Alexander Duals of Symmetric Simplicial Complexes and Stanley-Reisner Ideals

Uwe Nagel

It is known that any ascending chain $(I_n)_{n\in\mathbb{N}}$ of related squarefree monomial ideals, where I_n is invariant under the action of the symmetric group $Sym(n)$ on n letters, enjoys strong stabilization properties. For example, there are finitely many polynomials whose $Sym(n)$-orbits generate I_n if n is sufficiently large. We discuss properties of the corresponding chain of Alexander duals $(I_n^\vee)_{n\in\mathbb{N}}$. It does not have the same stabilization properties. However, it turns out the minimal generating set of $I_n^\vee$ can be described explicitly and that the number of orbit generators is given by a polynomial in n for sufficiently large n. As an application, one obtains that, for each $i \geq 0$, the number of i-dimensional faces of the associated Stanley-Reisner complexes of I_n is also given by a polynomial in n for large n. The needed arguments include a novel combinatorial tool, which we call *avoidance up to symmetry*, and methods from discrete geometry for counting lattice points in polyhedra. The talk is based on joint work with Ayah Almousa, Kaitlin Bruegge, Martina Juhnke-Kubitzke and Alexandra Pevzner.