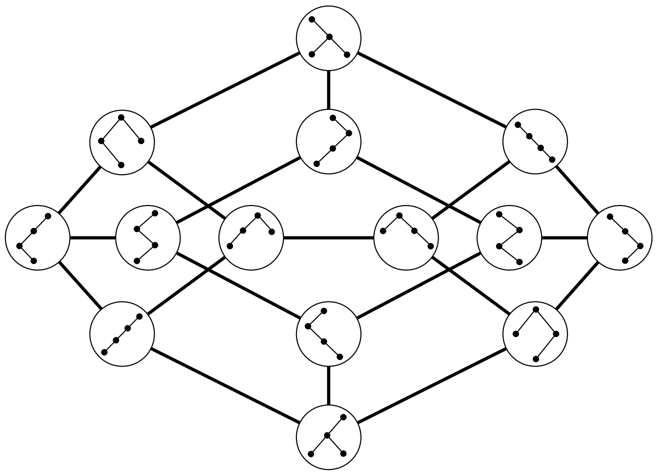

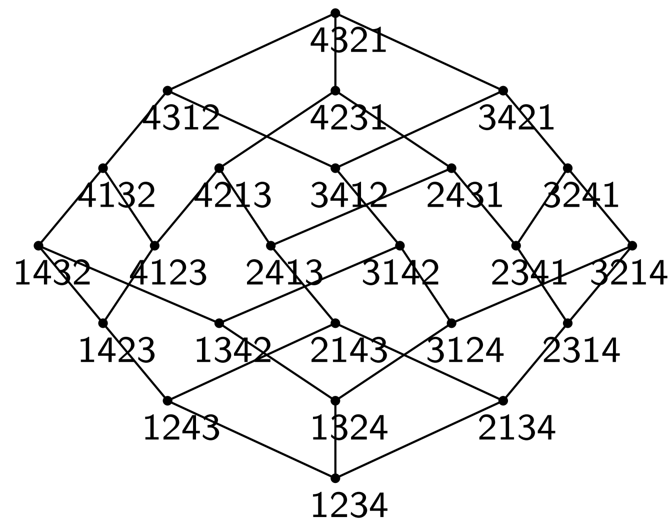


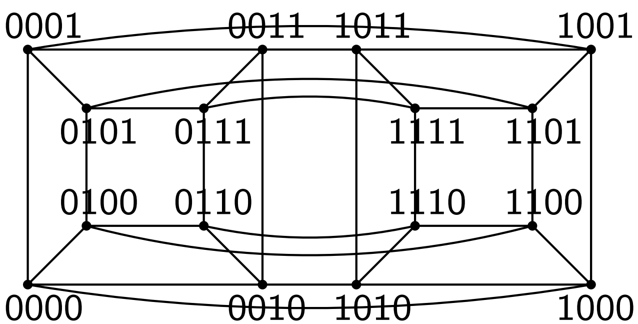


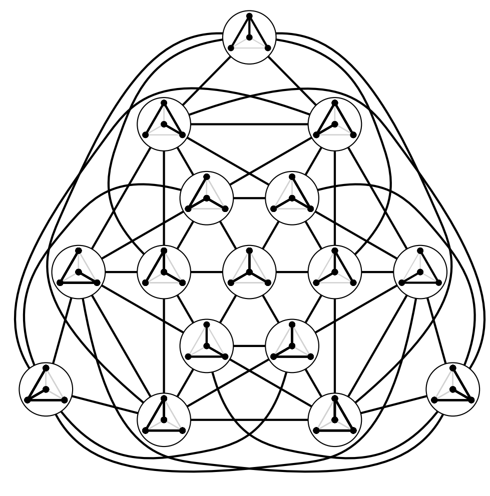

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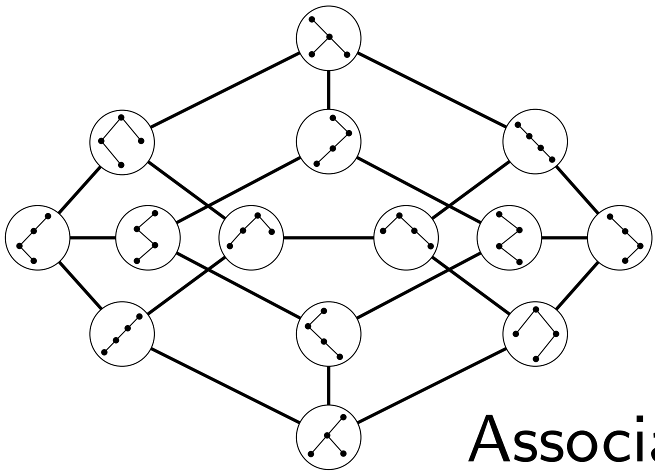
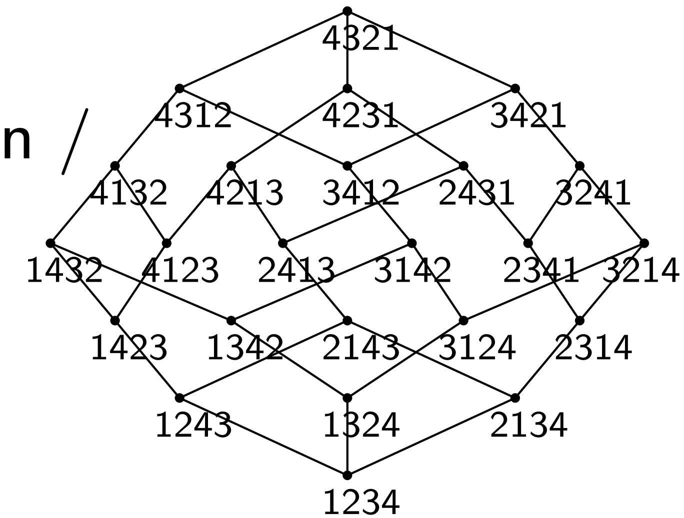
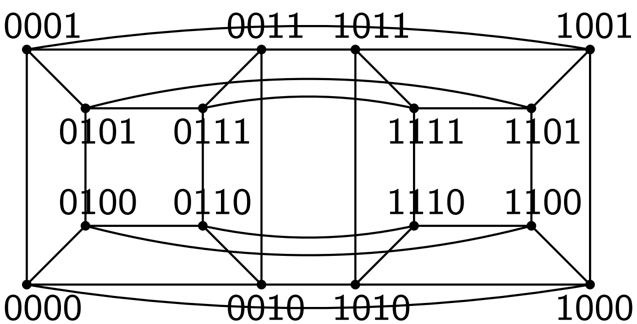


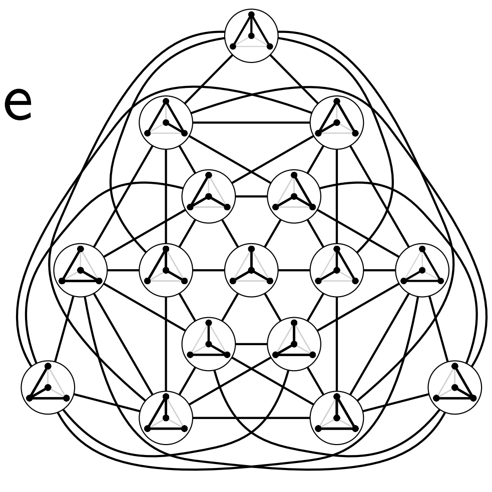
Associahedron / Tamari lattice




Permutahedron / weak order



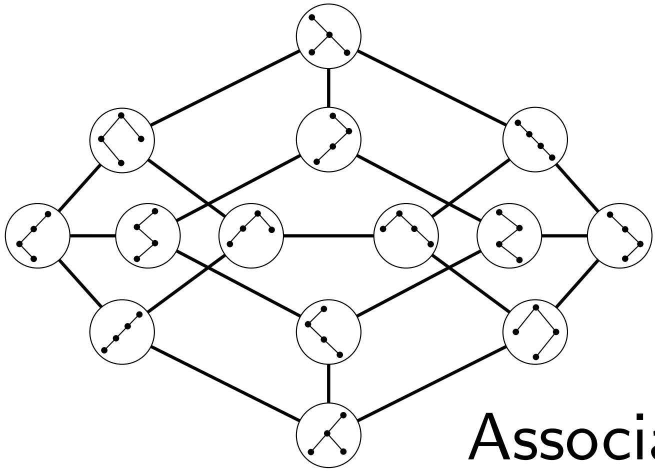

Hypercube / Boolean lattice



Base polytope

# Flip graphs and polytopes

- exhaustive generation → **Hamiltonian path/cycle**



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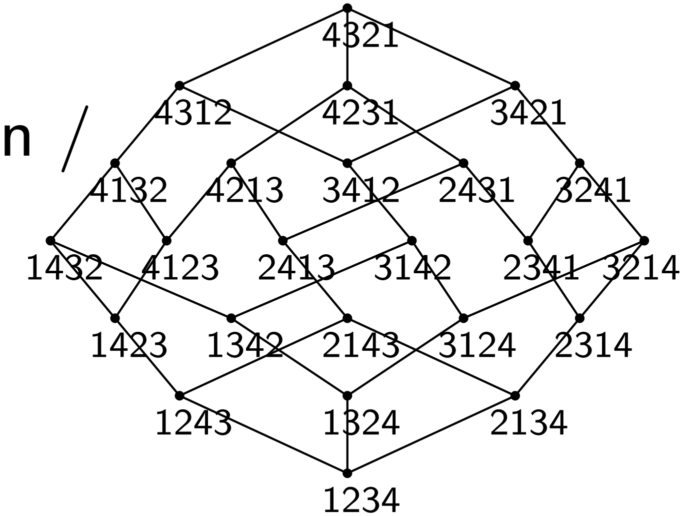


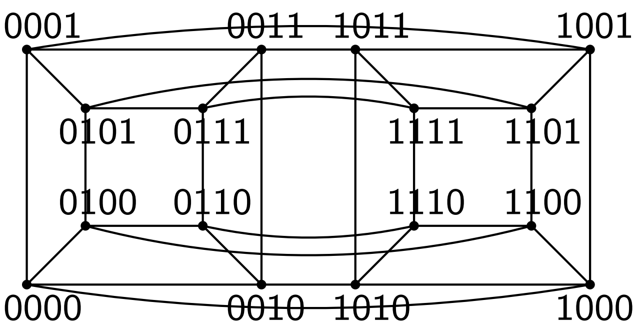


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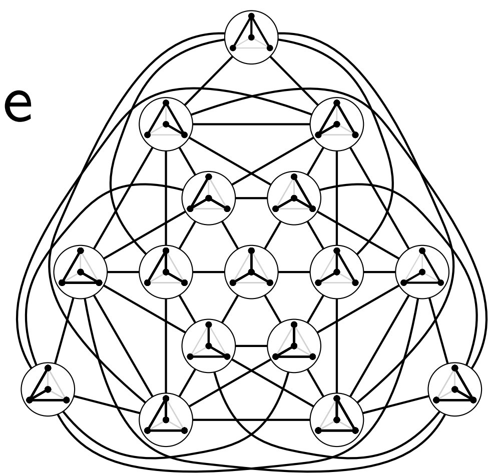

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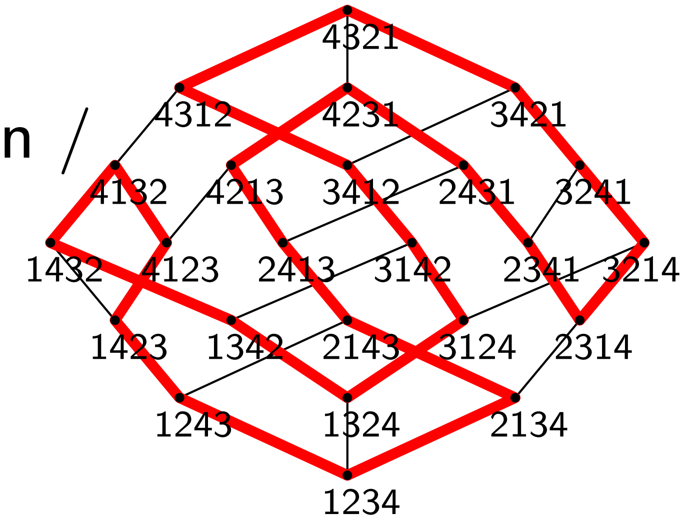


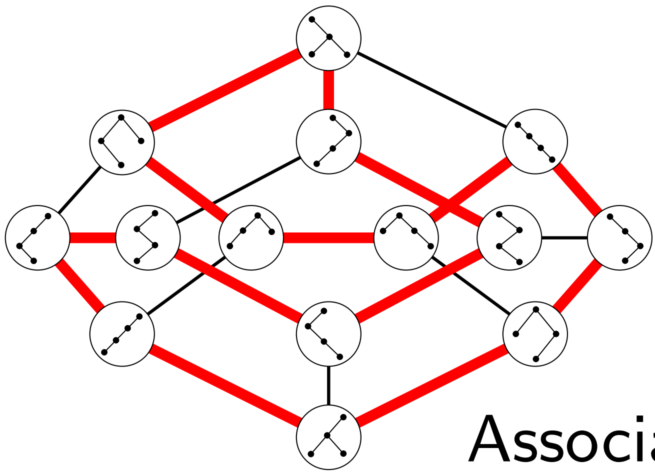

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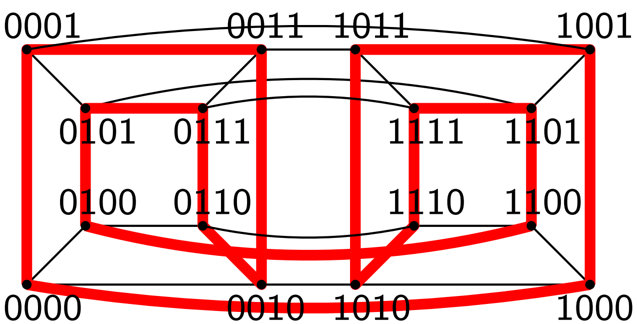


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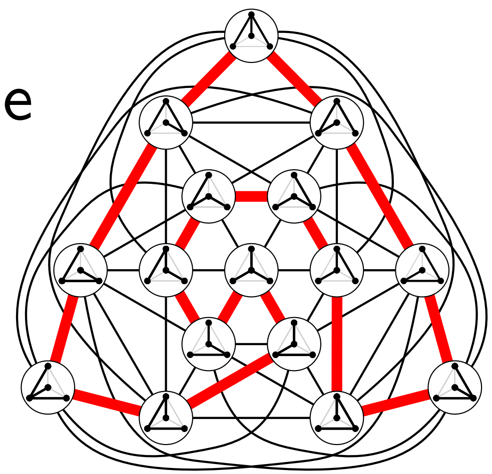

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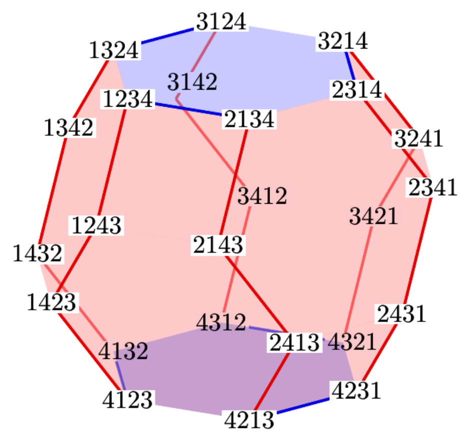

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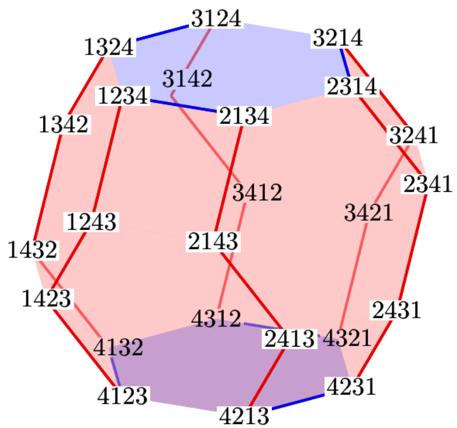


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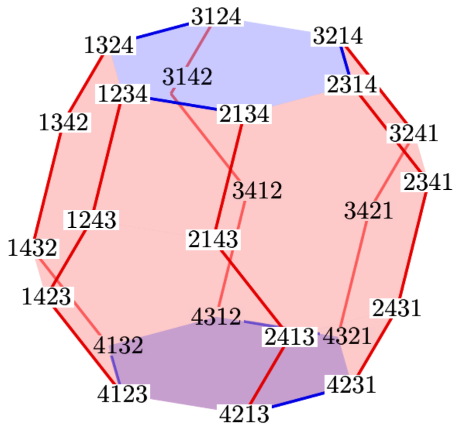


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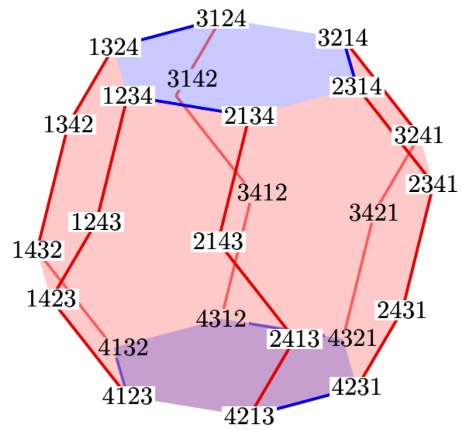


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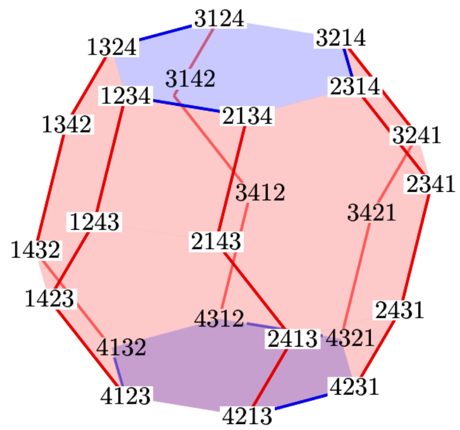


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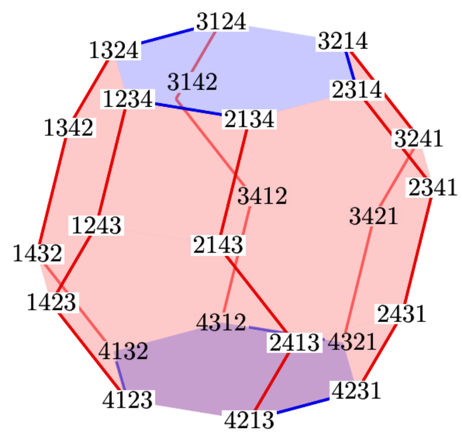


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  - acyclic orientations of chordal (hyper)graphs; quotients of acyclic reorientation lattices [Cardinal, Hoang, Merino, Mička, M. 23]

