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Title: On the Fractional and Integral Parts of Geometric Progression

Abstract: Let $\xi \neq 0$ and $\alpha > 1$ be two real numbers. We review some results concerning the distribution of the sequence $\xi\alpha^n$, $n = 0, 1, 2, \ldots$, which is an infinite geometric progression. The distribution of this sequence modulo 1 and the divisibility properties of its integral parts $\lfloor \xi\alpha^n \rfloor$ have been investigated for a long time. There is almost no progress in both problems for transcendental numbers α (except for the metrical results of Weyl (1916) and Koksma (1935)), so we will only consider algebraic numbers α. Although the sequence of fractional parts should be nicely (in fact, uniformly) distributed for 'most' ξ and α, its behaviour can be quite chaotic for some pairs ξ, α. In particular, some large regions of the interval $[0, 1]$ can be free from the elements of this sequence at all. We are mainly interested in these two problems when α is a fixed rational number. Then, the main open question related to the sequence of fractional parts is that of Mahler (1968) when $\alpha = 3/2$ and ξ is a positive real number. In the case of the sequence of integral parts $\lfloor \xi\alpha^n \rfloor$, $n = 0, 1, 2, \ldots$, the divisibility questions become nontrivial already for rational integers $\alpha \geq 2$.