

**Speaker:** Mar Curco Iranzo

**Title:** Generalised Jacobians of modular curves and their  $\mathbb{Q}$ -rational torsion

**Abstract.** The Jacobian  $J_0(N)$  of the modular curve  $X_0(N)$  has received much attention within arithmetic geometry for its relation with cusp forms and elliptic curves. In particular, the group of  $\mathbb{Q}$ -rational points on  $X_0(N)$  controls the cyclic  $N$ -isogenies of elliptic curves. A conjecture of Ogg predicted that, for  $N$  prime, the torsion of this group comes all from the cusps. The statement was proved by Mazur and later generalised to arbitrary level  $N$  into what we call generalised Ogg's conjecture.

Consider now the generalised Jacobian  $J_0(N)_{\mathbf{m}}$  with respect to a modulus  $\mathbf{m}$ . This algebraic group also seems to be related to the arithmetic of  $X_0(N)$  through the theory of modular forms. In the talk we will present new results that compute the  $\mathbb{Q}$ -rational torsion of  $J_0(N)$  for  $N$  an odd integer with respect to a cuspidal modulus  $\mathbf{m}$ . These generalise previous results of Yamazaki, Yang and Wei. In doing so, we will also discuss how our results relate to generalised Ogg's conjecture.