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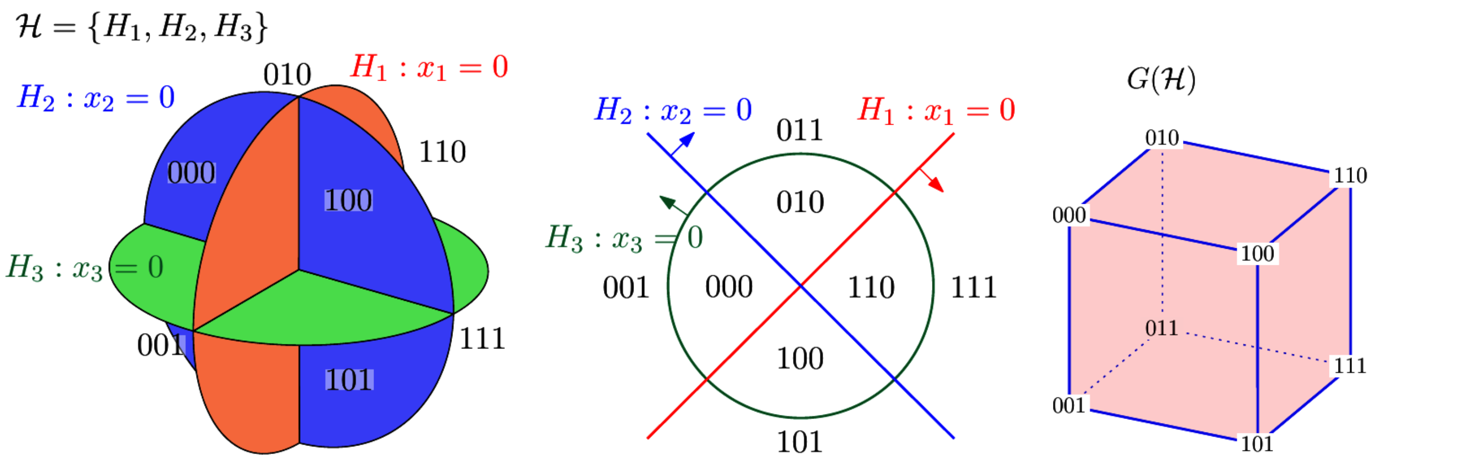

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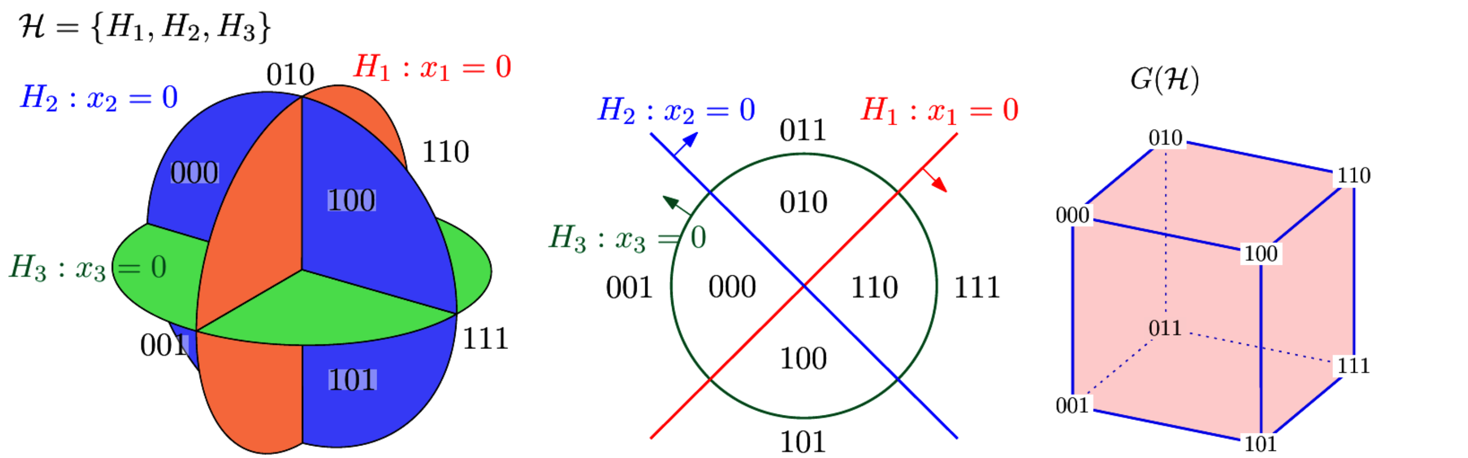
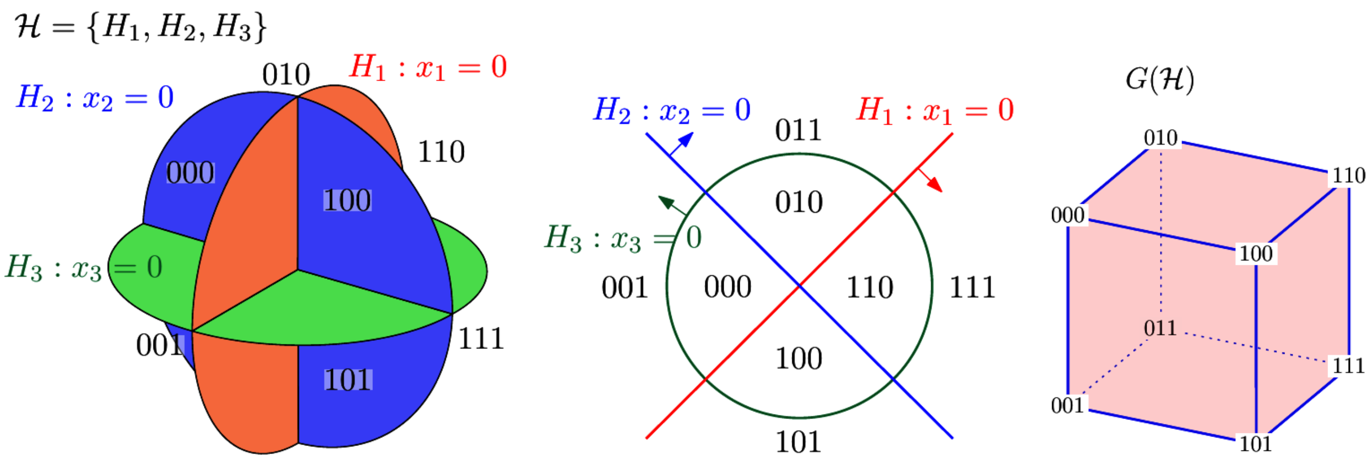



- **Goal:** Find Hamiltonian path/cycle in $G(\mathcal{H})$

# Examples

① **Coordinate arrangement**: $\{\vec{e_i} \mid 1 \le i \le n\}$



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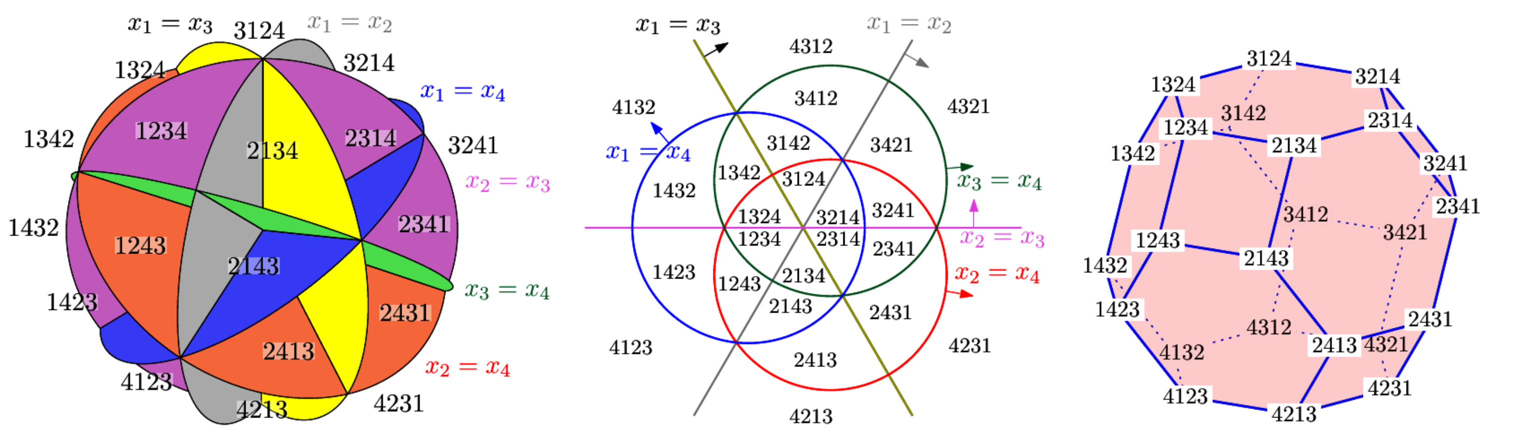

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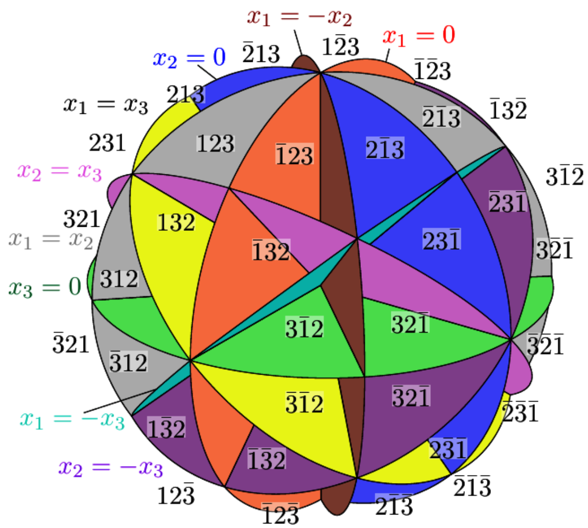

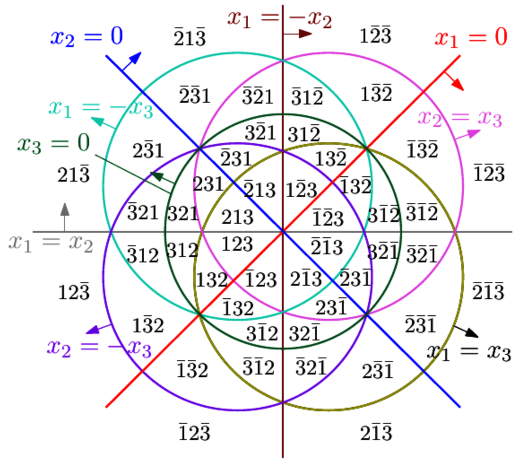

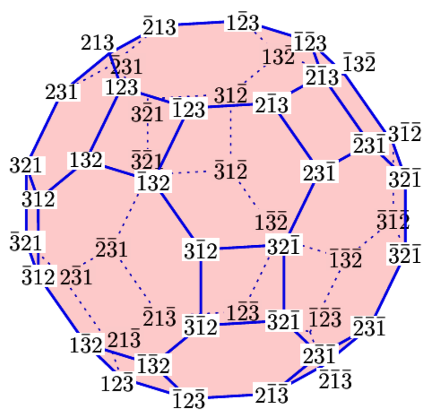

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④ **Graphic arrangement**: $F = ([n], E)$

$$\{\overrightarrow{e_i} - \overrightarrow{e_j} \mid \{i, j\} \in E\}$$

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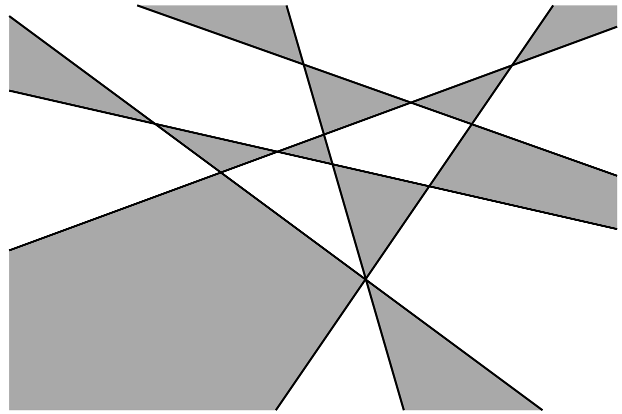

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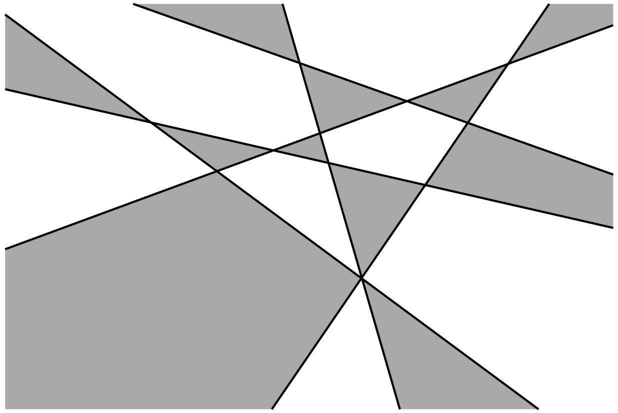

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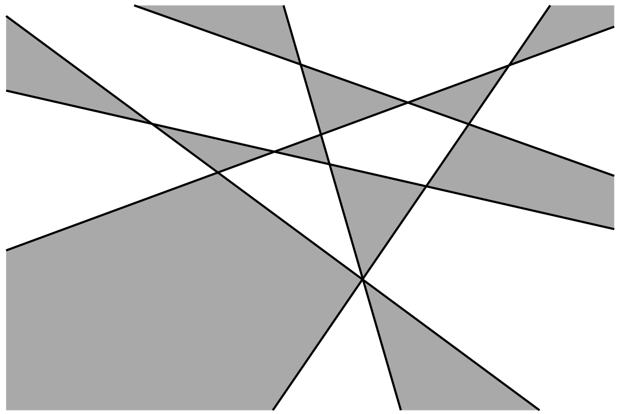

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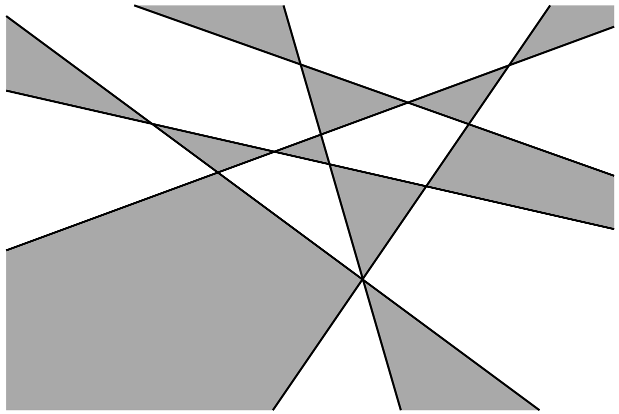

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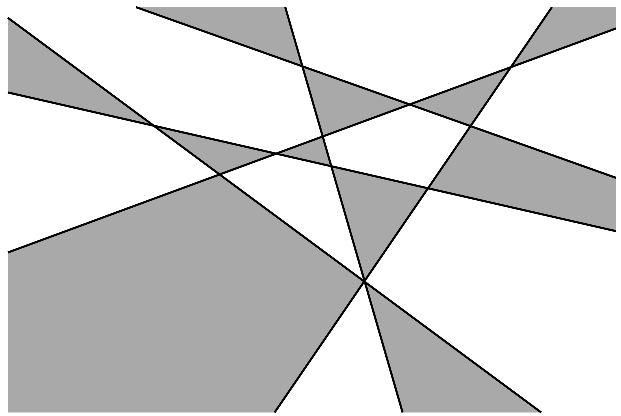

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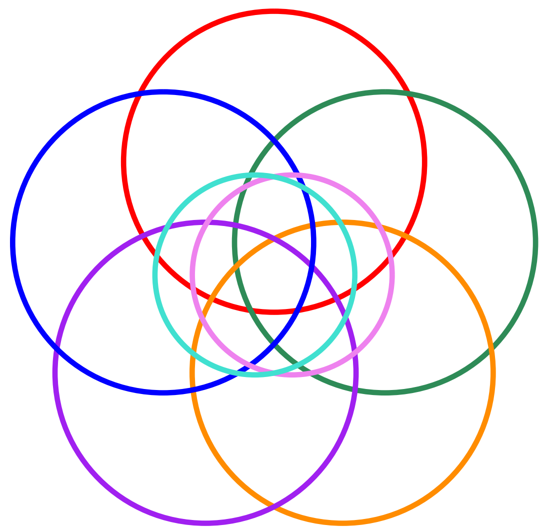

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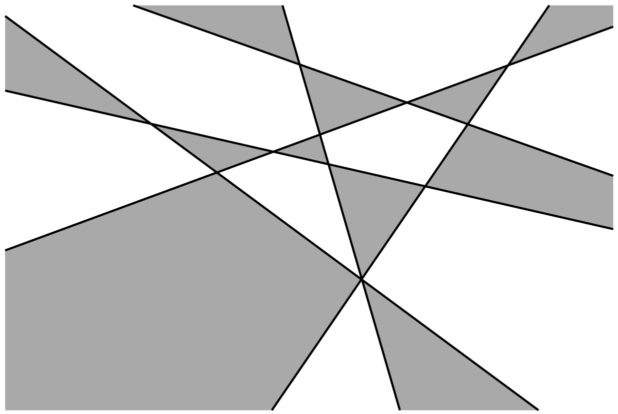

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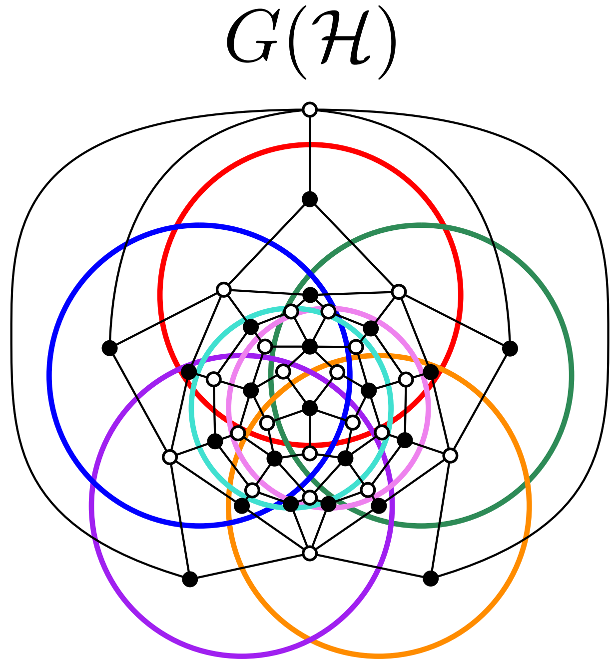

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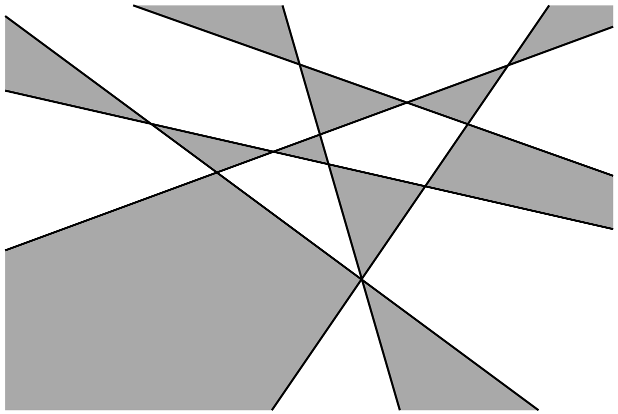

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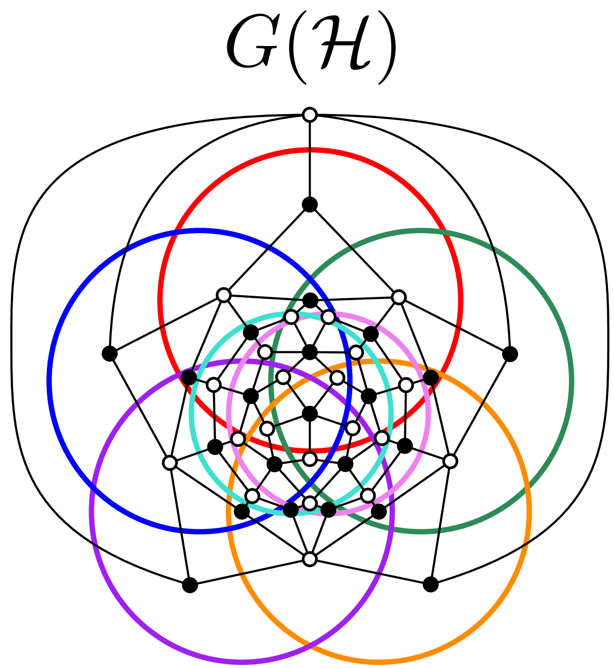
- $\mathbb{R}^3$: $G(\mathcal{H})$ is the dual graph of a **great-circle arrangement** on the sphere



