Faster algorithms for connectivity queries in unbounded real algebraic sets

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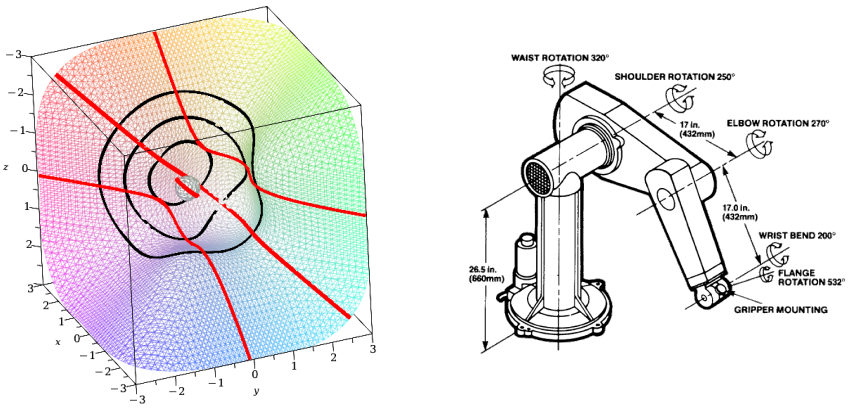
We address the problems of counting the number of connected components of sets of real solutions to systems of polynomial equations with real coefficients and of answering connectivity queries over such real solution sets. These problems are central in real algebraic geometry and find applications in robotics.

The chosen framework is the one of roadmaps, introduced by J. Canny in 1988: it consists in computing a curve, included in the solution set under consideration, which has a connected intersection with all its connected components. Describing the connectivity properties over the solution set is then reduced to describe the connectivity of the roadmap.

We design an algorithm which, under some regularity assumptions which are satisfied generically, computes such roadmaps in time subquadratic w.r.t. the output size. This latter quantity is *nearly optimal*. More precisely, it has output size $N = (n^2D)^{4n\log(d)+O(n)}$ and complexity $N^{1.5}$, where n is the number of unknowns, D the maximum degree of the defining equations of the set under consideration and d is its dimension. *This extends to non-compact situations the best complexity results known for such a computational problem.* We also show that, analyzing the connectivity of the roadmap (that is a curve), can be done in complexity N^3.

We will finish by an application of a prototype implementation of roadmap algorithms, to the analysis of the singularities of a PUMA-type robot. This has benefited from recent Gröbner bases softwares improvements.

This talk gather joint works with Md N. Islam, A. Poteaux, M. Safey El Din and É. Schost.



On the left, a roadmap of an unbounded solution-set; on the right a PUMA-type robot.