

Decoding Simultaneous Rational Evaluation Codes

Matteo Abbondati, Eleonora Guerrini and Romain Lebreton

We deal with the problem of simultaneous reconstruction of a vector of rational numbers, given modular reductions containing errors. Our methods apply as well to the simultaneous reconstruction of rational functions given evaluations containing errors. In the latter case, one can take advantage of techniques from coding theory and provide an algorithm that extends classical Reed-Solomon decoding. In recent works, interleaved Reed-Solomon codes are used to correct beyond the unique decoding capability in the case of random errors at the price of positive but small failure probability [2]. Our first contribution is to extend these works to the simultaneous reconstruction with errors of rational numbers instead of functions. Thus, considering rational number codes [1], we provide an algorithm decoding beyond the unique decoding capability and, we analyze in detail its failure probability. Our analysis generalizes for the first time the best known analysis for interleaved Reed-Solomon codes [2] to the problem of simultaneous rational function reconstruction with errors, improving on the existing bound [3], to interleaved Chinese remainder codes, also improving the known bound [4], and finally for the first time to the simultaneous reconstruction of rational numbers with errors.

References

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