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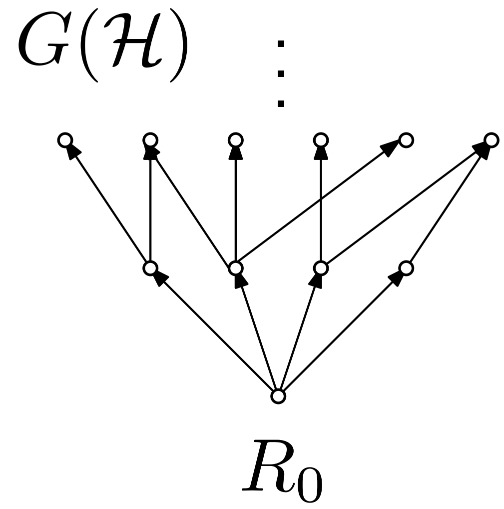

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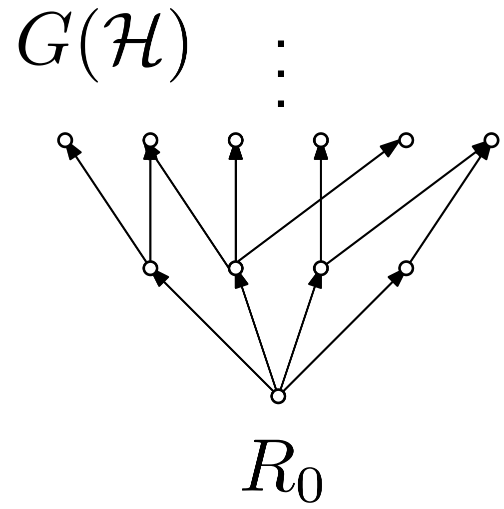

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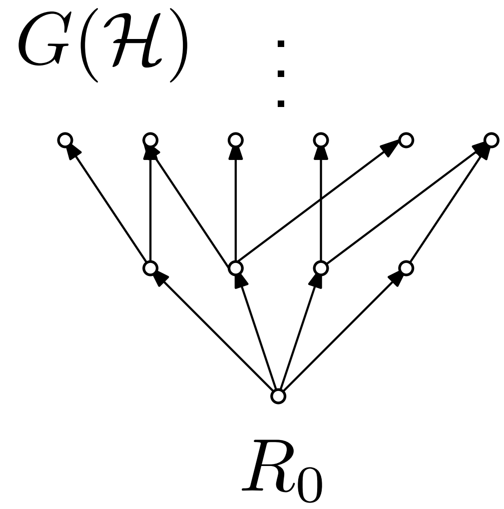

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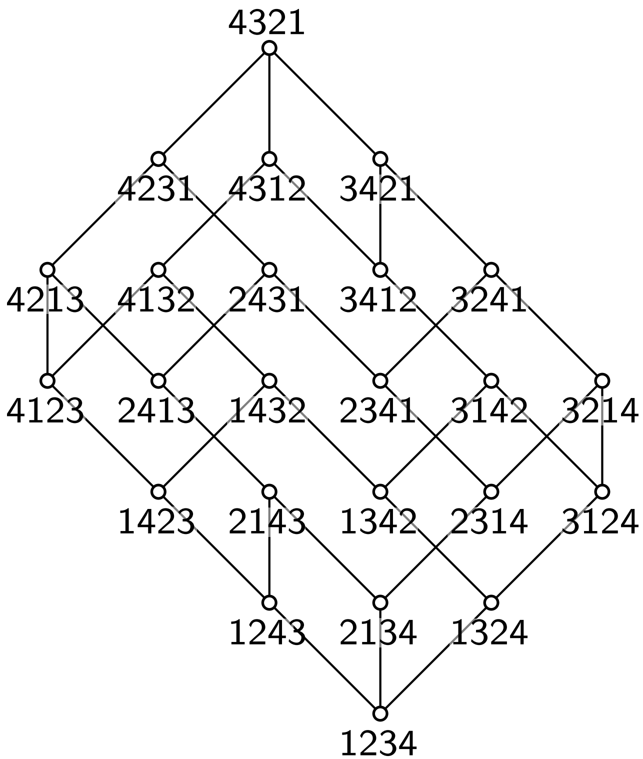

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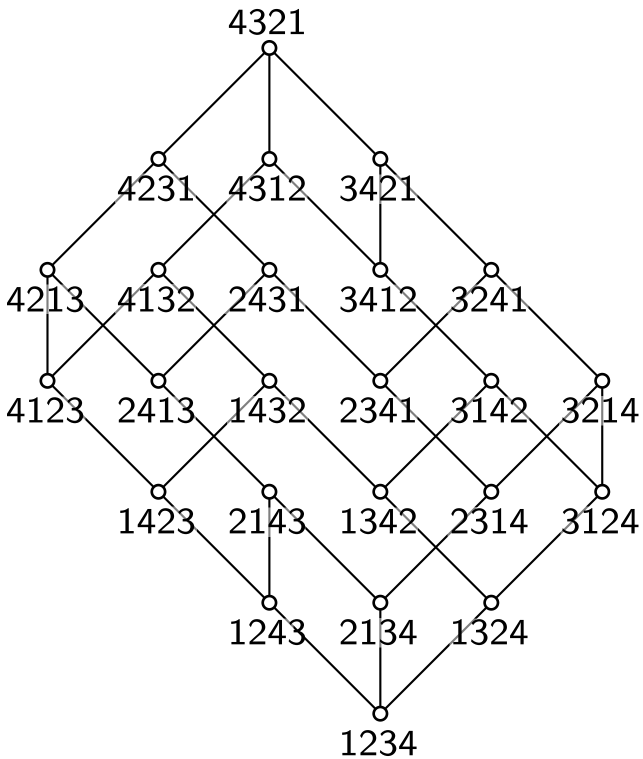

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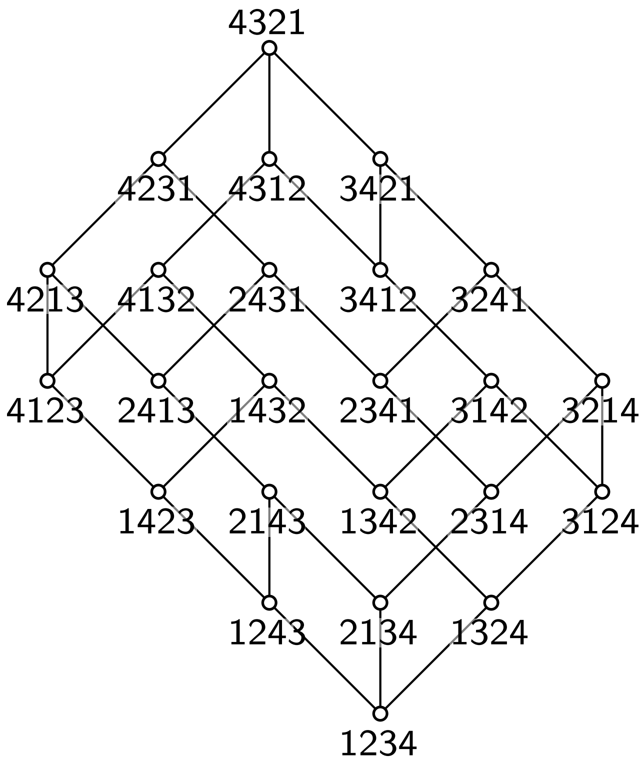

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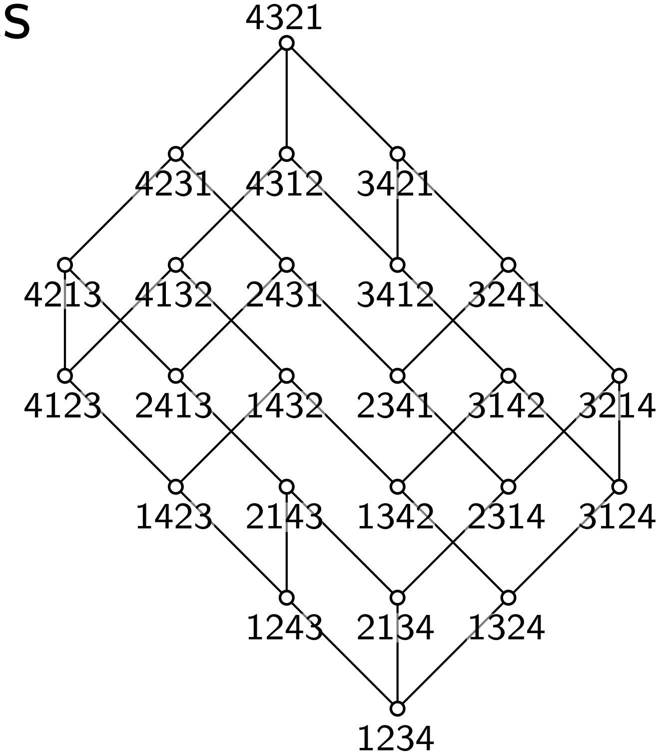

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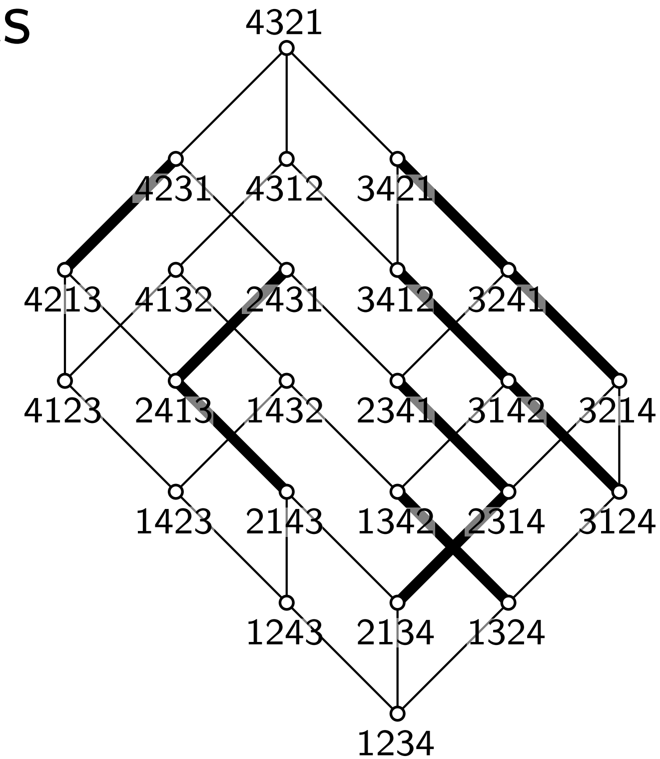

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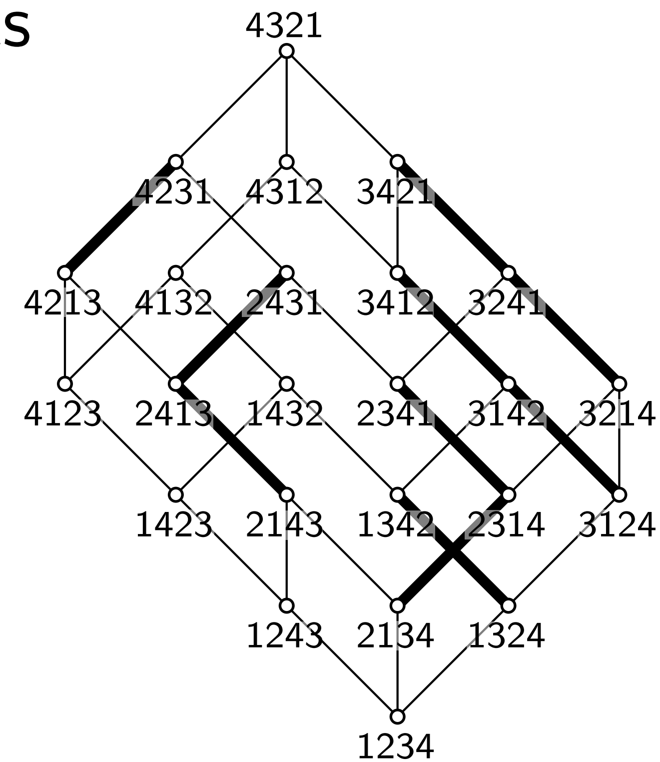


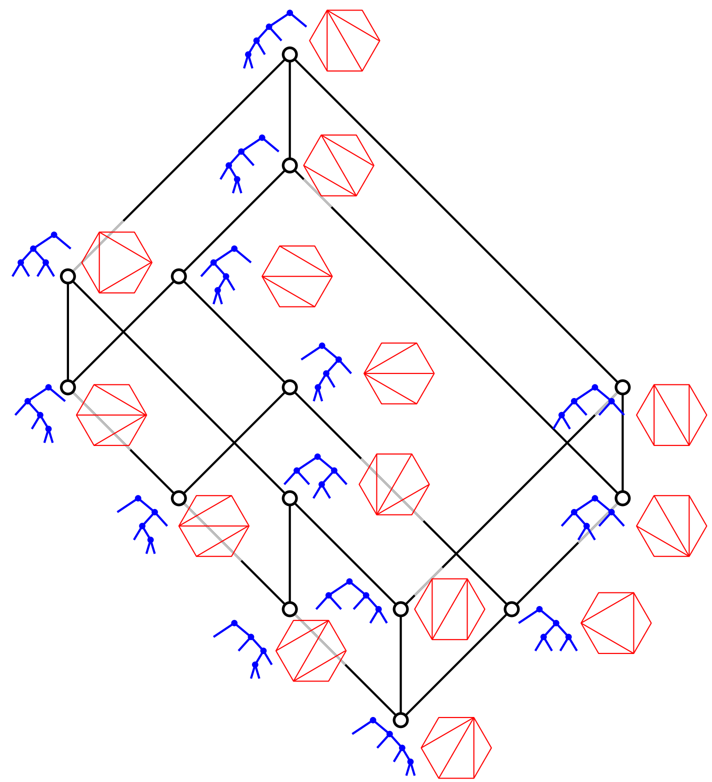

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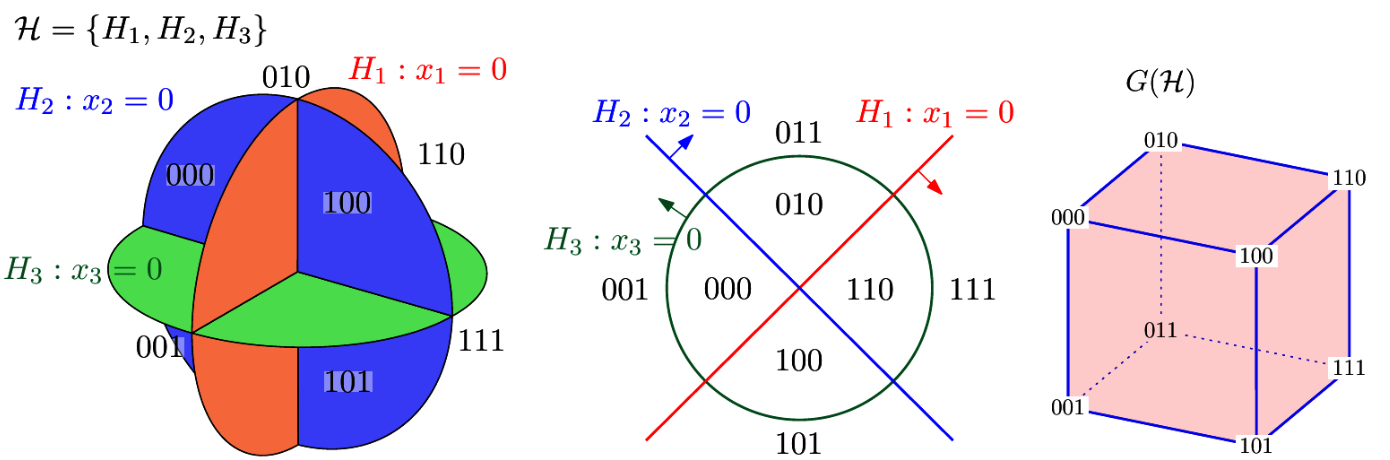

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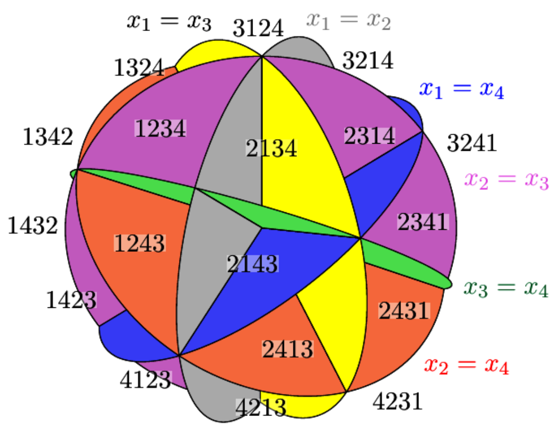


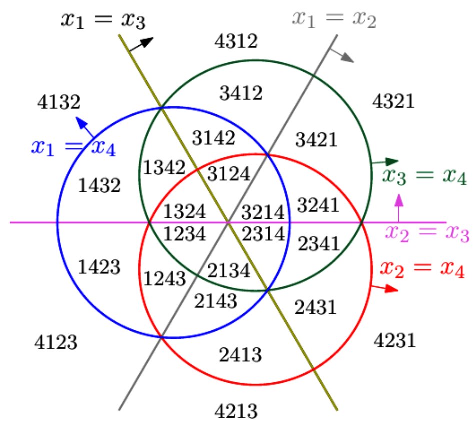


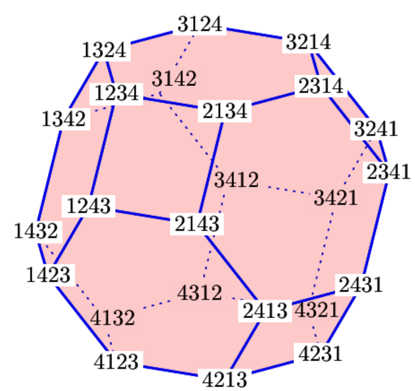

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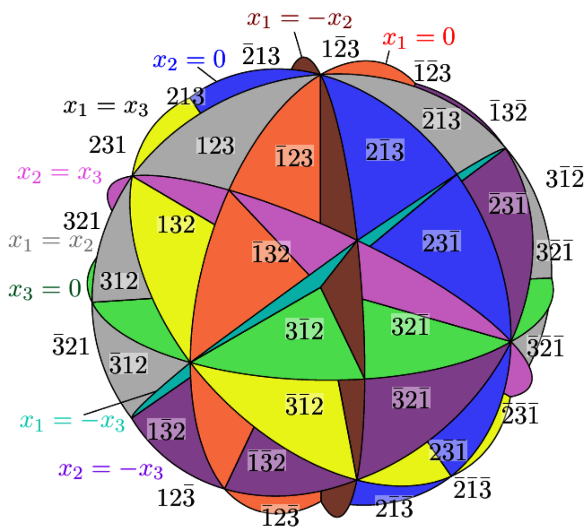


