On Bounding The Degree Of Irreducible Darboux Polynomials

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Given a polynomial ordinary differential equation (ODE), we devise generic polynomial reduction algorithms to investigate the intertwined relationship between the degree of (nontrivial) Darboux polynomials and the polynomials defining the ODE. We further present a light-weight abstraction to reduce, in a principled way, the existence of irreducible Darboux polynomials to a mixed optimization problem over the integers and the constants of the selected field. We illustrate how our procedure can be used to restrict the shape of Darboux polynomials, pinpointing which monomials ought to be preserved by removing the superfluous ones. This in turn could be relevant to speed up already existing generation algorithms. In the best case, we show that we are able to certify in full generality the nonexistence of Darboux polynomials. We give in particular a new tool to prove (and formally certify) the fact that the limit cycle of the Van der Pol oscillator is not algebraic.