Optimal Communication Unbalanced Private Set Union

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Definition

Linearly Homomorphic Encryption scheme (LHE):

- $L.D_{sk}(L.E_{pk}(m)) = m$

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Notation: $\hat{m} \leftarrow L.E_{pk}(m)$.

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Constraints

- No test (Straight-line programs).
- No division, no inversion.
- No ciphertext multiplication.

Linearly homomorphic multi-point evaluation

Construction

1. $$\begin{pmatrix} a_{11} & \cdots & a_{1m} \\ & \cdots & \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \ltimes_L \begin{pmatrix} \widehat{b_1} \\ \cdots \\ \widehat{b_m} \end{pmatrix} = \begin{pmatrix} a_{11} \ltimes_L \widehat{b_1} +_L \cdots +_L a_{1m} \ltimes_L \widehat{b_m} \\ \cdots \\ a_{n1} \ltimes_L \widehat{b_1} +_L \cdots +_L a_{nm} \ltimes_L \widehat{b_m} \end{pmatrix}$$

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2. Clear-cipher polynomial product and middle product.
$$\widehat{B} \longmapsto A \ltimes_L \widehat{B} \; ; \; \widehat{C} \longmapsto A \ltimes_L^t \widehat{C}$$

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$$\Rightarrow MEv_L : \widehat{A}, \{y_1, ..., y_m\} \longmapsto \{\widehat{A}(y_1), ..., \widehat{A}(y_m)\}$$

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Lemma

There is an algorithm computing MEv_L in $\mathscr{M}_L(m) \log m + \tilde{O}(m)$ such that:

$$L.D_{sk}\left(MEv_L\left(L.E_{pk}(A), \{y_1, ..., y_m\}\right)\right) = \{A(y_1), ..., A(y_m)\}$$

Notation: $\mathscr{M}_L(d)$ is arith. cost of LHE poly. mult. of degree $\leq d$.

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Notation: $\tilde{m} \leftarrow F.E_{pk}(m)$.

Constraints

- No test.
- No division, no inversion.
- Ciphertext multiplication ✓ (but ⚠ circuit depth).

Fully homomorphic euclidean remainder

Construction

1. $A(Z) = B(Z)Q(Z) + R(Z)$

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1. $\overleftarrow{A}(Z) = \overleftarrow{B}(Z)\overleftarrow{Q}(Z) + \overleftarrow{R}(Z)Z^{n-m+1} \Rightarrow \overleftarrow{Q} = \overleftarrow{B}^{-1}\overleftarrow{A} \mod Z^{n-m+1}$

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2. Newton method: $U_l = \overleftarrow{B}^{-1} \mod Z^{2^l}$

$$(U) = \begin{cases} U_0 & = & 1 \\ U_{k+1} & = & U_k\left(2 - U_k\overleftarrow{B}\right) \mod Z^{2^{k+1}} \end{cases}$$

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Lemma

There is an algorithm computing $\%_F$ in $\frac{9}{2}\mathscr{M}_F(n-m) + O(n)$ such that:

$$F.D_{sk}(F.E_{pk}(A)\%_F F.E_{pk}(B)) = R$$

Notation: $\mathscr{M}_F(d)$ is arith. cost of FHE poly. mult. of degree $\leq d$.

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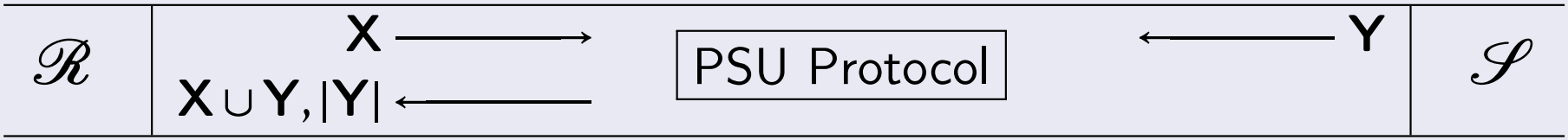