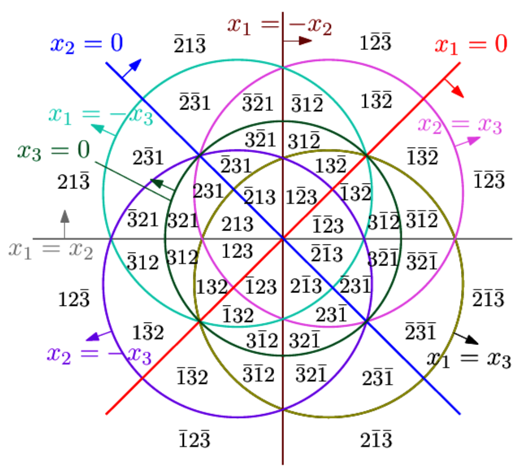


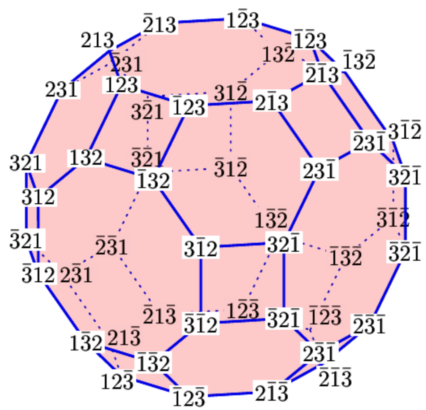

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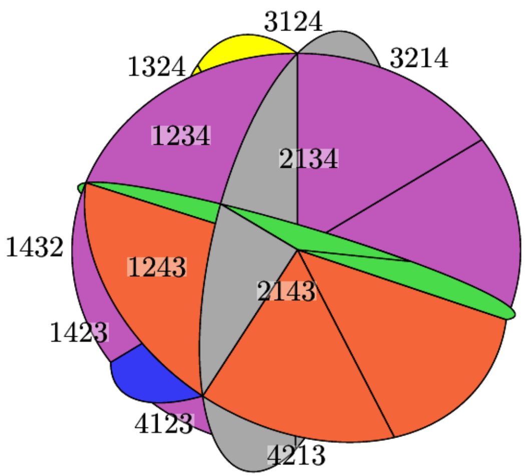


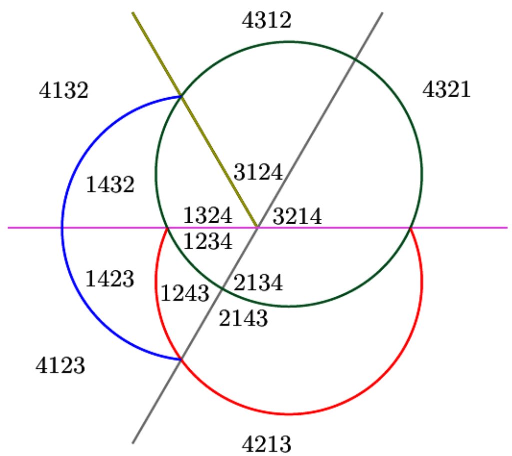


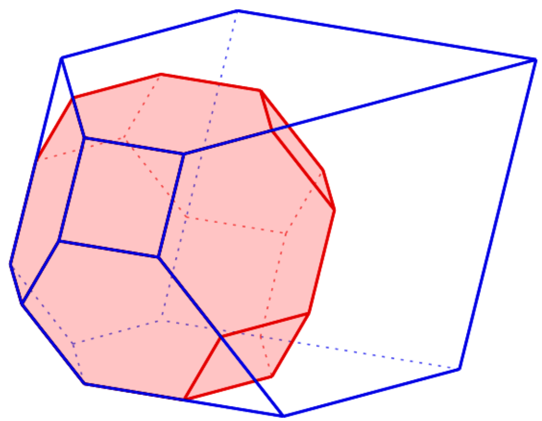

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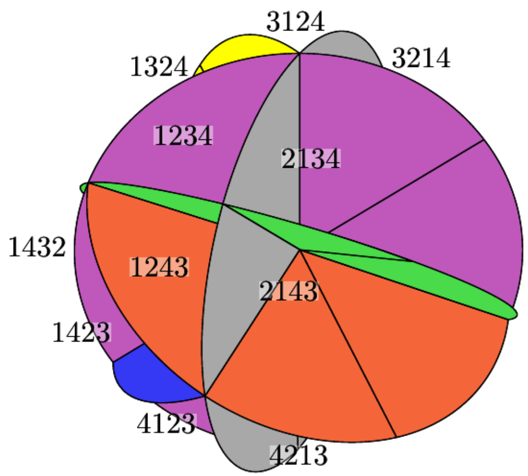


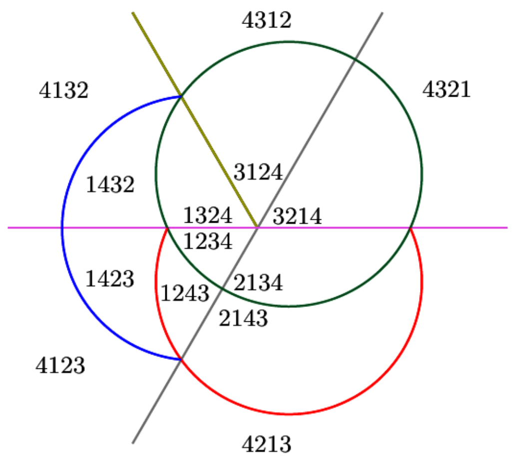


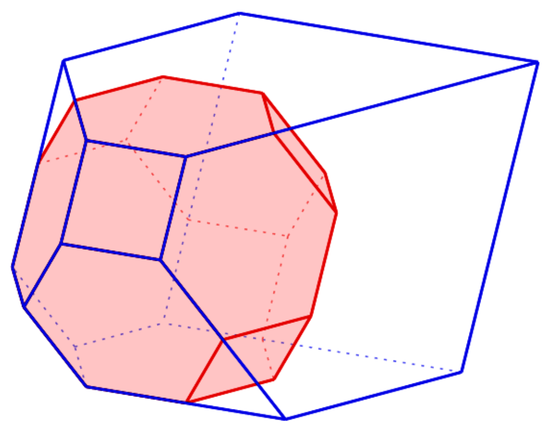

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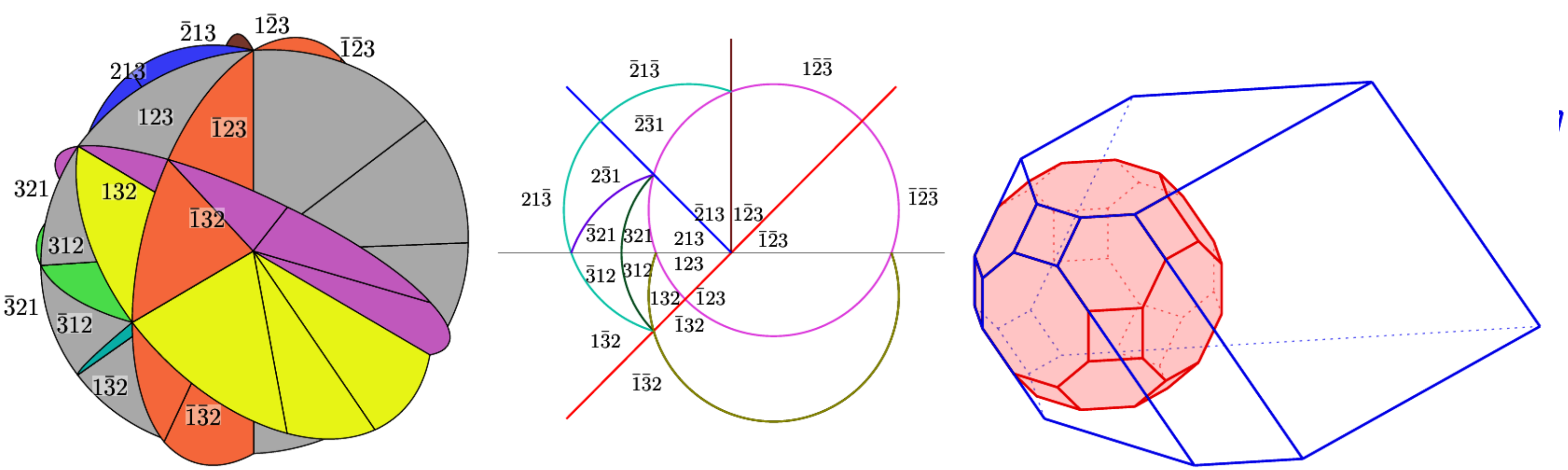

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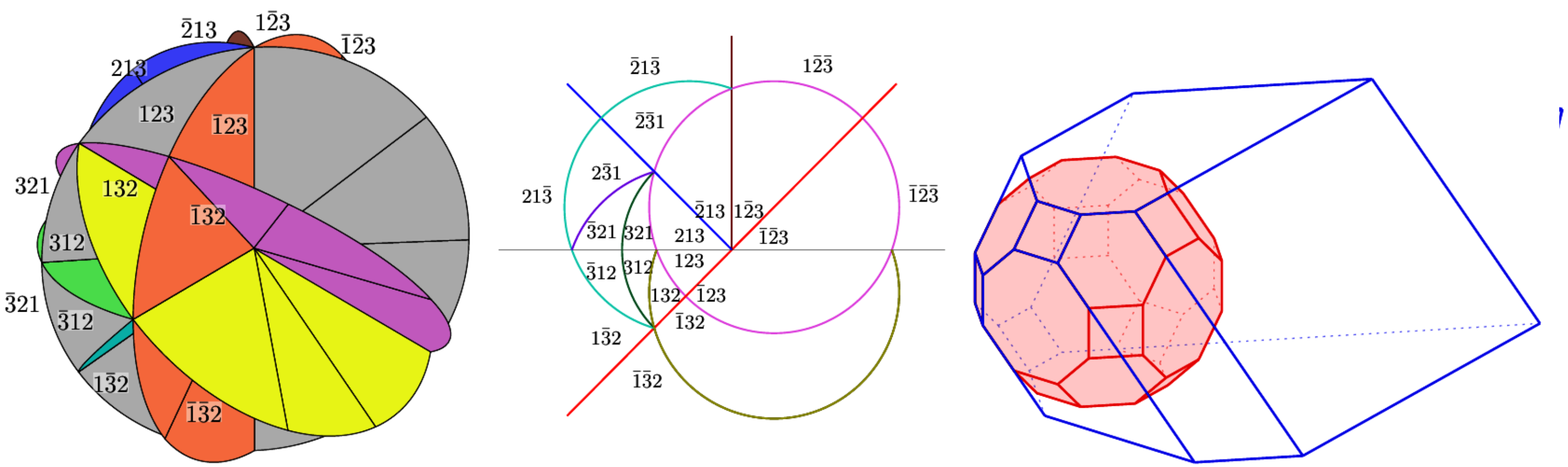

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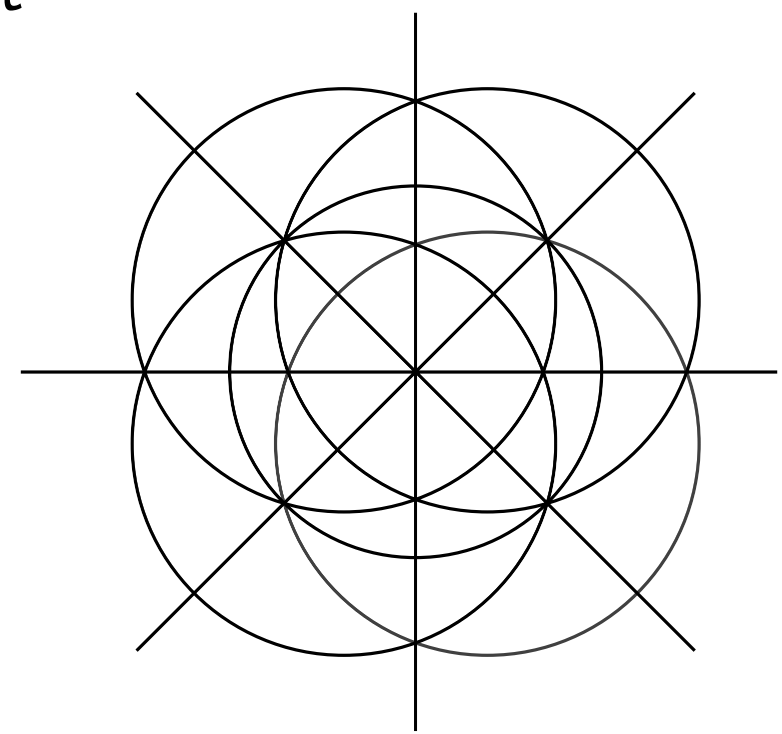

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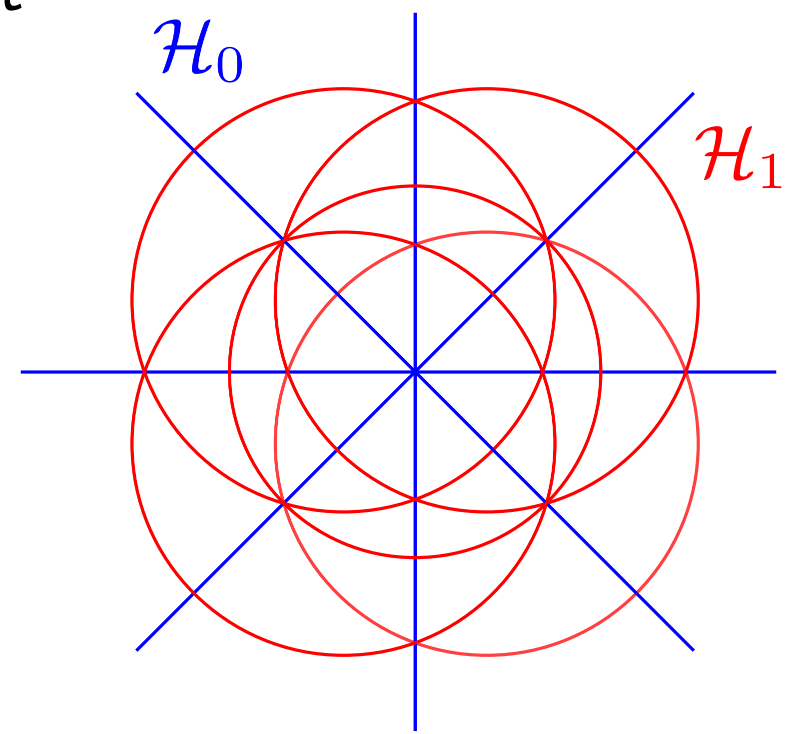

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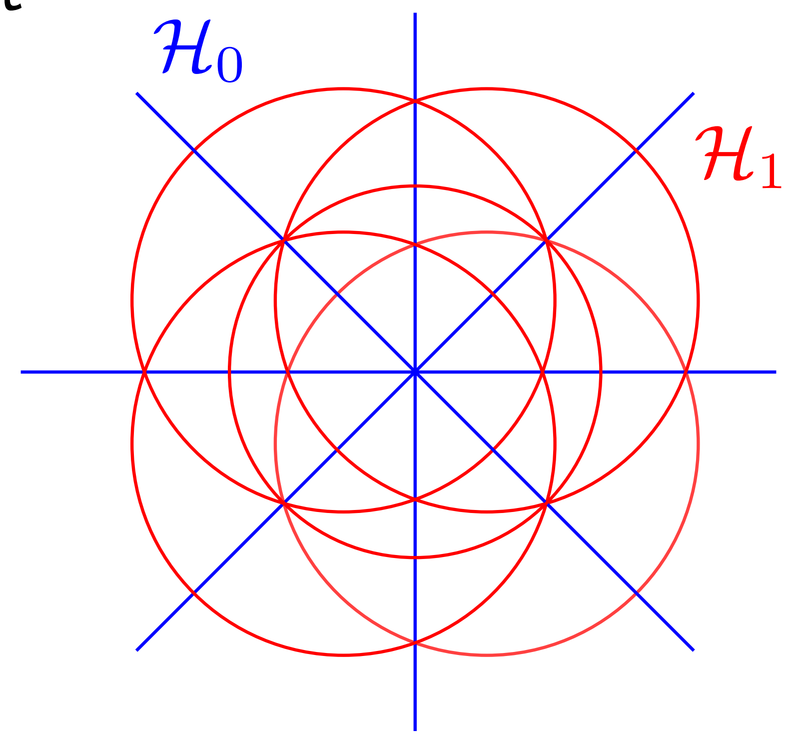

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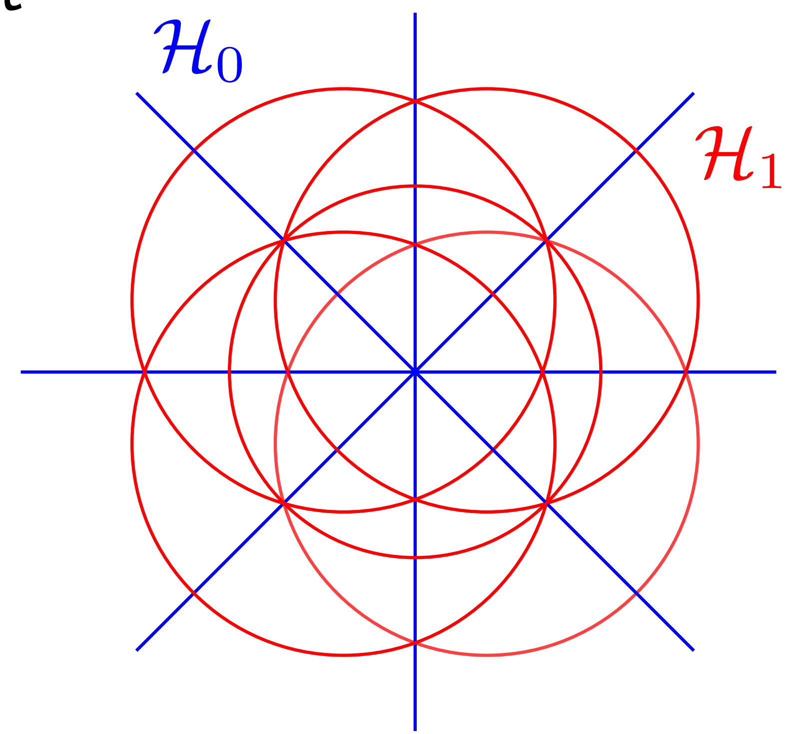

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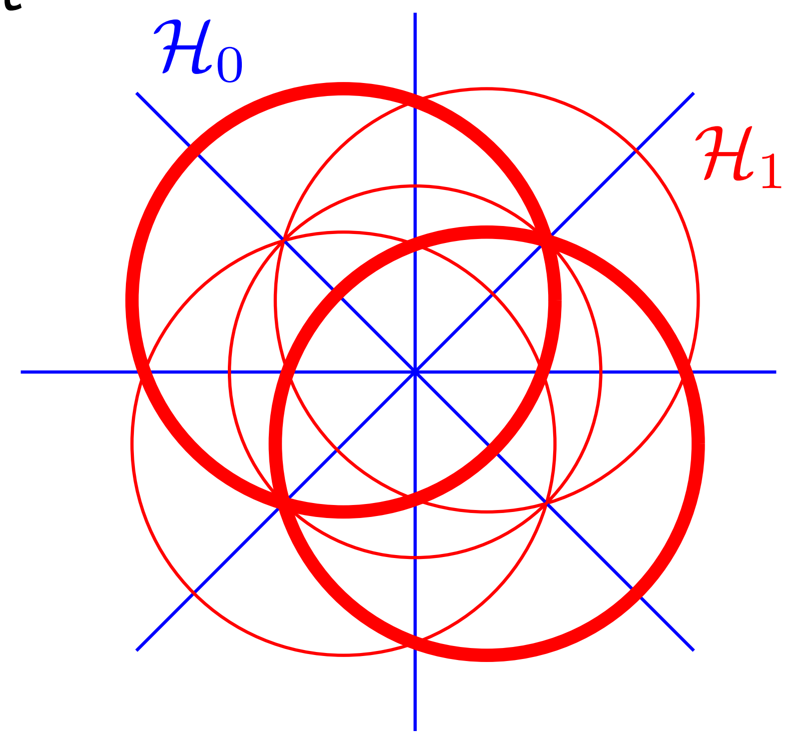

