Computing Generic Fibers of Polynomial Ideals Using FGLM and Hensel Lifting

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A *generic fiber* of a polynomial ideal I is the fiber over the generic point under a projection of I to an affine space of the same dimension as that of I. This generic fiber is given concretly as a zero-dimensional polynomial ideal over a function field. A Gröbner basis of such a generic fiber is classically computed using elimination monomial orders. Out of it, one can extract information about various decompositions of I, such as equidimensional decompositions or primary decompositions.

We give a new algorithm to compute these generic fibers, combining the FGLM algorithm, which computes a Gröbner basis for some monomial order given a Gröbner basis w.r.t. another monomial order, with a Hensel Lifting strategy. Similarly to classical Hensel Lifting, the algebraic complexity of our algorithm is quasi-linear in the number of lifting steps. Practical experiments with a proof-of-concept implementation show the efficiency of our algorithm. We also sketch another method to compute generic fibers, combining the F4 algorithm with so-called *tracers* and optimized modular lifting of LU-factorizations of matrices, and outline how both algorithms could be profitably combined.

This is joint work with Jérémy Berthomieu.