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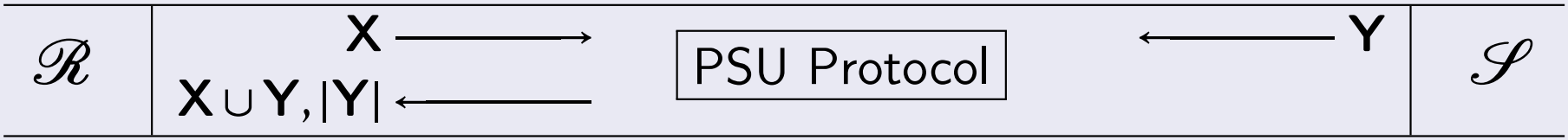
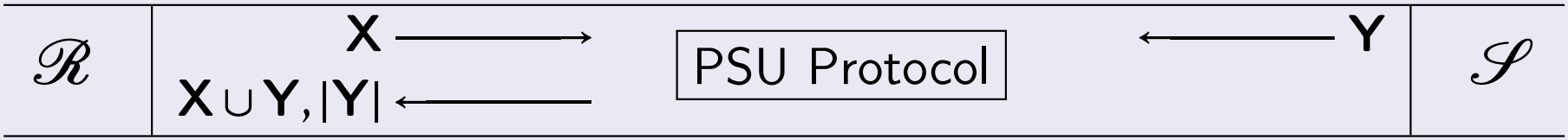


Security

In the honest-but-curious adversary model:

Unbalanced Private Set Union

Functionality



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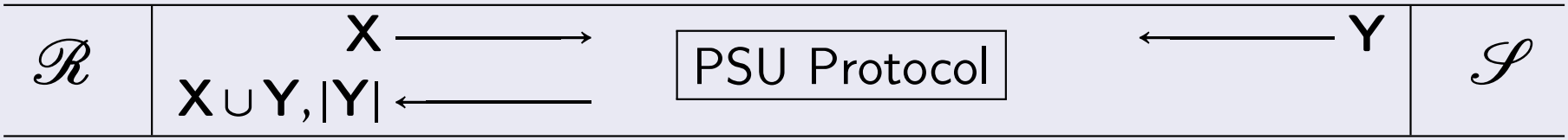
- $\mathscr{S}$ learns nothing about **X**.

Unbalanced Private Set Union

Functionality



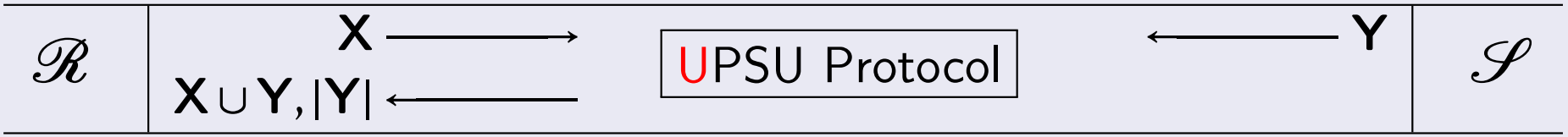
Security

In the honest-but-curious adversary model:

- $\mathscr{S}$ learns nothing about $\mathbf{X}$.
- $\mathscr{R}$ learns only $\mathbf{X} \cup \mathbf{Y}$ and $|\mathbf{Y}|$.

Unbalanced Private Set Union

Functionality



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Unbalanced situation goals

$|\mathbf{X}| = n$, $|\mathbf{Y}| = m$, $n \geq m$ (possibly $n \gg m$)

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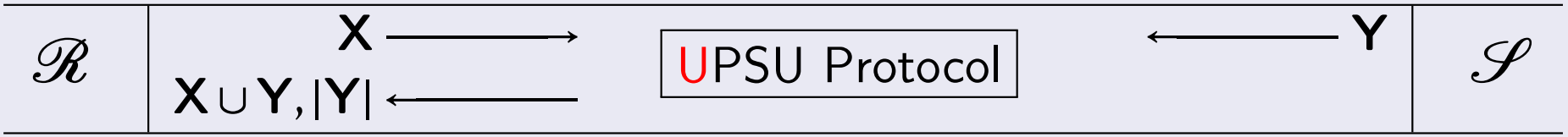

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- Communication volume proportional to m.

Unbalanced Private Set Union

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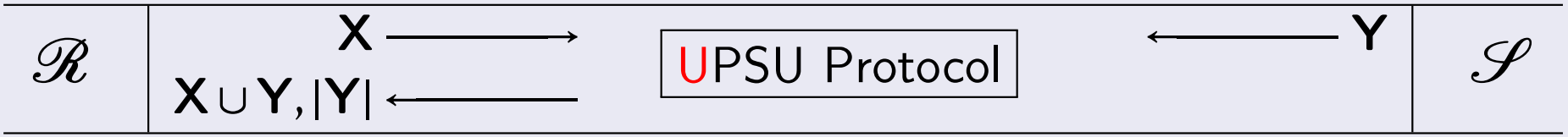

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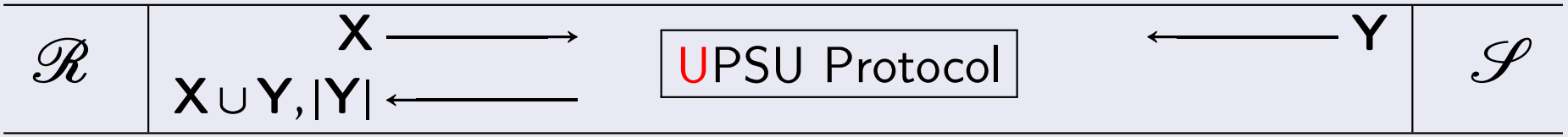

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- Communication volume proportional to m.
- Arithmetic cost for $\mathscr{S}$ independant from n.
- Arithmetic cost for $\mathscr{R}$ reasonable.