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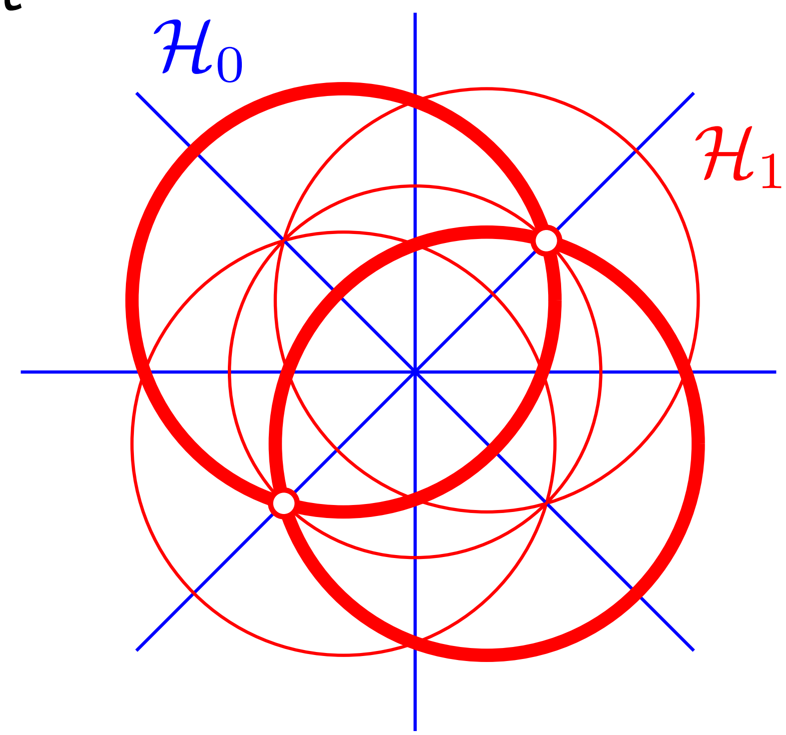

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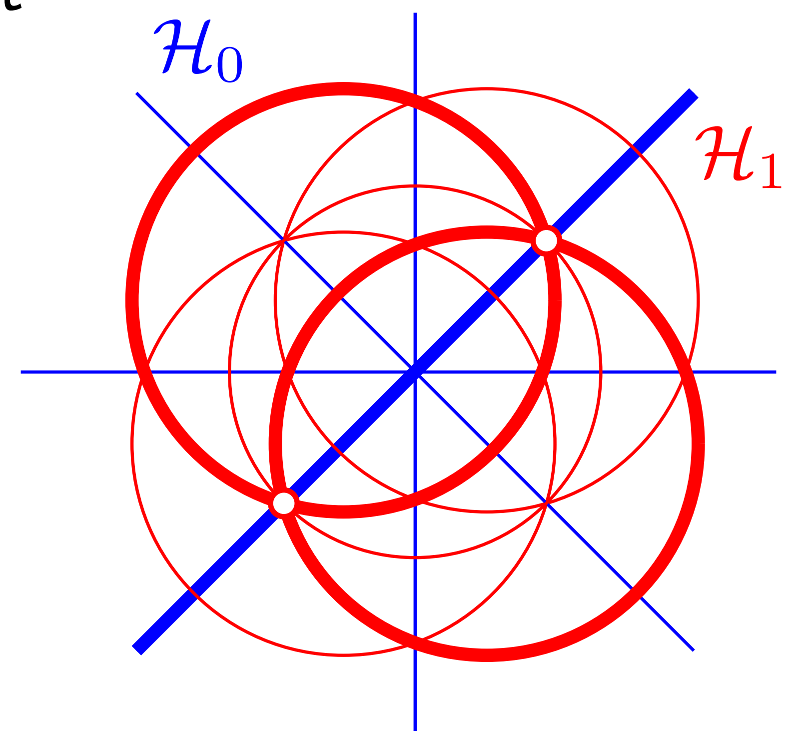

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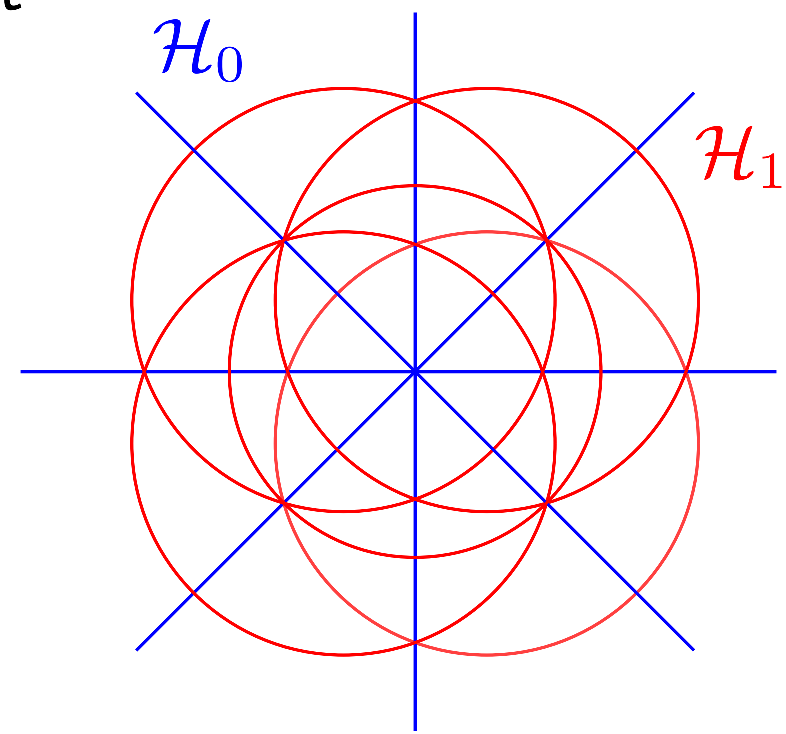

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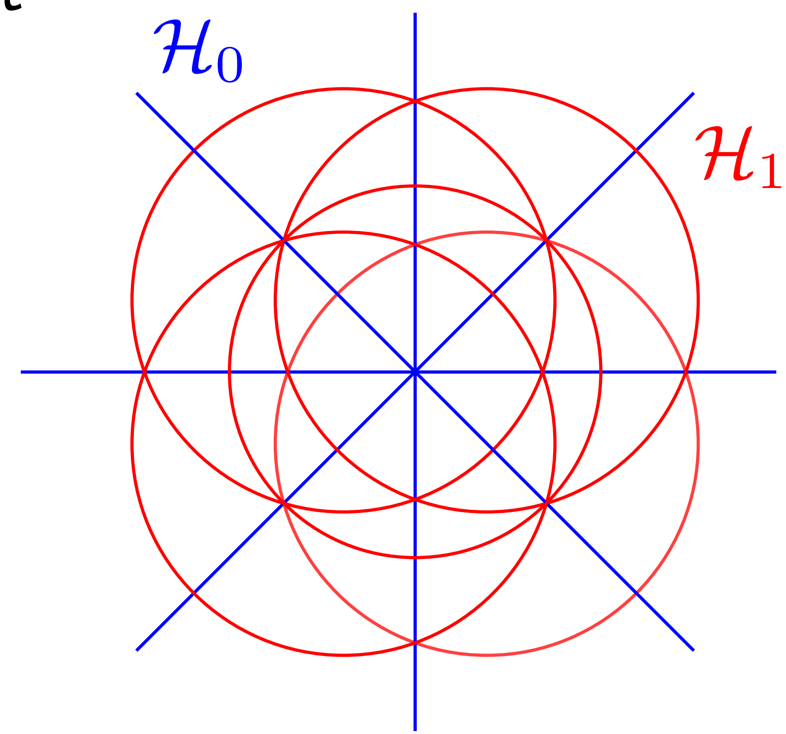
- **Examples:** coordinate arrangement, Type A+B Coxeter arrangements



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**Thm 2:** Let $\mathcal{H}$ be a **supersolvable** arrangement and $R_0 \in \mathcal{R}(\mathcal{H})$ a canonical base region. Then for any lattice congruence $\equiv$ on $L := P(\mathcal{H}, R_0)$, the cover graph of $L/\equiv$ has a **Hamilt. path**.

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- new Gray code for signed permutations

| $n$ | |
|---|---|
| 1 | $1, \bar{1}$ |
| 2 | $12, 21, \bar{2}1, 1\bar{2}, \bar{1}\bar{2}, \bar{2}\bar{1}, 2\bar{1}, \bar{1}2$ |
| 3 | $123, 132, 312, \bar{3}12, 1\bar{3}2, 12\bar{3},$ |
| | $21\bar{3}, 2\bar{3}1, \bar{3}21, 321, 231, 213,$ |
| | $\bar{2}13, \bar{2}31, 3\bar{2}1, \bar{3}\bar{2}1, \bar{2}\bar{3}1, \bar{2}1\bar{3},$ |
| | $1\bar{2}\bar{3}, 1\bar{3}\bar{2}, \bar{3}1\bar{2}, 31\bar{2}, 13\bar{2}, 1\bar{2}3,$ |
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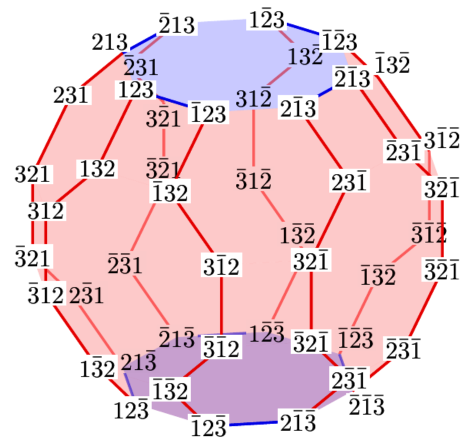

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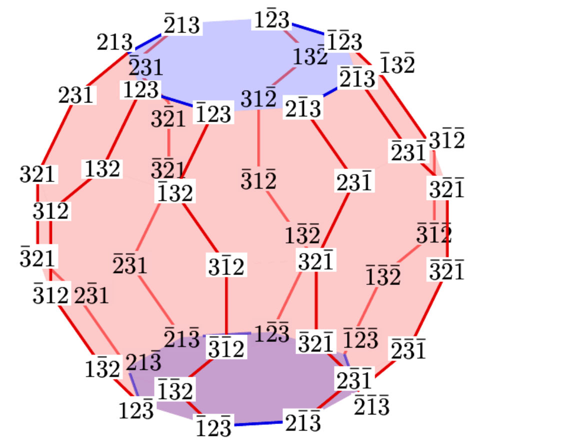

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$\rightarrow$ Gray codes for **quotients** of acyclic reorientation lattices

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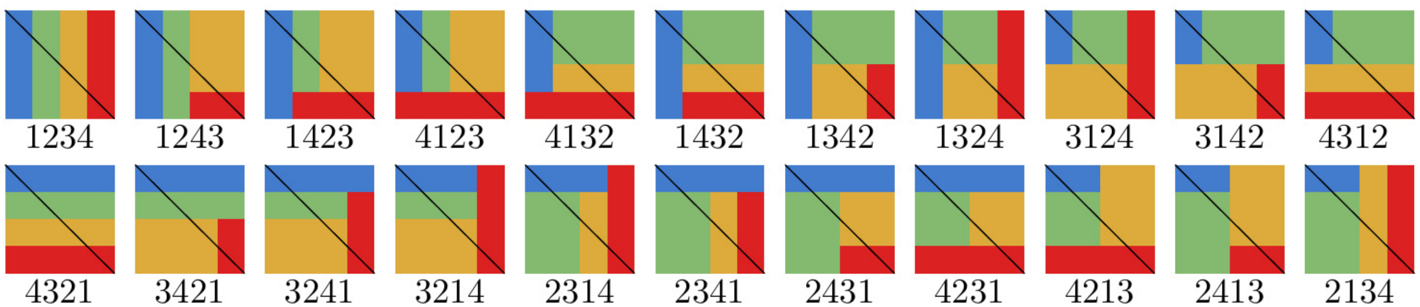
→ Hamiltonian paths on **rectangulotopes** introduced by **[Cardinal, Pilaud 25]** / Gray codes for rectangulations **[Merino, M. 23]**



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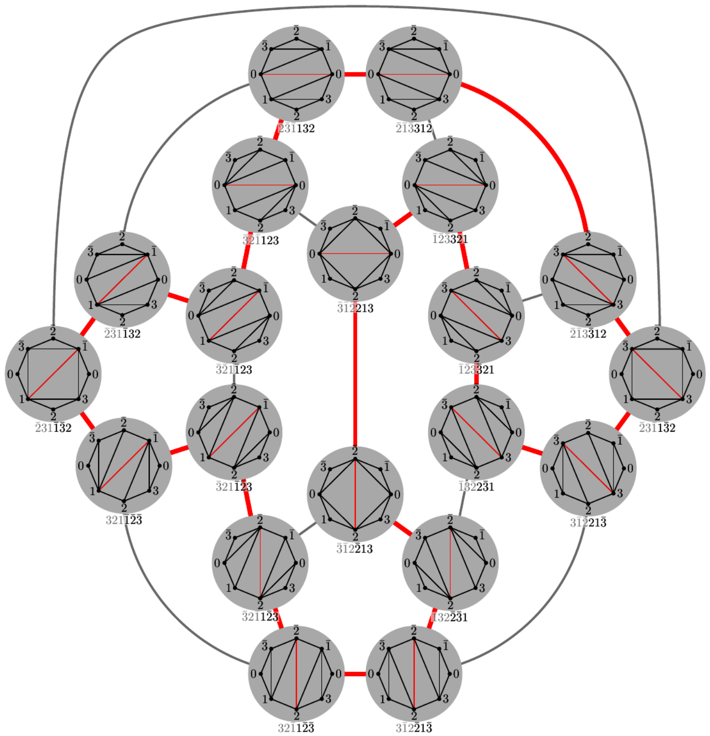
→ Hamiltonian cycles on $B$-associahedron / Gray code for symmetric triangulations



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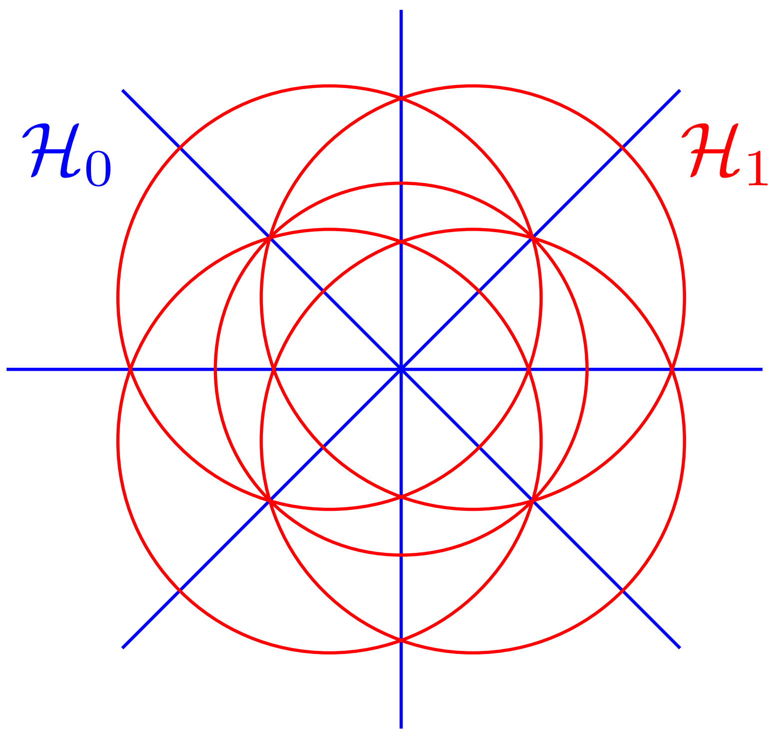

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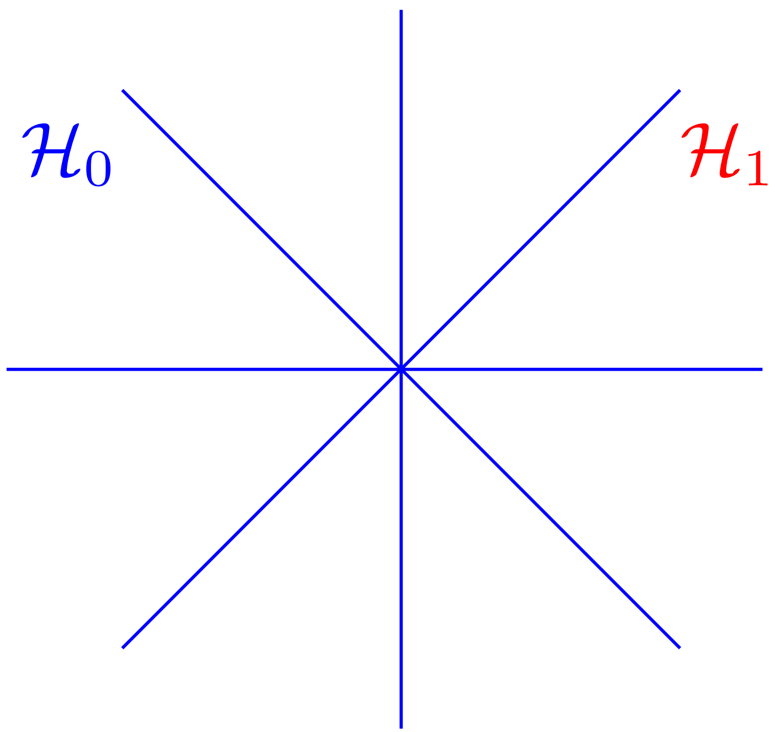

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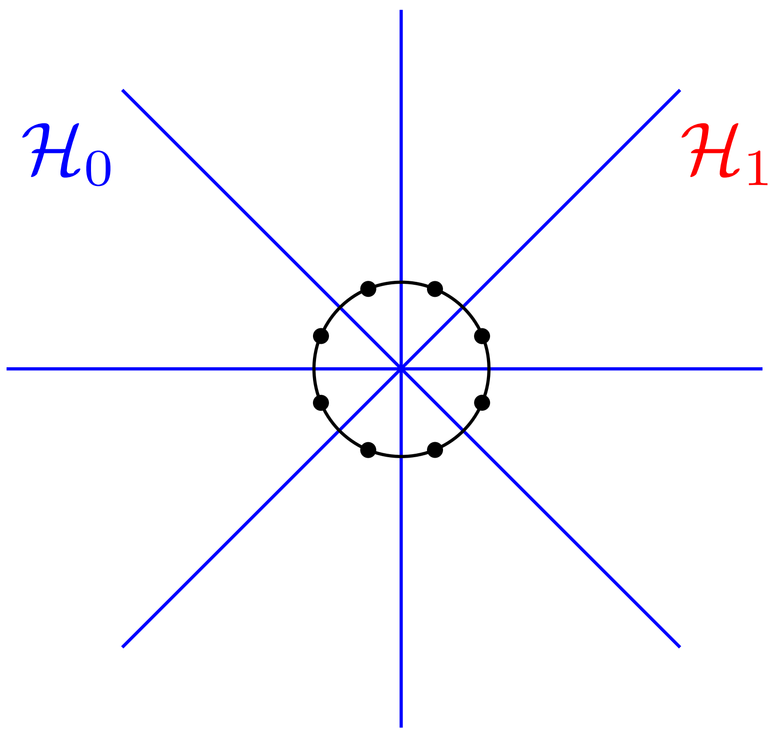

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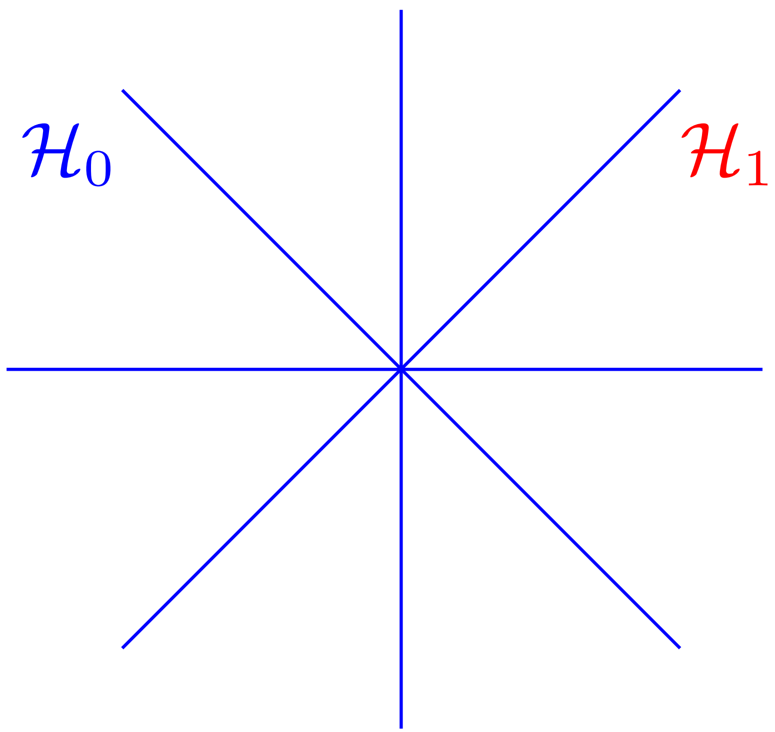

