Some simple conditions for entire functions to have only real zeros

Thu Hien Nguyen

The Laguerre–Pólya class is a special class of entire functions, which appears to be the analytic closure of sets of univariate hyperbolic polynomials. We present some simple necessary and sufficient conditions for entire functions to belong to the Laguerre–Pólya class, or to have only real zeros, in terms of their Taylor coefficients. For an entire function $f(z) = \sum_{k=0}^{\infty} a_k z^k$, we define the second quotients of Taylor coefficients as $q_n(f) := \frac{a_{n-1}^2}{a_{n-2}a_n}$, $n \geq 2$ and find conditions on $q_n(f)$ for f to belong to the Laguerre–Pólya class. We also discuss the operators that preserve the Laguerre–Pólya class and its relation to the generating functions of totally positive sequences. This is joint work with Anna Vishnyakova.