Symmetry reduction to optimize a graph-based polynomial from queuing theory

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For given integers n and d, both at least 2, we consider a homogeneous multivariate polynomial of degree d in variables indexed by the edges of the complete graph on n vertices and coefficients depending on cardinalities of certain unions of edges.

Cardinaels, Borst, van Leeuwaarden [Oper. Res. Lett. 50 (2022) 699–706] conjectured that the polynomial (which arises in a model of job-occupancy in redundancy scheduling) attains its minimum over the standard simplex at the uniform probability vector. Brosch, Laurent and Steenkamp [SIAM J. Optim. 31 (2021) 2227–2254] showed that the polynomial is convex if $d = 2$ and $d = 3$, which implies that the conjecture holds for these d.

We use elementary representation theory to show that for fixed d, the polynomial is convex (for all n at least 2) if and only if a constant number of constant matrices (with size and coefficients independent of n) are positive semidefinite. This result is then used in combination with a computer-assisted verification to show that the polynomial is convex for d at most 9.

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