Isogeny-based cryptography on the (abelian) surface

Yan Bo **Ti**, DSO Singapore

DSO Singapore

8 June 2023

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What to do with your life when SIDH breaks?

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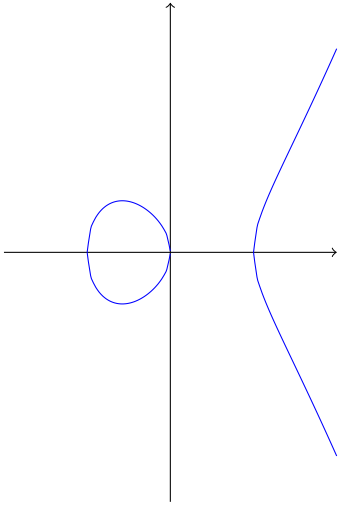
- Arithmétique et Géométrie
  - Hyperelliptic curves
  - Isogenies
  - Isogeny graphs
- Cryptographie
  - Hash function
  - Cryptanalysis of hash function
  - State of genus-two isogeny cryptography
  - Cryptanalysis of FESTA
  - What is next?

Arithmétique et Géométrie

An *elliptic curve* E is a curve in $\mathbb{P}^2(k)$ given by

$$E : y^2 = \text{cubic in } x$$



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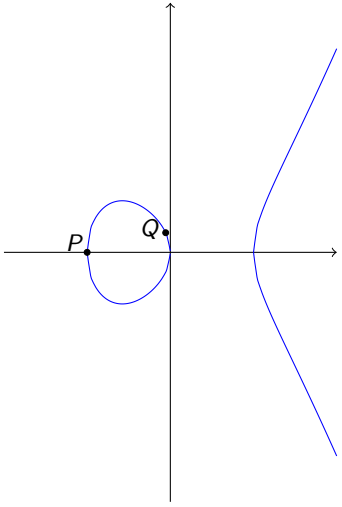

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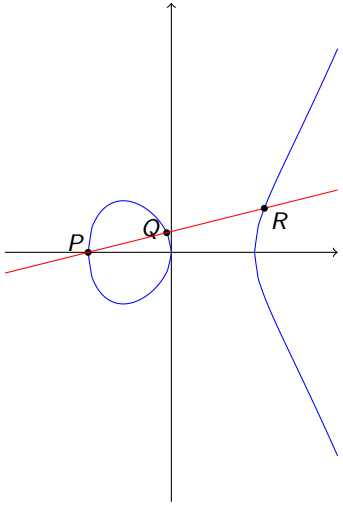

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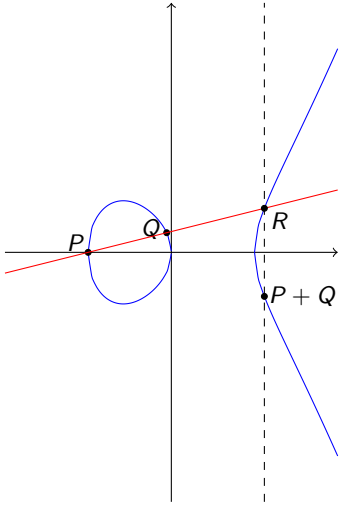

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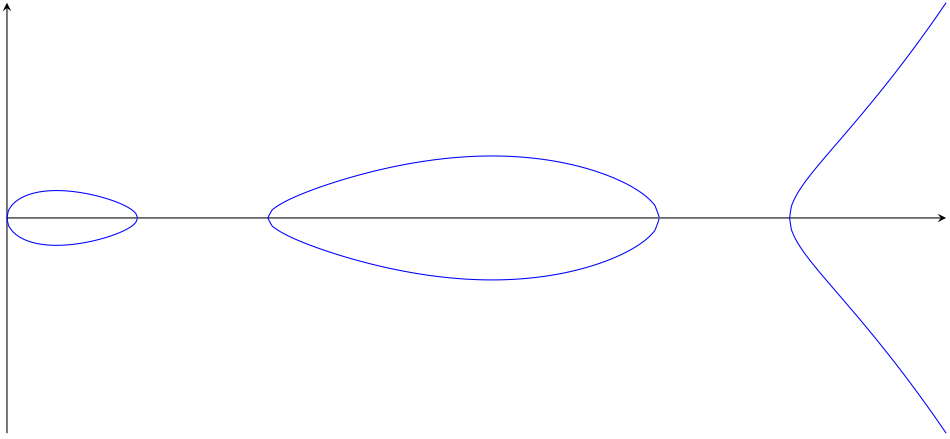

