Finding the distribution of random multiplicative functions in short intervals

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Consider the sum $\sum_{x<n\leq x+y} f(n)$ of a random multiplicative function in a short interval (where $y = o(x)$ as x tends to infinity). Thanks to work of Chatterjee—Soundararajan and Soundararajan—Xu, it is known that these sums have a Gaussian limiting distribution when rescaled by their standard deviation, provided x/y is at least a certain power of $\log x$. On the other hand, work of Harper and of Caich implies that these sums will converge to zero when rescaled by their standard deviation, if y is "close" to x. I will report on joint work (in preparation) of myself, Soundararajan and Xu on this problem. We find that on the full range $y = o(x)$, the sums have a Gaussian limiting distribution when rescaled properly, but the correct scaling factor changes as y approaches x. In contrast, when $y \asymp x$ there is no rescaling under which the sums have a (non-degenerate) Gaussian limit.