Comment certifier la transcendance de fonctions D-finies? How to certify transcendence of D-finite functions?

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Deciding whether a "function", in a wide sense, satisfies an equation from a certain class is always interesting, because it gives a new way to represent and manipulate the function, as well as easy access to important information on the function. In this work, we consider the problem of deciding whether a D-finite function, that is, a function solution of a linear differential equation with polynomial coefficients, satisfies an algebraic equation.

In theory, this problem is solved, with an algorithm due to Singer deciding whether a given differential equation admits algebraic solutions. In practice however, this algorithm is impractical for all but the most trivial examples, and besides a few restricted cases with dedicated algorithms, proving transcendence of D-finite functions remains a challenge.

Instead, the question is usually answered by checking certain sufficient conditions proving algebraicity or transcendence. Some of those conditions are actually proofs, such as the existence of logarithmic or exponential singularities for the function, while others are just very convincing indications, such as the p-curvature of the equation.

In this work, we propose a new exact certificate for transcendence, using the property that non-constant algebraic functions must have poles. This allows to certify the existence of transcendent solutions for differential equations for which the existing exact tests fail. The certificate can be computed using integral bases, making it not only effective both in theory and in practice.