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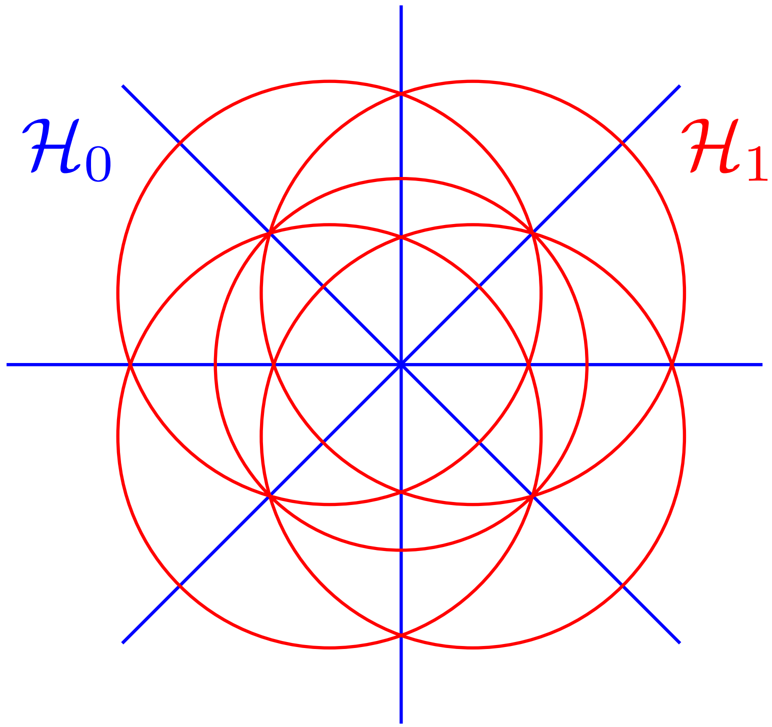

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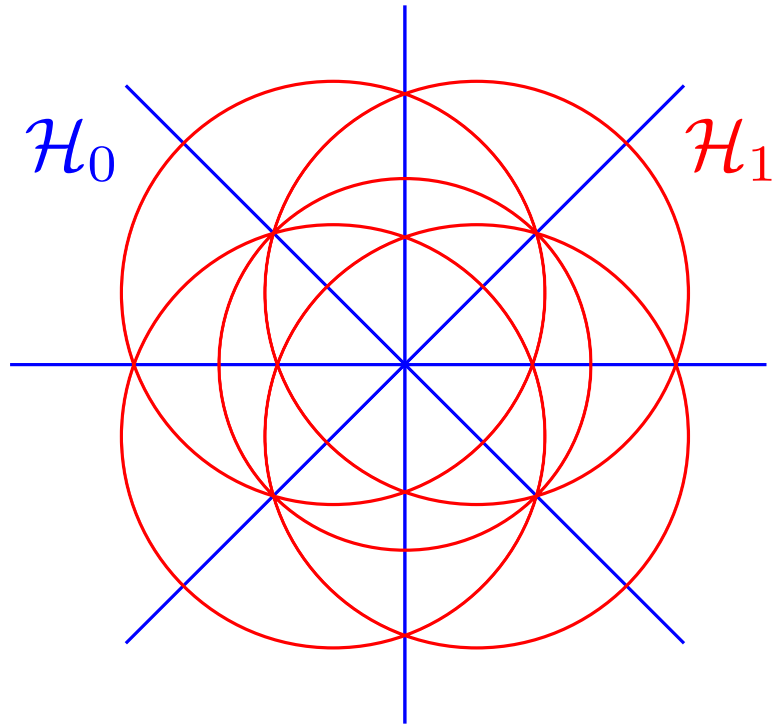


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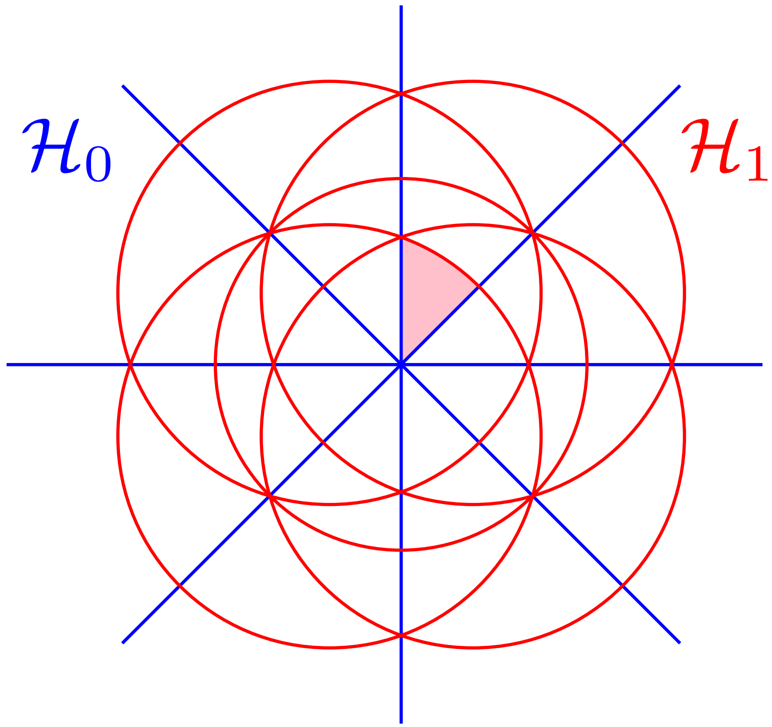

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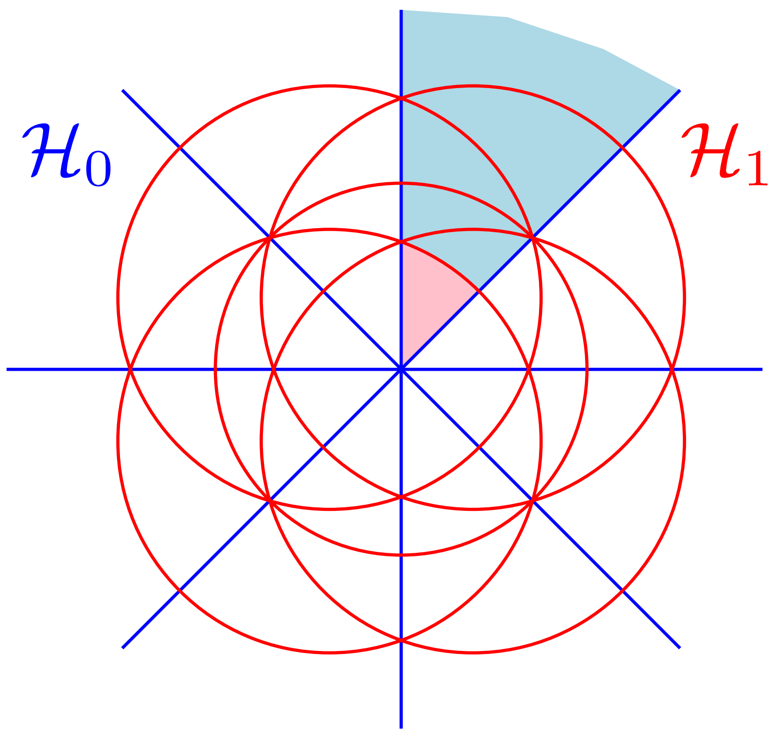

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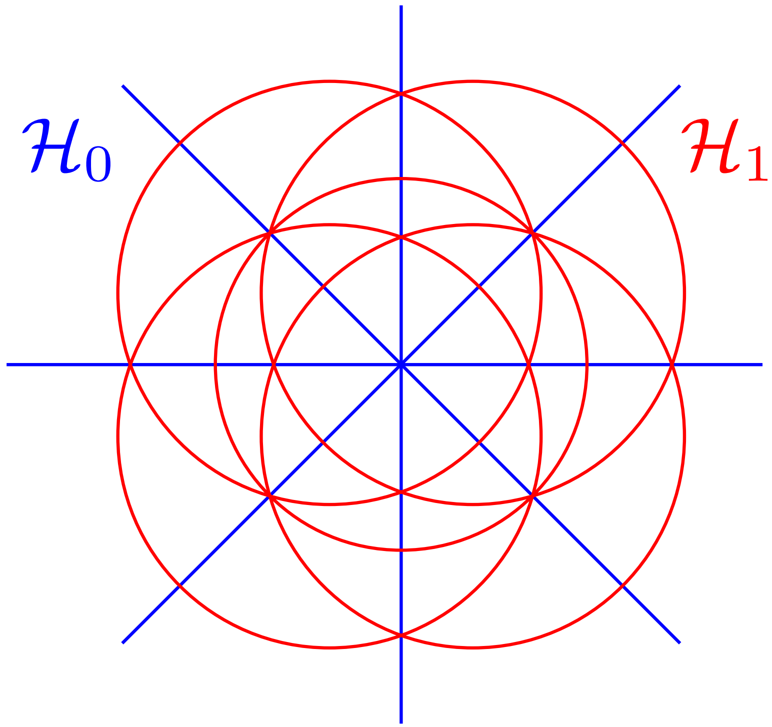


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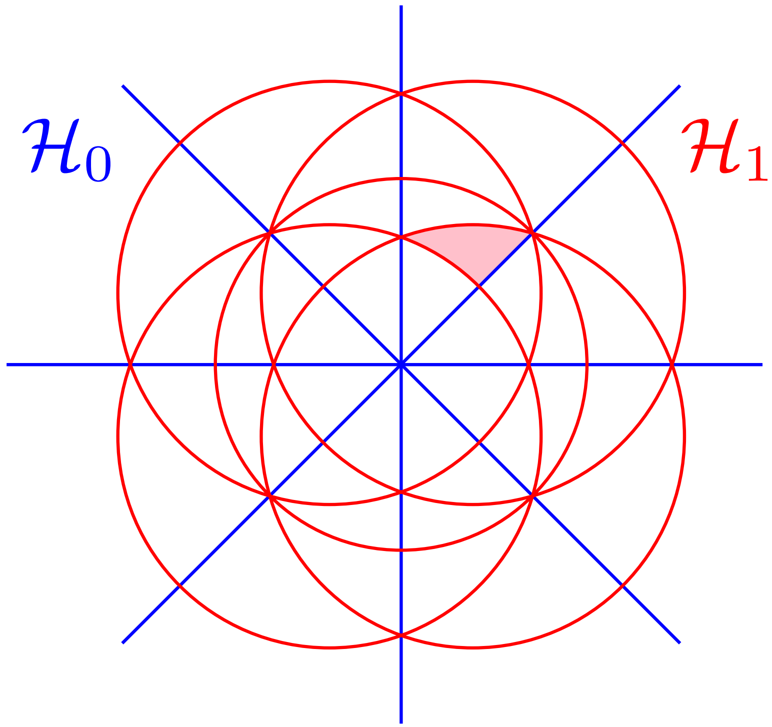

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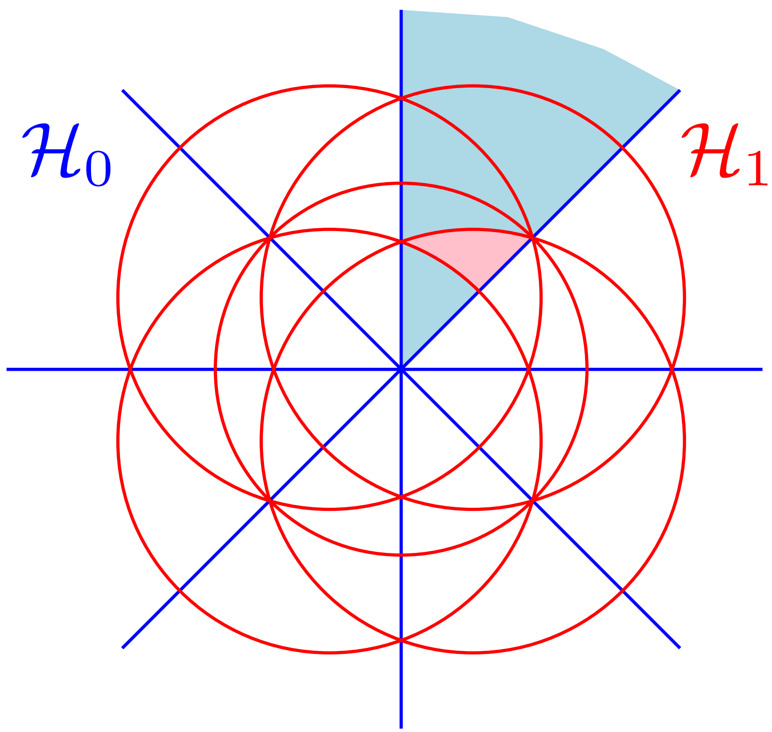

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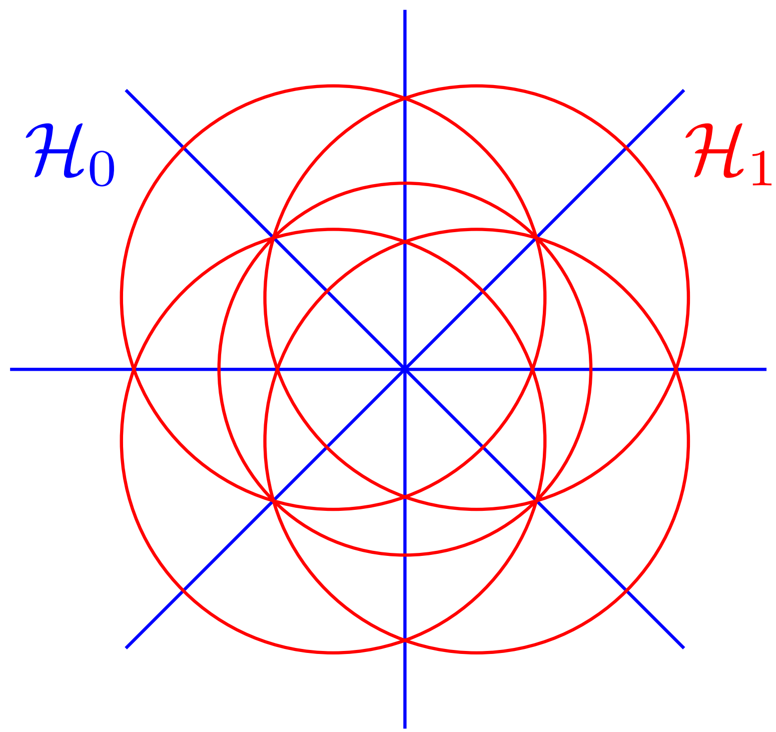


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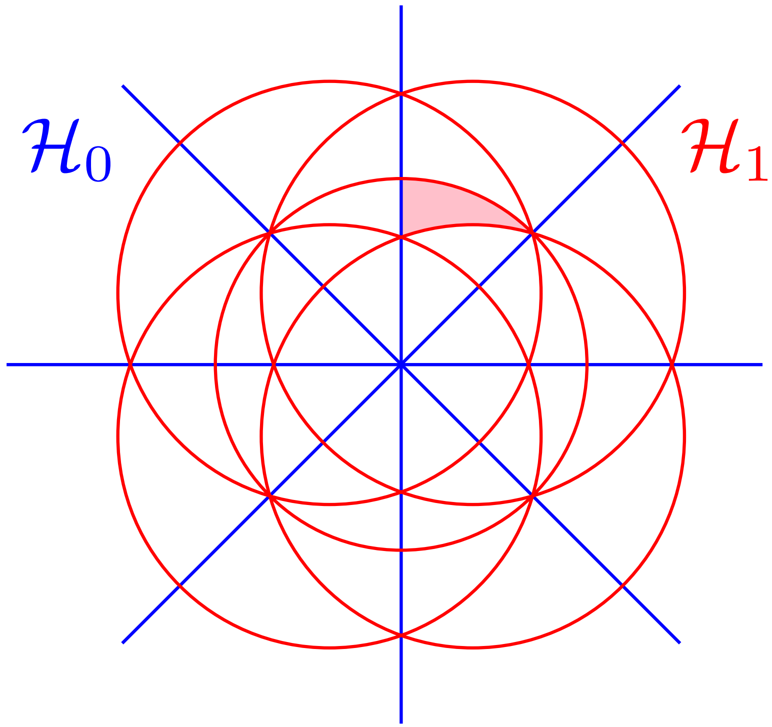


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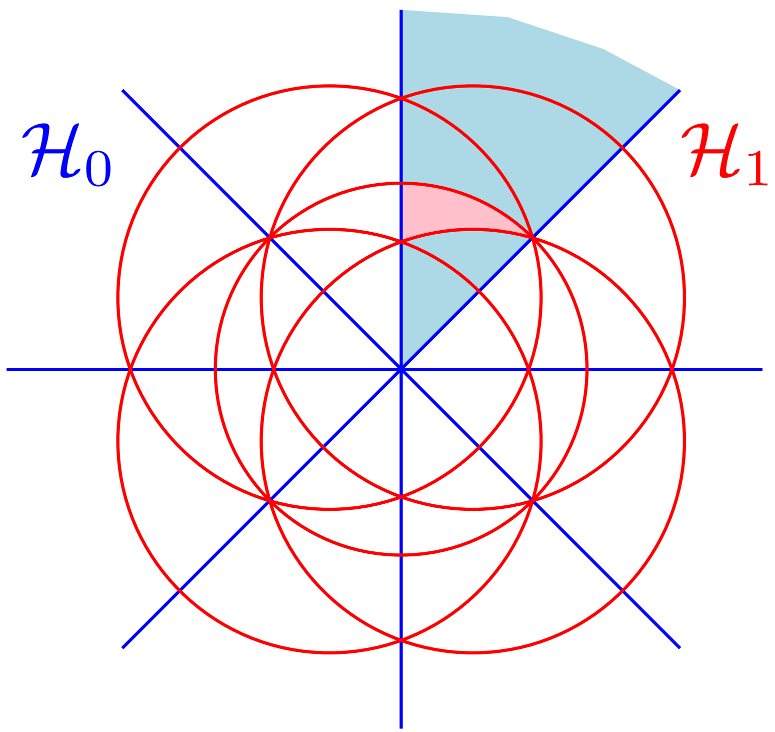