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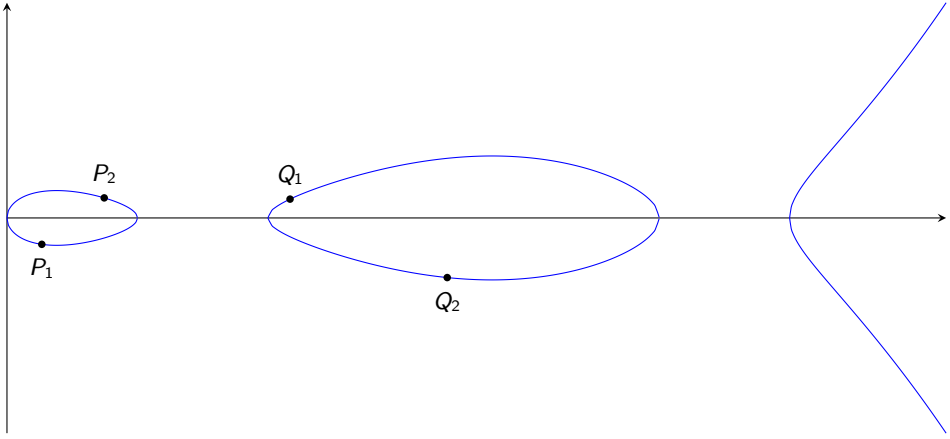

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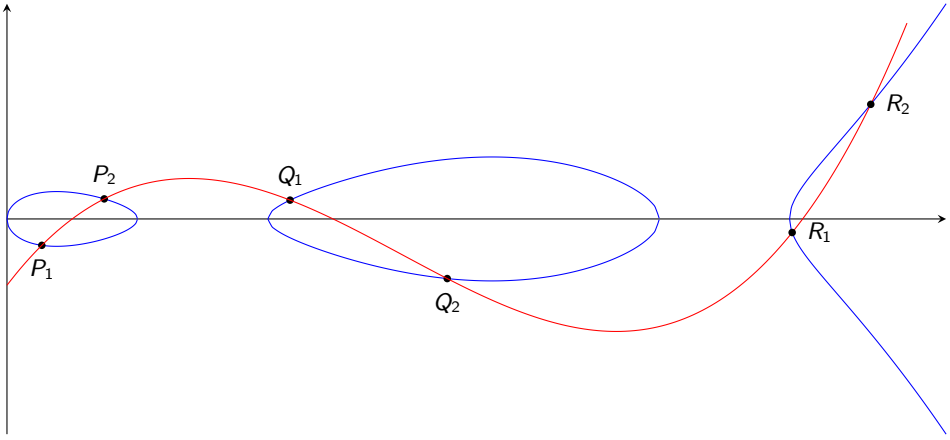

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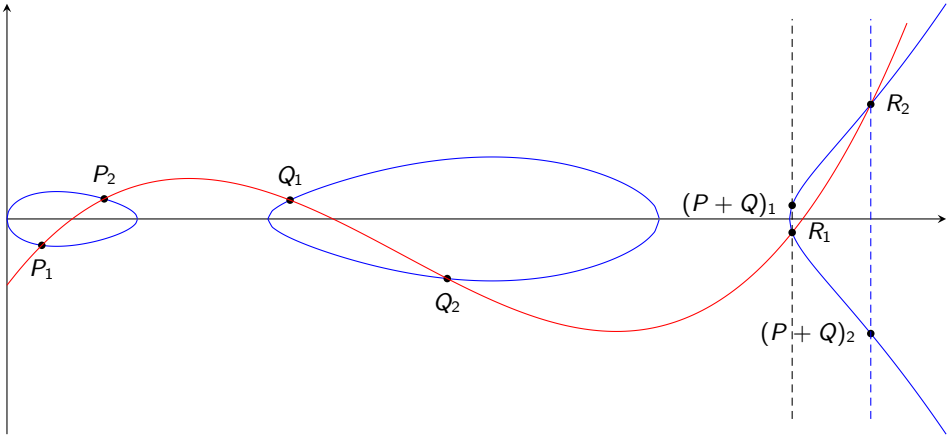

