ERAPs: state of art in 1d and future perspectives

Matteo D'ACHILLE

Random Geometry–Géométrie Aléatoire, CIRM Luminy, 16 – 20 January, 2023

Abstract

An Euclidean Random Assignment Problem (ERAP) [4] is the study of the random optimal energy of assigning n points (blues) to n points (reds) randomly distributed on a metric space (Ω, D) depending on the choice of (Ω, D), on the law(s) of the points and on an energy-distance exponent $p \in \mathbb{R}$.

ERAPs have attracted interest over the past forty years since they are toy models of finite-dimensional spin-glasses (Mézard–Parisi [5]) and are discrete versions of the continuous Monge–Kantorovich problem in transportation theory (Villani [6], Brezis [1]). However, in spite of such a simple formulation, ERAPs remain comparatively little understood.

After quoting some selected results for general (Ω, D), in this talk I will first focus on "one-dimensional" cases, such as $\Omega =$ the real line, $\Omega =$ the unit circle or $\Omega =$ a tree, endowed with $D =$ their natural geodesic distances. I will then show how the value of p characterizes the combinatorial properties of the optimal assignment, providing a simple description of the scaling limit of the solution in terms of Brownian processes if $p \geqslant 1$.

Finally, I will propose two research questions related to $1d$ ERAPs: a self-contained problem related to functionals of Brownian processes and some signature of the scaling limits of the solution for $p < 1$. The general reference for both problems is [4], Chapter 2 (available on this link); the references for the two problems are respectively [2] and [3].

References

[1] H. Brezis. Remarks on the Monge–Kantorovich problem in the discrete setting. *Comptes Rendus Mathematique*, 356(2):207–213, 2018.

[2] S. Caracciolo, M. D'Achille, and G. Sicuro. Random Euclidean matching problems in one dimension. *Physical Review E*, 96(4):042102, oct 2017.

[3] S. Caracciolo, M. P. D'Achille, V. Erba, and A. Sportiello. The Dyck bound in the concave 1-dimensional random assignment model. *Journal of Physics A: Mathematical and Theoretical*, 53(6):064001, feb 2020.

[4] M. D'Achille. *Statistical Properties of the Euclidean Random Assignment Problem.* Theses, Université Paris-Saclay, oct 2020.

[5] M. Mézard and G. Parisi. The euclidean matching problem. *Journal de Physique*, 49(12):2019–2025, 1988.

[6] C. Villani. *Optimal Transport, old and new*, volume 338 of *Grundlehren der mathematischen Wissenschaften.* Springer Berlin Heidelberg, Berlin, Heidelberg, springer edition, 2009.