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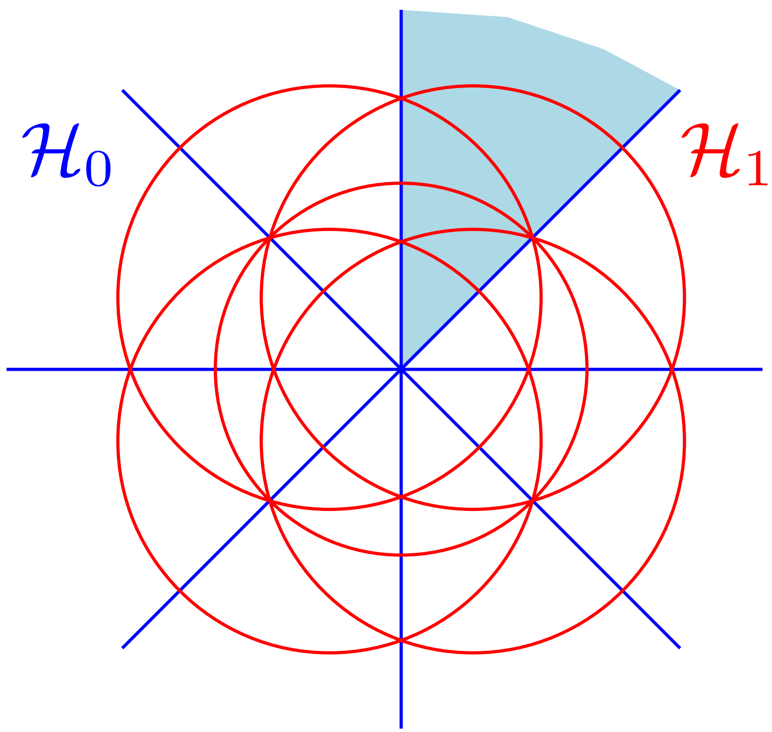

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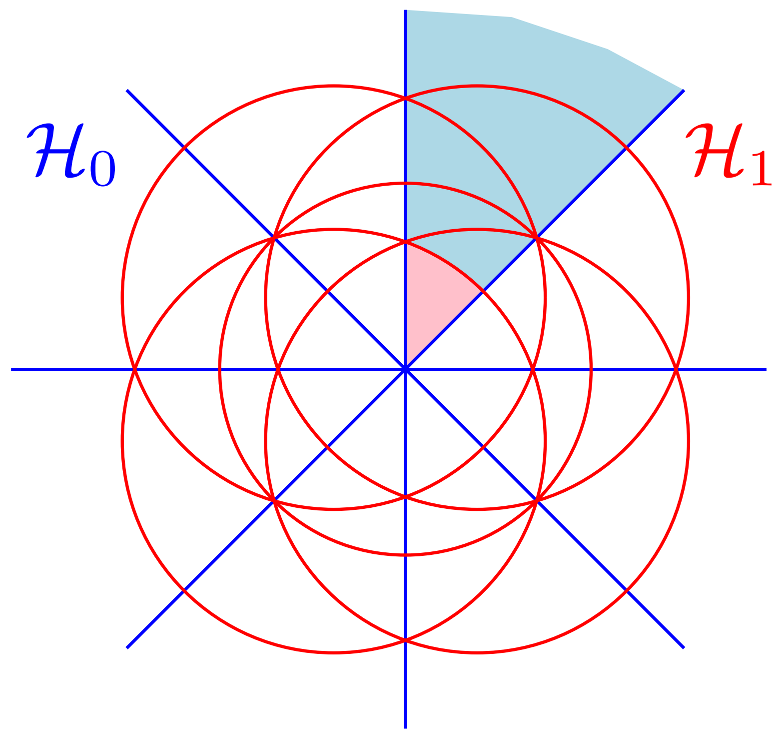


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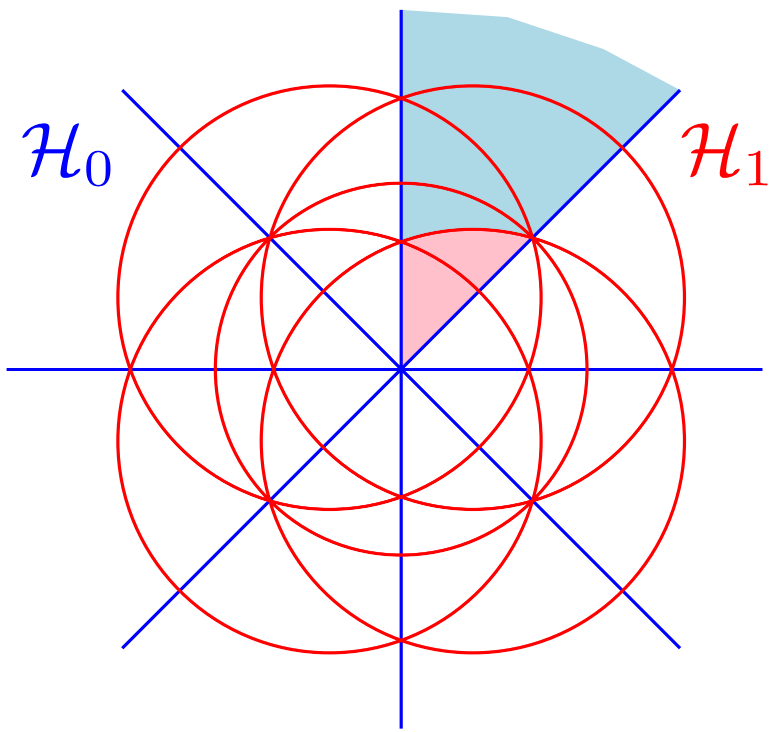


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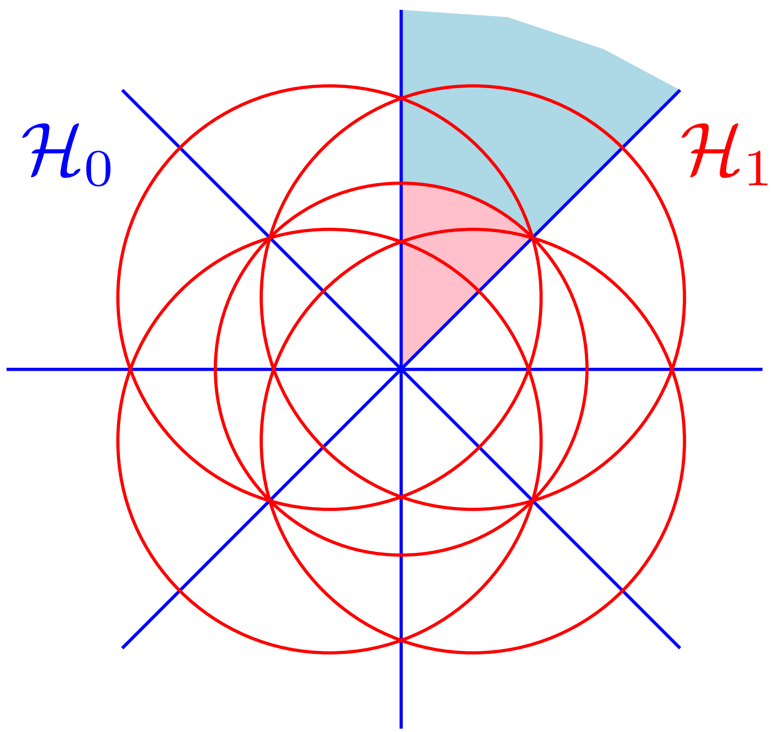


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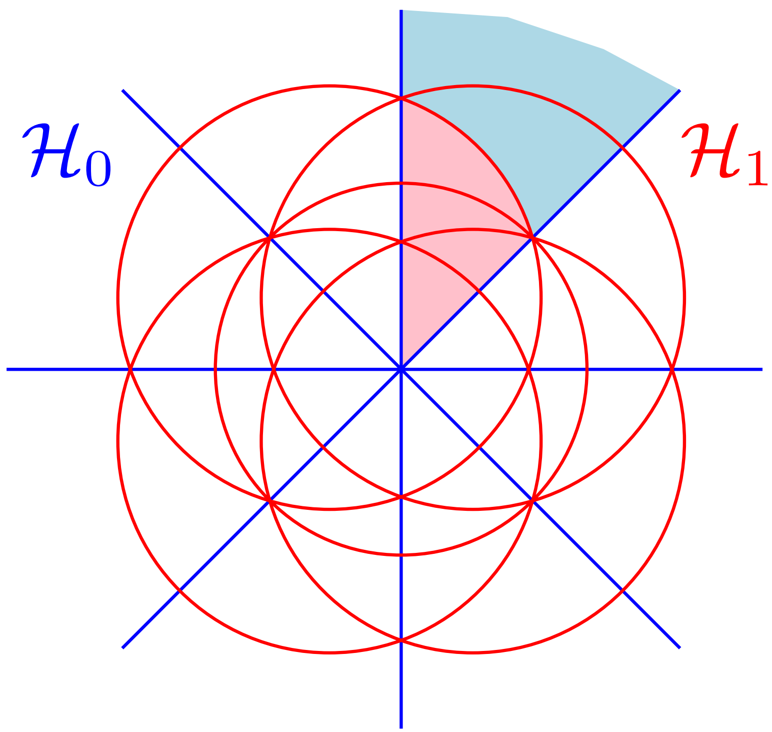


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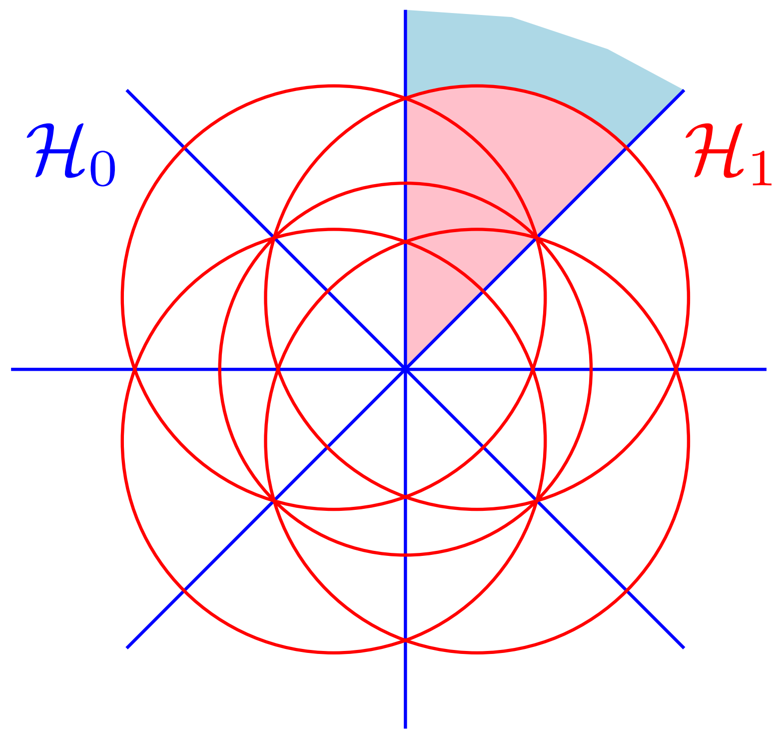


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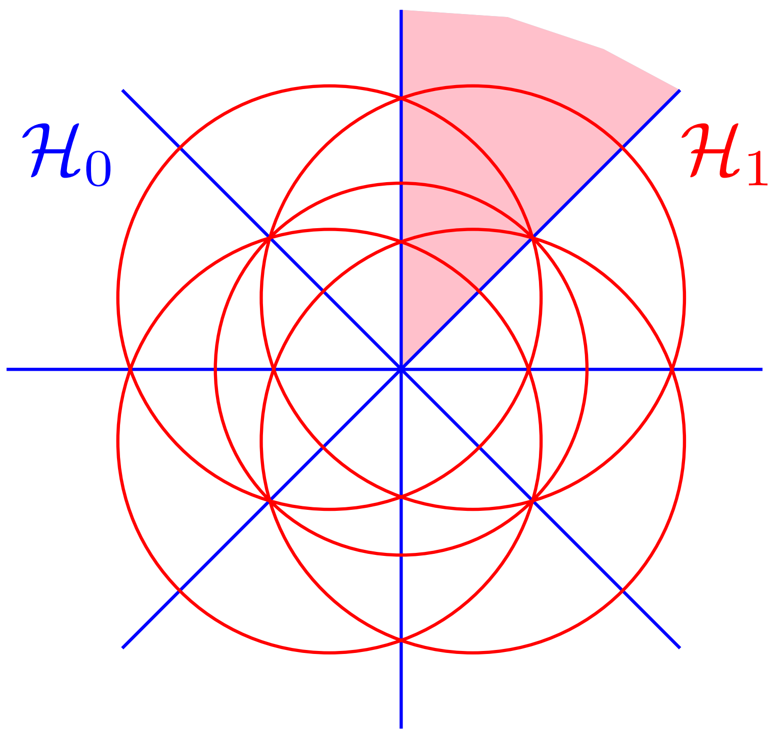


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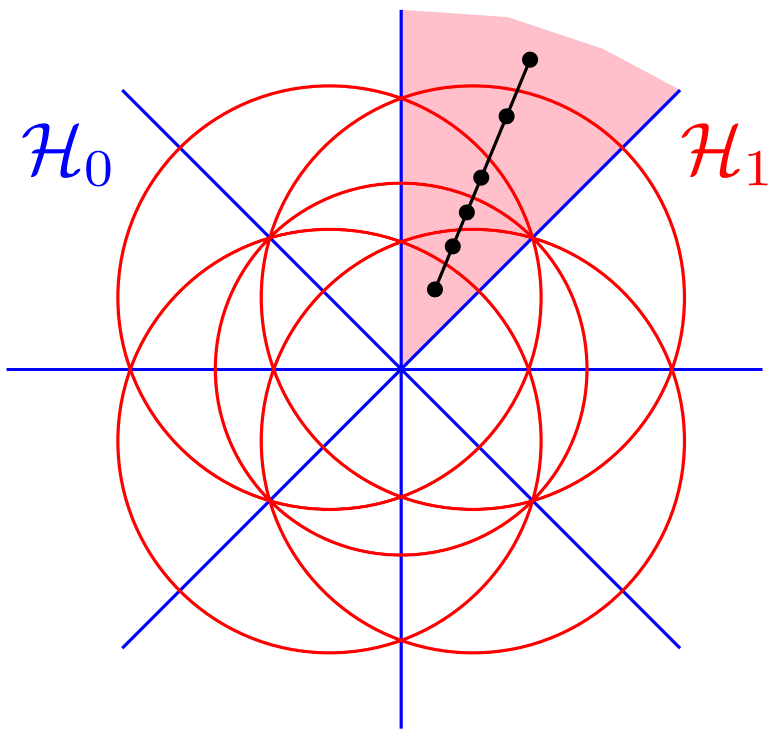


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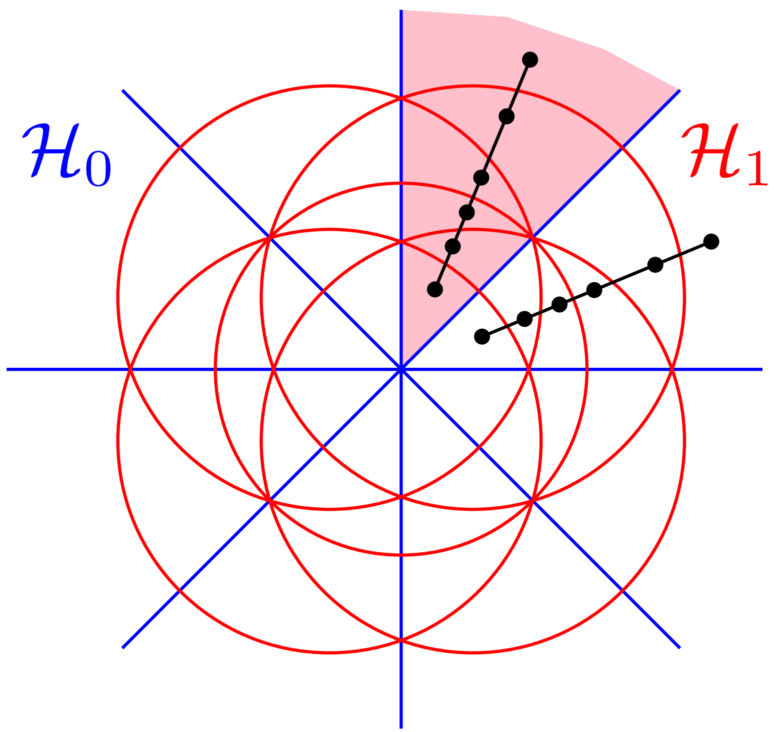


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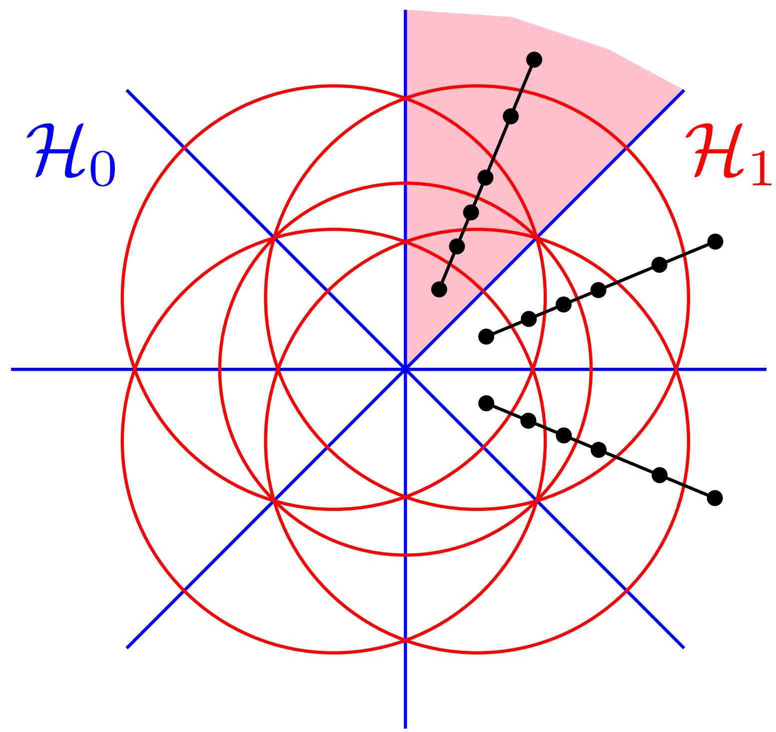


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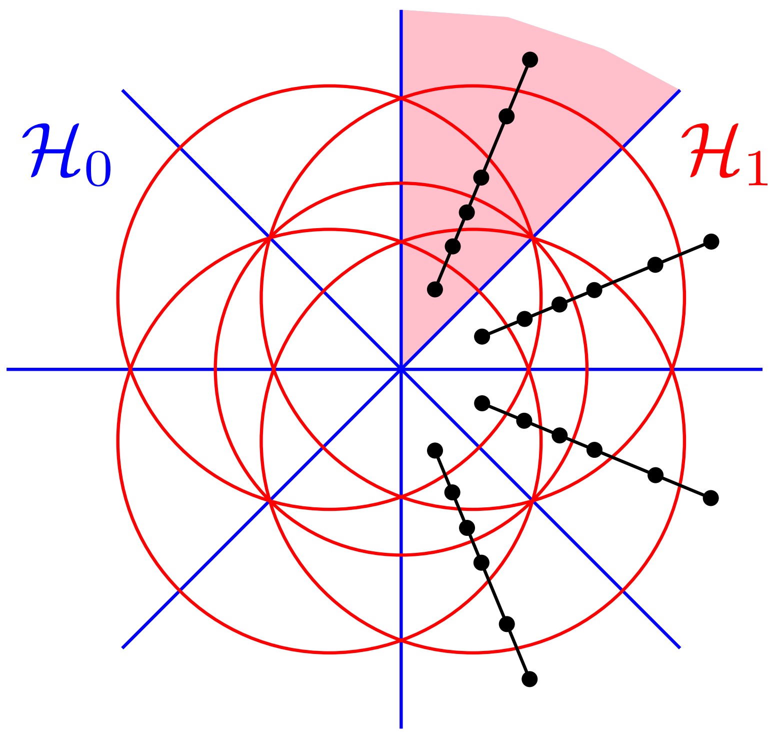


