Detecting Rigid Symmetries in Digital Objects: The case of 2D gray-scale images

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A gray-scale digital image is defined by gray scale values $\omega_i \in \mathbb{R}_{\geq 0}$ at pixel positions $p_i \in \mathbb{R}^d$, $i = 1, \ldots, n$. We consider the problem of determining if such an image is unchanged after some rotation or reflection. A solution is presented for the two dimensional case $d = 2$. Our solution involves constructing a hierarchy of polynomial representations, using the radon transform, to represent the image at different levels of details. The coefficients of these polynomials correspond to the Discrete Fourier Transform coefficients of the projection of the image on a line rotating around the center of mass of the image. We find that a k-fold rotational symmetry correspond to the vanishing of all the coefficients except the ones corresponding to the period of the symmetry. Reflexion symmetry are also characterized. This work has been applied to automatically detect HAZMAT signs in smart-phone images taken by first responders.