Title: Directional Expansiveness for tiling dynamical

Expansiveness is an important property in topological dynamics. A homeomorphism f of a compact metric space (X, d) (or equivalently, a continuous $\mathbb{Z}$-action on (X, d)) is ***expansive*** if there exists $\delta > 0$ so if $d(f^n(x), f^n(y)) < \delta$ for all $n \in \mathbb{N}$, then $x = y$. For $d > 1$, however, the case of $\mathbb{Z}^d$-actions offers additional possibilities. Expansiveness for flows ($\mathbb{R}$-actions) was first studied by Bowen and Walters [1]. The definition of expansive flow easily generalizes to $\mathbb{R}^d$-actions, and was studied under the name of strong expansiveness by Frank and Sadun [2], who also introduced a weaker notion that they called ***weak expansiveness.*** We adopt Frank and Sadun's terminology, studying both weak and strong expansiveness for $\mathbb{R}^d$-actions where which is the natural generalization of Bowen and Walters expansiveness for flows. Also, they show for FLC tiling dynamical systems weak expansiveness is the same as strong expansiveness.

In this presentation, we study directional version of Frank and Sadun's weak and strong expansiveness for $\mathbb{R}^d$-actions which are the same for FLC tiling dynamical systems. Once the basic theory of directional expansiveness for $\mathbb{R}^d$ actions is established, we move to our main topic: the expansive directions for Penrose tiling dynamical systems and its generalizations. We show that the Penrose tiling dynamical system has five directions that are not weakly and thus not strongly expansive: the directions parallel to the 5th roots of unity. These are the directions with Conway "worms". We show that all other directions are strongly expansive.

References

[1] Bowen, Rufus; Walters, Peter, Expansive one-parameter flows, ***Journal of differential Equations***, 12, (1972), no. 1, 180-193.

[2] Priebe Frank, Natalie; Sadun, Lorenzo Fusion tilings with infinite local complexity. Topology Proc. 43 (2014), 235âĂŞ276.