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Theorem

The map

$$\sigma : \mathrm{Pic}^0(E) \to E$$
$$D \sim (P) - (\mathcal{O}) \mapsto P$$

is an isomorphism.

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- We want to preserve this.
- There are other properties one can choose to keep that can be useful for other cryptographic schemes.

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Vertices:

Edges:

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ℓ-Isogeny graphs:

Vertices: Isomorphism classes of PPASs

Edges: (ℓ, ℓ)-isogenies

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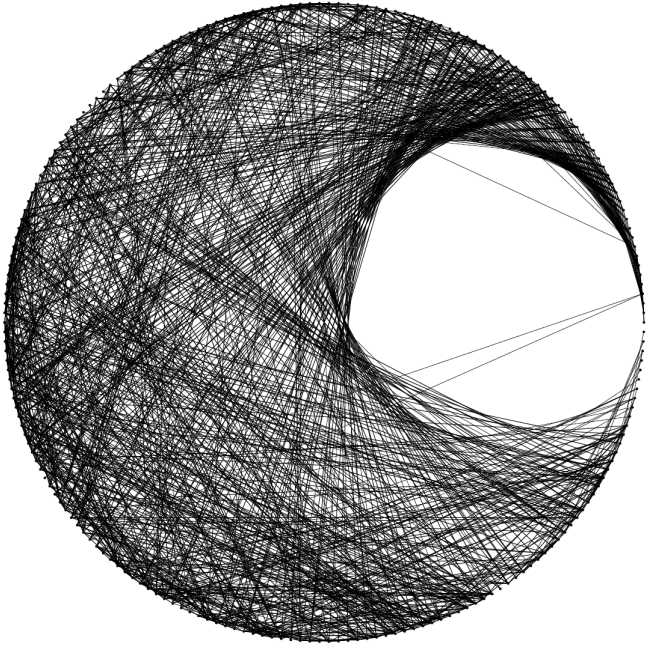

Cryptographie

- Hash function

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- Preview of hash function on isogeny graph:

Input	String of bits.
Ouput	Vertex on graph.
Method	Use input as random walk with random starting vertex.

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Proposition

Let H be a hyperelliptic curve of genus 2 over $\mathbb{F}_q$. Let K be a finite, non-trivial, $\mathbb{F}_q$-rational subgroup of $J_H(\mathbb{F}_q)$. There exists a PPAS A over $\mathbb{F}_q$, and an isogeny $\phi : J_H \to A$ with kernel K, if and only if K is a maximal ℓ-isotropic subgroup of $J_H[\ell]$ for some positive integer ℓ.

- Isogenies can be studied by looking at their kernels.

Proposition ([FT19])

Let A be a PPAS. The maximal ℓ^n-isotropic subgroups of $A[\ell^n] \cong C_{\ell^n}^4$ are isomorphic to

$$C_{\ell^n} \times C_{\ell^n} \quad \text{or} \quad C_{\ell^n} \times C_{\ell^{n-k}} \times C_{\ell^k} \quad \text{or } A[\ell^m]$$

where $1 \leq k \leq \lfloor n/2 \rfloor$ and $m \leq n$.

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Proof.

If K is cyclic, then $K \cong C_\ell \subseteq C_\ell \times C_\ell$, hence not maximal.
For rank 2: Use maximality of subgroups.
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How does structure of subgroup affect isogenies?

