Fast Algorithms for Discrete Differential Equations

Hadrien Notarantonio

Discrete Differential Equations (DDEs) are functional equations that relate polynomially a power series $F(t, u)$ in t with polynomial coefficients in a "catalytic" variable u and the specializations, say at $u = 1$, of $F(t, u)$ and of some of its partial derivatives in u. DDEs occur frequently in combinatorics, especially in map enumeration. If a DDE is of fixed-point type then its solution $F(t, u)$ is unique, and a general result by Popescu (1986) implies that $F(t, u)$ is an *algebraic* power series. Constructive proofs of algebraicity for solutions of fixed-point type DDEs were proposed by Bousquet-Mélou and Jehanne (2006). Bostan et. al (2022) initiated a systematic algorithmic study of such DDEs of order 1. We generalize this study to DDEs of arbitrary order. First, we propose nontrivial extensions of algorithms based on polynomial elimination and on the guess-and-prove paradigm. Second, we design two brand-new algorithms that exploit the special structure of the underlying polynomial systems. Last, but not least, we report on implementations that are able to solve highly challenging DDEs with a combinatorial origin.

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