Title: Entropy, pressure, and densities for $\mathbb{Z}^d$-SOFT

Abstract: 1. Subshift of finite type (SOFT) on $\mathcal{S} \subset [n]^{\mathbb{Z}^d}$ can be coded as a near neighbor SOFT (NNSOFT). Here $n \geqslant 2$ is the number of particles, and $[n] = \{1, \ldots, n\}$. For $d = 1$ NNSOFT is a binifinite walk on a digraph $\vec{G} = ([n], \vec{E})$. If $\vec{G}$ is an undirected graph G, then we call $\vec{G}$ symmetric. For $d \geqslant 2$ NNSOFT is given by d digraphs $\vec{G}_1, \ldots, \vec{G}_d$. That is, on a line in direction the coordinate i through a point with integer coordinates in $\mathbb{R}^d$, the location of particles in $[n]$ correspond to a binifinite walk on $\vec{G}_i$. For the case $d = 2$ we will discuss a simple equivalence of NNSOFT and the Wang tiling.
2. Next we discuss three kinds of entropies: combinatorial entropy (capacity), topological entropy, and periodic entropy. Capacity=topological entropy, denoted as $h_{\mathcal{S}}$. We give upper bounds on capacity, which in limit gives the capacity. For general d capacity=periodic entropy if at least $d-1$ of the digraphs $\vec{G}_1, \ldots, \vec{G}_d$ are symmetric. For nonperiodic Wang tiling this result does not apply. If time permits, we will bring some examples from statisitical mechanics for SOFT.
3. We define a pressure function $P_{\mathcal{S}}(\mathbf{u})$ on $\mathbb{R}^n$. It is convex and and with Lipschitz constant $\leqslant 1$. A probability vector $\mathbf{p} = (p_1, \ldots, p_n)^\top$ is a density point of $\mathcal{S}$, if it is a limit point of densities of the n particles on an increasing sequence of boxes in $\mathbb{Z}^d$. Denote by $\Pi_{\mathcal{S}}$ the set of density points of $\mathcal{S}$. This allows to define the entropy $h_{\mathcal{S}}(\mathbf{p})$. One has the maximal characterization: $P_{\mathcal{S}}(\mathbf{u}) = \max_{\mathbf{p} \in \Pi_{\mathcal{S}}} (\mathbf{p}^\top \mathbf{u} + h_{\mathcal{S}}(\mathbf{p}))$. Hence, $h_{\mathcal{S}} = P_{\mathcal{S}}(\mathbf{0}) = \max_{\mathbf{p} \in \Pi_{\mathcal{S}}} h_{\mathcal{S}}(\mathbf{p})$. There are nice connections between $h_{\mathcal{S}}(\mathbf{p})$ and the conjugate convex function $P^*_{\mathcal{S}}$ (the Legendre-Fenchel function). A first order phase transmission can be attributed to a jump in corresponding directional derivative of the pressure function.

References

[1] S. Friedland, Multi-dimensional capacity, pressure and Hausdorff dimension, Mathematical System Theory in Biology, Communication, Computation and Finance, edited by D. Gilliam and J. Rosenthal, IMA vol. 134, Springer, 2003, 183-222.

[2] S. Friedland and U. Peled,The pressure, densities and first order phase transitions associated with multidimensional SOFT, jointly with U.N. Peled, Notions of Positivity and the Geometry of Polynomials, Trends in Mathematics, 179-220, 2011, Springer, Basel AG.