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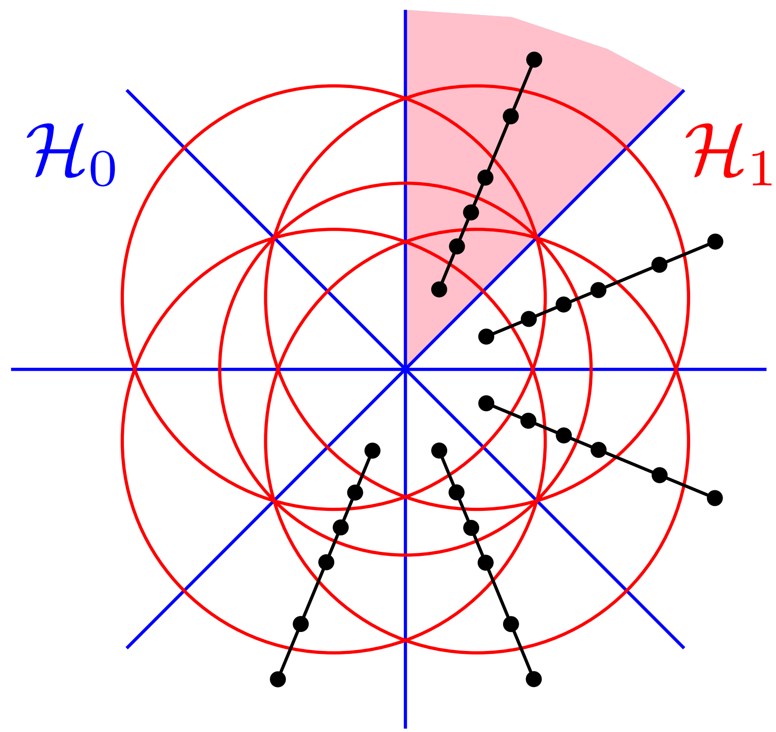


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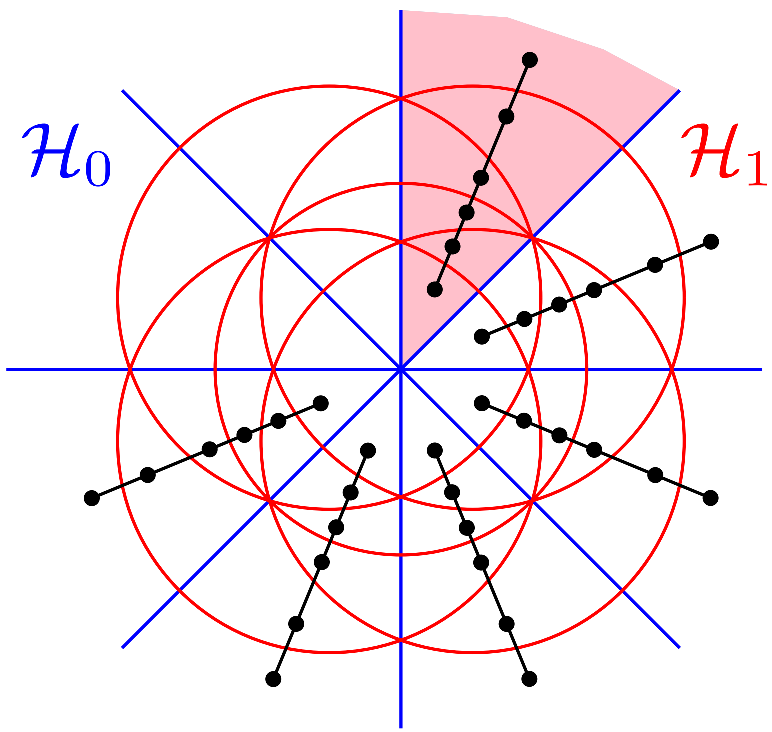


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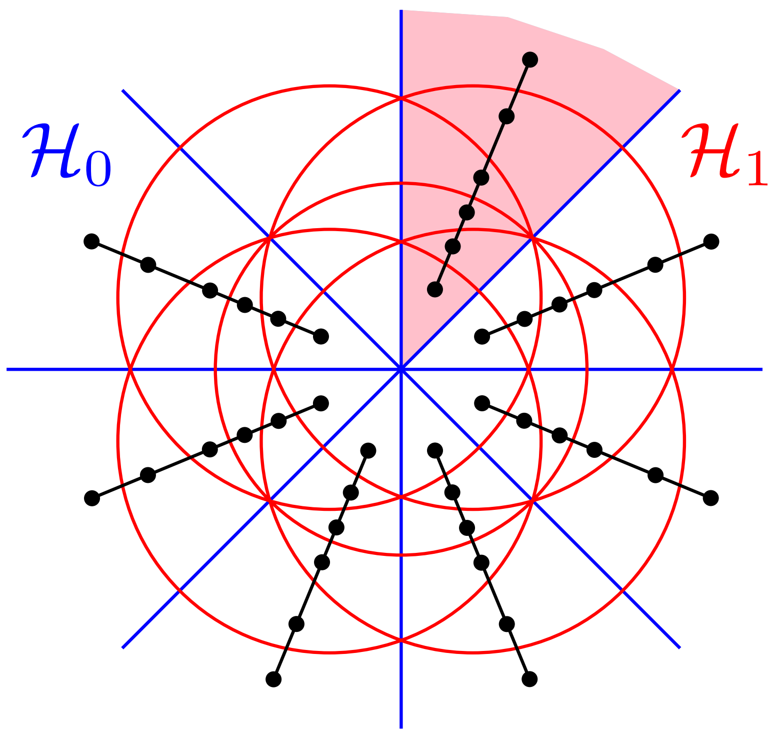


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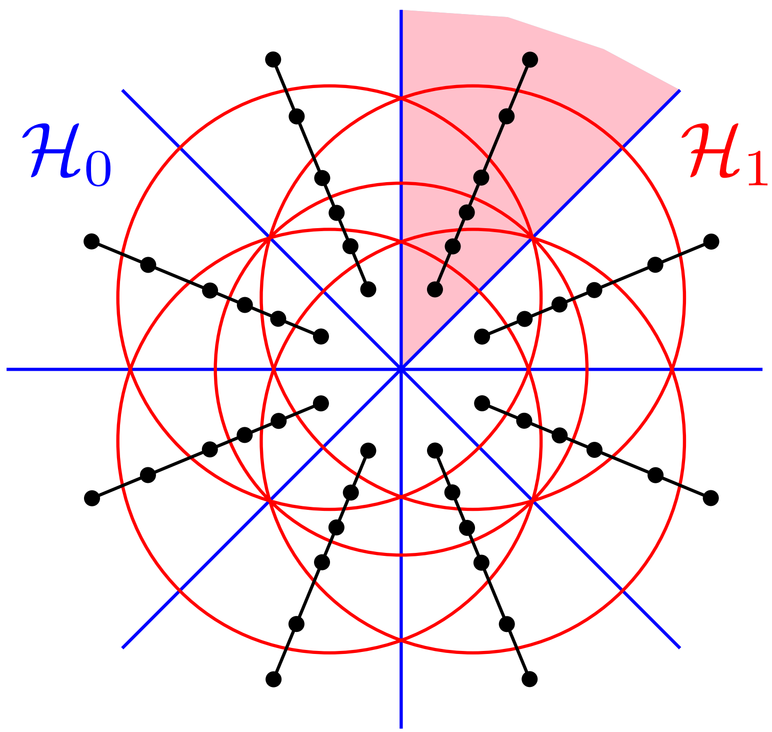

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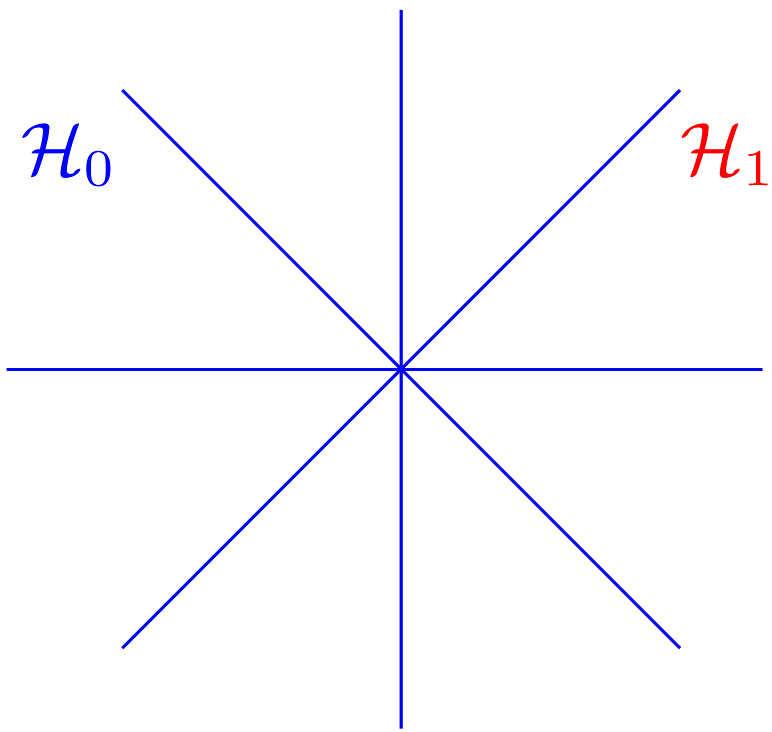


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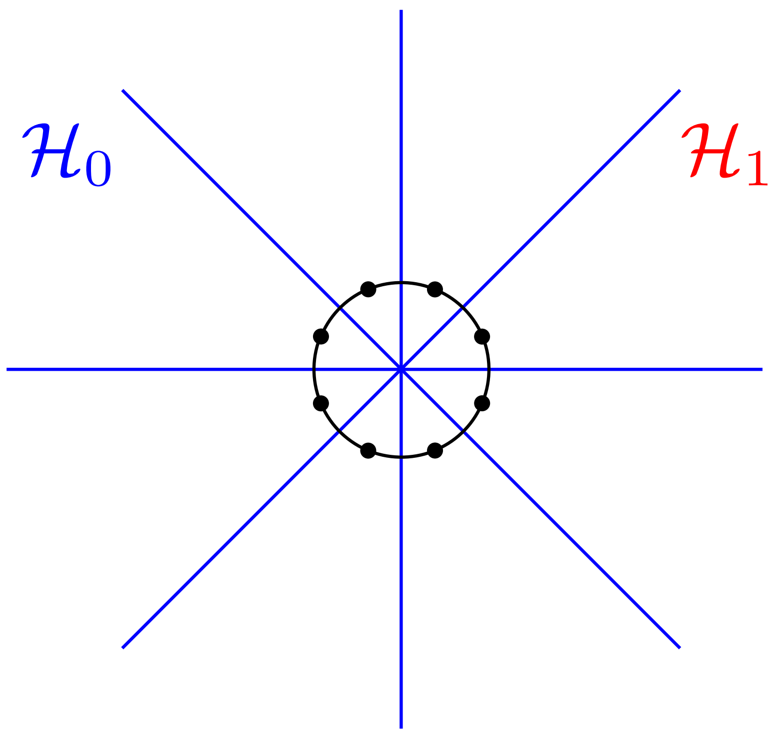


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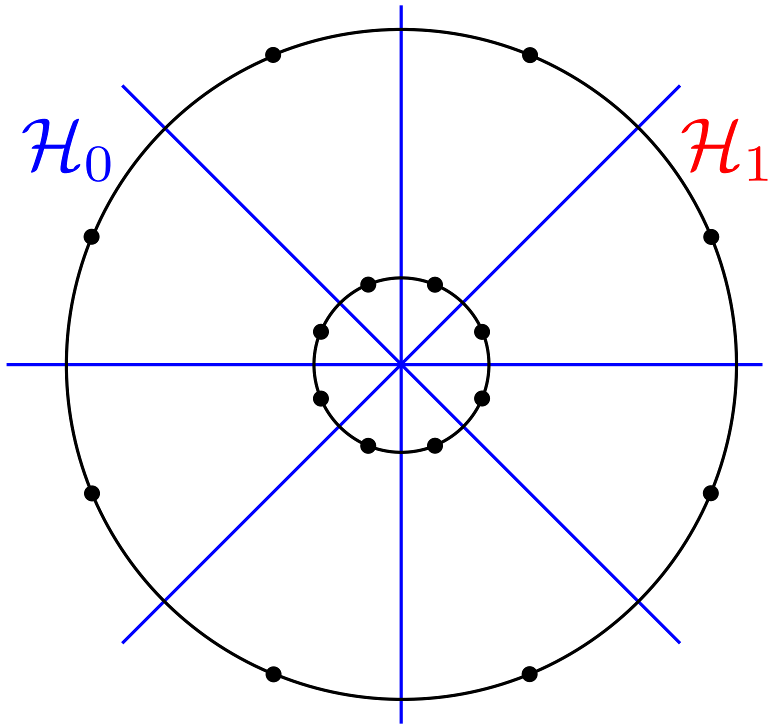


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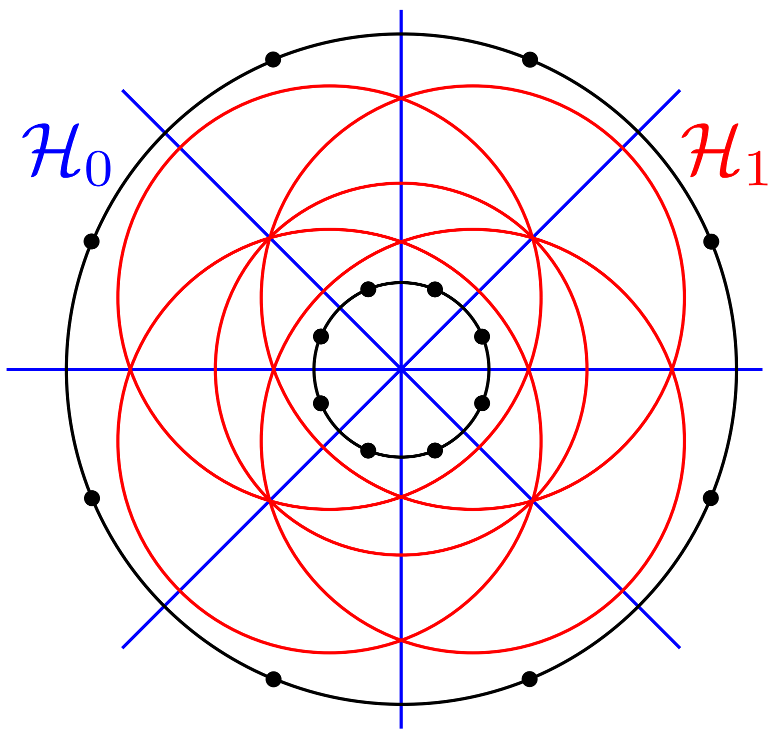


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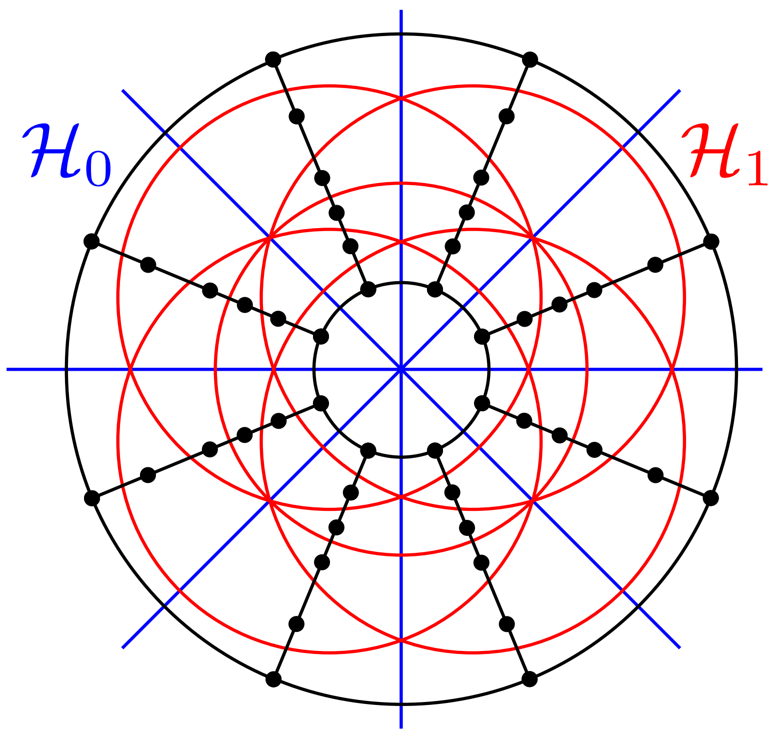


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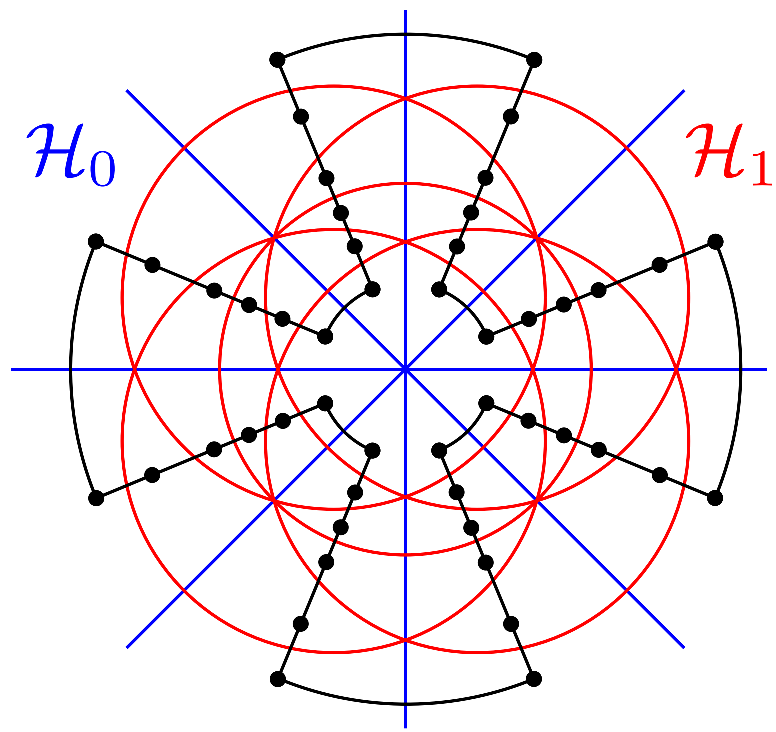


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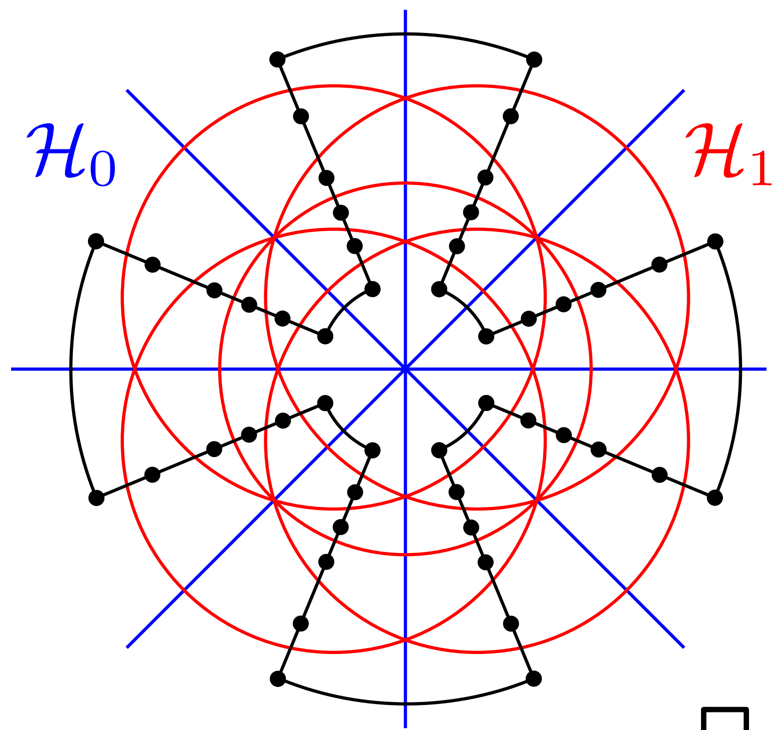


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