Proposition ([FT19])

Let $\mathcal{G}_{p,\ell}$ be the (ℓ,ℓ)-isogeny graph of PPAS over $\overline{F}_p$. Then the number of elements in the n-sphere, where $n > 2$, centred around an arbitrary vertex is

$$\ell^{2n-3}(\ell^2+1)(\ell+1)\left(\ell^n + \ell\frac{\ell^{n-2}-1}{\ell-1}+1\right)$$

if n is even, and

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Proof.

- Count number of ℓ^n-maximal isotropic subgroups.
- Sum them together.

□

- Fix primes p and ℓ, and a PPAS A.
- Consider kernel $K \subseteq A[\ell^n]$, i.e. fix a ℓ^n-maximal isotropic subgroup.
- How many ways can we get from $A \to A/K$?

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The key observation is that the number of $C_\ell \times C_\ell$ isotropic subgroups of K corresponds with the number choices for the first isogeny.

- Fix p, and a PPAS A.
- Let $\ell = 2$ and let $K = \langle P, Q, R \rangle \cong C_4 \times C_2 \times C_2$.
- K has order 16, so we expect $A \to A/K$ to be a sequence of 2 $(2,2)$-isogenies.

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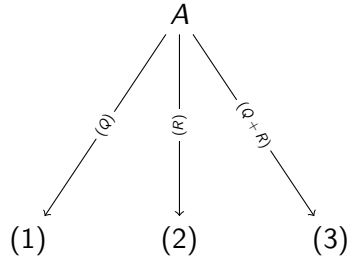
- First step: $\langle [2]P, Q \rangle$, $\langle [2]P, R \rangle$, $\langle [2]P, Q + R \rangle$.



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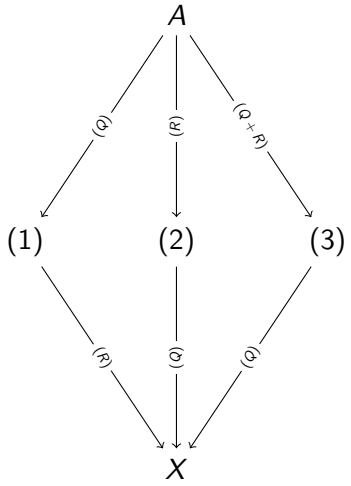
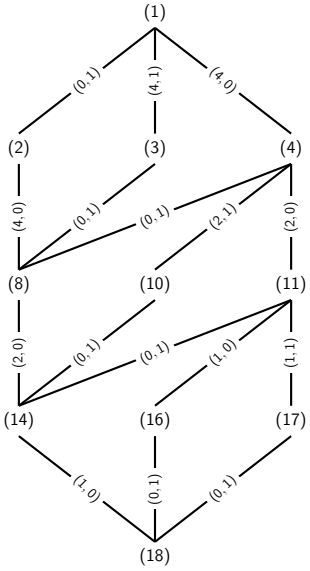
- First step: $\langle [2]P, Q \rangle$, $\langle [2]P, R \rangle$, $\langle [2]P, Q + R \rangle$.
- Second step: No choices.



Example: $C_{16} \times C_8 \times C_2$



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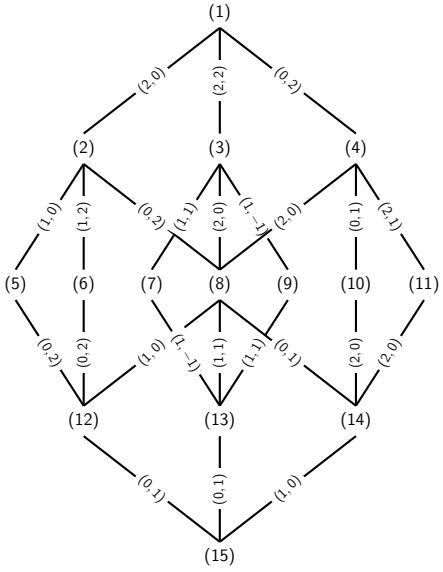

