CONNECTING TAMENESS AND LEARNABILITY

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Abstract. While the axiomatic framework for tame geometry has been developed within the field of Model Theory, several key results, such as the o-minimality of the real exponential field, have had an impact on other areas of mathematics. One such application concerns the Fundamental Theorem of Statistical Learning, which characterises PAC learnability (a theoretical underpinning for classification problems in Machine Learning) of hypothesis classes in terms of their finite VC dimension. It was first observed in [2] that a finite VC dimension is always obtained if the hypothesis class is uniformly defined over an NIP structure. Since o-minimality implies NIP, a common fallacy in this context is to deduce that any hypothesis class uniformly definable over an o-minimal structure, such as the real exponential field, is automatically PAC learnable. This seemingly straightforward argument overlooks the fact that PAC learnability requires more than just a finite VC dimension, namely the measurability of several sets and functions arising from the hypothesis class.

In my talk, I will present the model theoretic main result of joint work with Laura Wirth [1], which re-evaluates PAC learnability of hypothesis classes definable over o-minimal expansions of the reals. Thanks to o-minimal cell decomposition, some of the measurability conditions needed for PAC learnability are readily verified. This subsequently leads to the question: how much of the theory can be extended to the more general setting of NIP structures, as opposed to just o-minimal ones.

[1] L. S. Krapp and L. Wirth, 'Measurability in the Fundamental Theorem of Statistical Learning', Preprint, 2024, arXiv:2410.10243.

[2] M. C. Laskowski, 'Vapnik–Chervonenkis Classes of Definable Sets', J. Lond. Math. Soc., II. Ser. 45 (1992) 377–384.