Jean-François Dat

Finiteness properties of Hecke algebras of p-adic groups.

Abstract : if K is a compact open subgroup of a p-adic group G, the fact that any double K-coset in G is the union of finitely many left K-cosets allows one to define the Hecke ring $H(G,K)$ of the pair (G,K). In the most crucial case where K is hyperspecial, this ring is commutative and finitely generated. In the next most interesting cases (Iwahori subgroup and its pro-radical), several presentations of this ring are known, from which one can prove that it is finitely generated as a module over its center and that the latter is a finitely generated ring. It is very plausible that this property remains true for any K, although it seems hard to deduce from the explicit generators and relations that are known e.g. for congruence subgroups of Iwahori subgroups. On the other hand, Bernstein proved such a result after extending scalars to $\mathbf{C}$, using representation theoretic methods. Unfortunately, Bernstein's approach does not work if one replaces $\mathbf{C}$ by a field of positive (non banal) characteristic. In this talk, I will explain a proof of these finiteness properties of $H(G,K)$ after "only" inverting p, using Fargues and Scholze geometric constructions, and some of Bernstein's old ideas. This is joint with Helm, Kurinczuk and Moss.