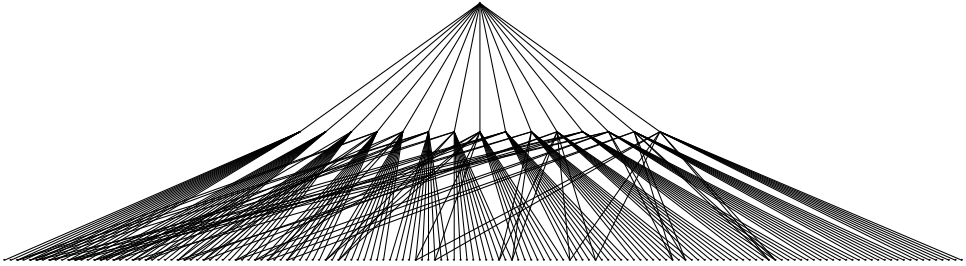

Proposition ([FT19])

Let $P(n, a)$ be the number of paths in a $(C_{\ell^n} \times C_{\ell^{n-a}} \times C_{\ell^a})$-isogeny. Then $P(n, a)$ satisfies the following recursive equation:

$$P(n, a) = \ell P(n-1, a-1) + P(n-1, a)\,,$$

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Proof.

Similar to diamond example: consider the number of choices available as the first step, then obtain the recursive relation. □

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- SSECs have three 2-isogenies

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Problem: What happens when you hit a product?

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Genus 2 the sweet spot? Can we remove products?

Definition

Let $k = \mathbb{F}_{p^n}$, then E/k is *supersingular* if any one (hence all) of the following is true:

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Theorem (Shioda, Deligne, Oort)

Let A be an abelian variety over a field of characteristic p and of dimension $g \geq 2$, and let $E^g \to A$ be an isogeny of degree d, where E is a supersingular elliptic curve. If $p \nmid d$, then $A \cong E^g$.

Let k be a finite field of characteristic p. Consider the finite group schemes

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f	a	$A[p]$	Type	Codim.
2	0	L^2	Ordinary	0
1	1	$L \oplus I_{1,1}$	Non-ordinary	1
0	1	$I_{2,1}$	Supersingular	2
0	2	$I_{1,1}^2$	Superspecial	3

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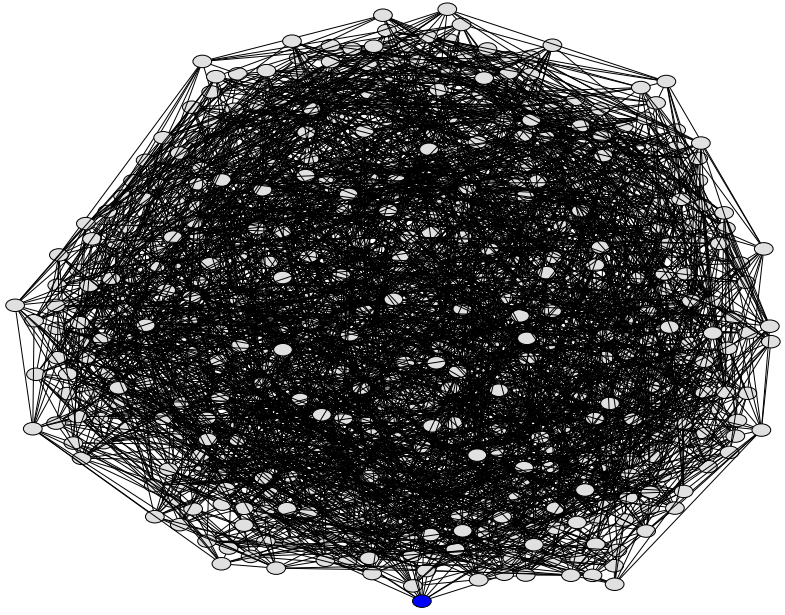
See [LTZ22] for more information.



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- Oracle model

$$O(c, m) = \begin{cases} \perp & \text{if } \mathrm{Dec}(sk, c) \neq m, \\ 1 & \text{if } \mathrm{Dec}(sk, c) = m. \end{cases}$$

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- Note that H has many generators:

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Thank you and questions?


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