State of the art

Notations

- $\mathscr{R}$ owns $\mathbf{X} \subset \mathbb{F}$, $|\mathbf{X}| = n$.
- $\mathscr{S}$ owns $\mathbf{Y} \subset \mathbb{F}$, $|\mathbf{Y}| = m$.
- $n \gg m$.
- Comm. Vol. : "number of field element sent".
- Arith. Cost : "number of field operations".

	Comm. Vol.	Arith. Cost $\mathscr{S}$	Arith. Cost $\mathscr{R}$

State of the art

Main ideas in different protocols

- **[Frikken, 2007]**: Polynomial set representation and LHE evaluation.

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$$\rightsquigarrow \mathbf{X} \longleftrightarrow P_{\mathbf{X}}(Z) = \prod_{x \in \mathbf{X}} (Z - x)$$

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Frikken	$O(n)$	$O(nm)$	$O(n^{1+\epsilon})$

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Main ideas in different protocols

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- **[Davidson and Cid, 2017]**: Bloom filter set representation and LHE sum.

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Dav. & Cid	$O(n)$	$O(m)$	$O(n)$

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- **[Frikken, 2007]**: Polynomial set representation and LHE evaluation.
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	Comm. Vol.	Arith. Cost $\mathscr{S}$	Arith. Cost $\mathscr{R}$
Frikken	$O(n)$	$O(nm)$	$O(n^{1+\epsilon})$
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Construction of our protocol

Notations

- $\mathscr{R}$ owns $\mathbf{X} \subset \mathbb{F}$, $\mathscr{S}$ owns $\mathbf{Y} \subset \mathbb{F}$.
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$$\widehat{R}(y) = \widehat{0} \Leftrightarrow y \in \mathbf{X};$$

Optimal communication UPSU protocol

$\mathscr{R}$ $(\mathbf{X} = \{x_1, ..., x_n\}, pk_F, sk_L, pk_L)$

$P_{\mathbf{X}} = \prod(Z - x_i)$

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$P_{\mathbf{X}} = \prod(Z - x_i)$		$P_{\mathbf{Y}} = \prod(Z - y_i)$
	$\xleftarrow{\widetilde{P_{\mathbf{Y}}}}$	$\widetilde{P_{\mathbf{Y}}} \leftarrow F.E_{pk_F}(P_{\mathbf{Y}})$

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$\widetilde{P_{\mathbf{Y}}}$ ⟵

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M random mask	$\xleftarrow{\widetilde{P_{\mathbf{Y}}}}$	$\widetilde{P_{\mathbf{Y}}} \leftarrow F.E_{pk_F}(P_{\mathbf{Y}})$
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$\widetilde{H} \leftarrow \widetilde{R} +_F \widetilde{M}$	$\xrightarrow{\widetilde{H}, \widehat{M}}$	

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$\widetilde{H} \leftarrow \widetilde{R} +_F \widetilde{M}$	$\xrightarrow{\widetilde{H}, \widehat{M}}$	$H = R + M \leftarrow F.D_{sk_F}(\widetilde{H})$ $\{h_i\} \leftarrow MEv(H, \mathbf{Y})$ $\{\widehat{m_i}\} \leftarrow MEv_L(\widehat{M}, \mathbf{Y})$

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$\begin{cases} r_i = 0 & \Rightarrow \perp \\ r_i \neq 0 & \Rightarrow y_i = \rho_i r_i^{-1} \end{cases}$		

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Comparison to the state of the art

Our protocol Asymptotic

- Communication volume : $O(m)$.
- Arithmetic cost for $\mathscr{S}$: $\mathscr{M}_L(m)\log m + \tilde{O}(m) \in O(m^{1+\epsilon})$.
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Tu et al.	$O(m\log n)$	$O(m^{1+\epsilon})$	$O(n^{1+\epsilon})$	leaky
Ours	$O(m)$	$O(m^{1+\epsilon})$	$O(n^{1+\epsilon})$	✓

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Thank you for your attention !
Any question ?