Solving efficiently connectivity queries on real algebraic curves

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Using the notion of roadmaps introduced by Canny in 1988, one can reduce connectivity queries in real algebraic sets of arbitrary dimension to such queries in real algebraic space curves, making this a topical problem in computational real algebraic geometry. Algorithms for computing roadmaps, on input a defining system for a real algebraic set, has been continuously improved in a series of recent works, making now tractable challenging problems in applications such as kinematic singularity analysis or cuspidality decision in robotics. As a natural continuation of these works, we focus here on the problem of answering connectivity queries on a real algebraic curve.

More precisely, we consider a curve given as the real trace of an algebraic curve, assumed to be in generic position, and being defined by some rational parametrization. The query points are given by a zero-dimensional parametrization. We design an algorithm which counts the number of connected components of the real curve under study, and decides which query point lie in which connected component, in time log-linear in N^6, where N is the maximum of the degrees and coefficient bitsizes of the polynomials given as input. This matches the currently best-known bound for computing the topology of real plane curves and is much lower than the cost of computing the full topology of the space curve.

The main novelty of this algorithm is the avoidance of the computation of the complete topology of the curve.