Characterizing Submodules in $H^2(\mathbb{D}^2)$ Using the Core Function

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Abstract

It is well known that $H^2(\mathbb{D}^2)$ is a RKHS with the reproducing kernel $K(\lambda, z) = \frac{1}{(1-\overline{\lambda_1}z_1)(1-\overline{\lambda_2}z_2)}$ and that for any submodule $M \subseteq H^2(\mathbb{D}^2)$ its reproducing kernel is $K^M(\lambda, z) = P_M K(\lambda, z)$ where P_M is the orthogonal projection onto M. Associated with any submodule M are the core function $G^M(\lambda, z) = \frac{K^M(\lambda,z)}{K(\lambda,z)}$ and the core operator C_M, an integral transform on $H^2(\mathbb{D}^2)$ with kernel function G^M. The utility of these constructions for better understanding the structure of a given submodule is evident from the various works in the past 20 years. In this talk we will discuss the relationship between the rank, codimension, etc. of a given submodule and the properties of its core function and core operator. In particular, we will discuss the longstanding open question regarding whether we can characterize all submodules whose core function is bounded. This is a joint project with Rongwei Yang.